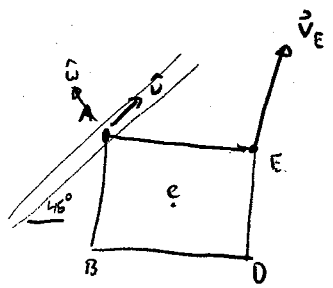
Lecture 10) Announcements

Mid-semester check-up : Post privately with any concerns about groups and/or group project

Constrained Motion



Suppose

$\vec{r}_{AE} = 4\hat{\imath} - \hat{\jmath}$

$\vec{v}_E = \hat{\imath} + 7\hat{\jmath}$

Point A constrained to slot

Two ways to quantify the constraint

① What direction can A move? $\hat{u}$ $\Rightarrow$ $\vec{v}_A = v_u \hat{u}$ one unknown

② What direction can't A move? $\hat{\omega}$ $\Rightarrow$ $\vec{v}_A = v_x \hat{\imath} + v_y \hat{\jmath}$ two unknowns (extra unknown)

$\vec{v}_A \cdot \hat{\omega} = 0$ extra eqn

① $\vec{V}_A = V_v \hat{v}$ $\hat{v} = \cos(\frac{\pi}{4})\hat{\imath} + \sin(\frac{\pi}{4})\hat{\jmath}$

$\vec{V}_A = \left(\frac{V_A}{\sqrt{2}}, \frac{V_A}{\sqrt{2}}, 0\right)$

$\vec{\omega} = (0, 0, \omega)$

$V_E = V_A + \omega \times r_{AE}$

$\hat{\imath} + 7\hat{\jmath} = \frac{V_A}{\sqrt{2}}\hat{\imath} + \frac{V_A}{\sqrt{2}}\hat{\jmath} + 4\omega\hat{\jmath} + \omega\hat{\imath}$ → $1 = \frac{V_A}{\sqrt{2}} + \omega$, $7 = \frac{V_A}{\sqrt{2}} + 4\omega$ $\Rightarrow$ $\omega = 2$, $V_A = -\sqrt{2}$

So $V_A = (-1, -1, 0)$ $\omega = 2\hat{k}$

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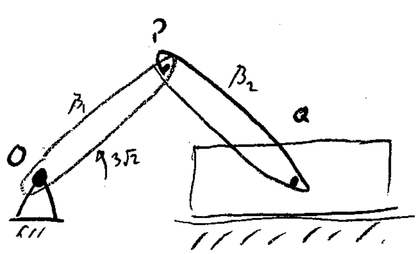
② $\vec{V}_A = V_x\hat{\imath} + V_y\hat{\jmath}$ $\hat{v} = \left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right)$

$\hat{\imath} + 7\hat{\jmath} = v_x\hat{\imath} + v_y\hat{\jmath} + 4\omega\hat{\jmath} + \omega\hat{\imath}$ 2 eqns

$-\frac{V_x}{\sqrt{2}} + \frac{V_y}{\sqrt{2}} = 0$ 1 eqn



$r_{OP} = 3\sqrt{2}$ $\vec{\omega}_1 = 4\hat{k}$

$r_{PQ} = 5$ Block slides along ground

Guess: $\omega_2 > 0$ $= 0$ < 0

$\vec{r}_{OP} = (3\sqrt{2})(\cos 45\,\hat\imath + \sin 45\,\hat\jmath) = \boxed{3\hat\imath + 3\hat\jmath}$

$|\vec{r}_{PQ}| = 5$ and $r_{PQ} \cdot \hat\jmath = -3$ so $\boxed{\vec{r}_{PQ} = 4\hat\imath - 3\hat\jmath}$

$\vec{\omega}_2 = \omega_2 k$ 1 unknown

$\vec{v}_P = v_{Px}\hat\imath + v_{Py}\hat\jmath$ 2 unknowns

$\vec{v}_Q = v_Q\hat\imath$ 1 unknown

$\vec{v}_P = \cancel{\vec{v}_O}^{\,0} + \vec{\omega}_1 \times \vec{r}_{OP}$ 2 eqns

$v_Q = \vec{v}_P + \omega_2 \times r_{PQ}$ 2 eqns

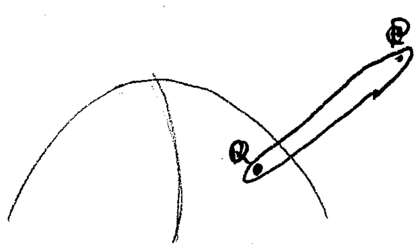
$v_P = 3k \times (3\hat\imath + 3\hat\jmath) = \boxed{-12\hat\imath + 12\hat\jmath}$

$v_Q\hat\imath = -12\hat\imath + 12\hat\jmath + 4\omega_2\hat\jmath + 3\omega_2\hat\imath$

$12 + 4\omega_2 = 0$ $\omega_2 = -3$

$v_Q = -12 + 3(-3) = \boxed{-21}$

When would we use the other method?



$\vec{r}_P = 10\hat\imath - 14\hat\jmath + 37\hat{k}$

$\vec{r}_Q = 9\hat\imath - 12\hat\jmath + 36\hat{k}$

$v_P = 3\hat\jmath - \hat{k}$ ω parallel to $2\hat\imath + 2\hat\jmath + 2\hat{k}$

Q constrained to the sphere of radius $R = 39$

What direction can Q move? $\hat{u}_1, \hat{u}_2$ hard to compute!

What direction can't Q move? $\boxed{\hat{r}_Q}$