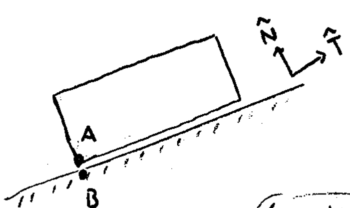
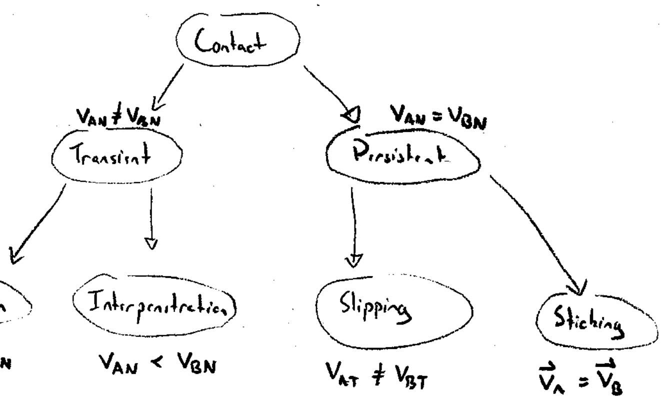
$\boxed{\text{Lecture 11}}$ Announcements

Reminder: Constraints provide

1. Elimination of certain unknowns
2. Extra equations for system

$\boxed{\text{Contact constraints}}$





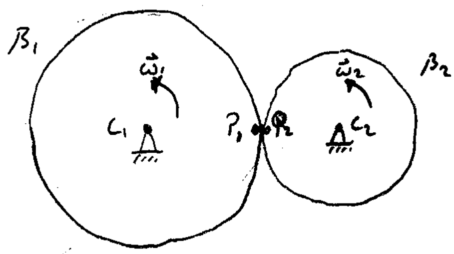
The type of contact determines what you know

A particular type of sticking - meshed gears

* Meshed gears ⇒ no slipping

* Fixed centers (for now)

Two meshed gears



P_1 is on β_1

P_2 is on β_2

Q is the position in space where contact happens

! $\vec{v}_Q = 0$ Contact location doesn't move

- ☐ $\vec{v}_{P_1} = \vec{v}_{P_2} = \vec{v}_Q$
- ☐ $\vec{v}_{P_1} = \vec{v}_Q \neq \vec{v}_{P_2}$
- ☐ $\vec{v}_{P_2} = \vec{v}_Q \neq \vec{v}_{P_1}$
- ☑ $\vec{v}_{P_1} = \vec{v}_{P_2} \neq \vec{v}_Q$
- ☐ All different

Use standard (right-handed) conventions

$$\vec{v}_{P_1} = \vec{v}_{C_1} + \vec{\omega}_1 \times \vec{r}_{C_1P_1} \qquad \vec{v}_{P_2} = \vec{v}_{C_2} + \vec{\omega}_2 \times \vec{r}_{C_2P_2}$$

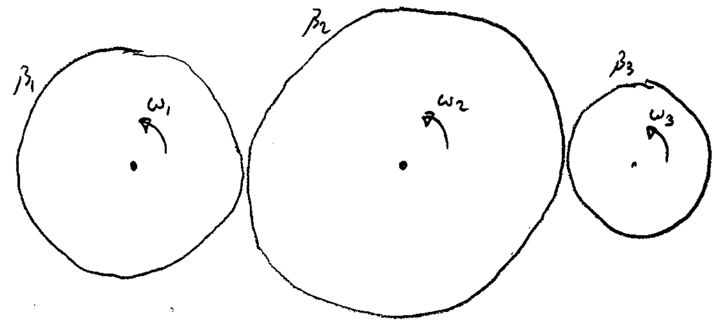
$$= 0 + \omega_1 \hat{k} \times (r_1 \hat{\imath}) \qquad = 0 + \omega_2 \hat{k} \times (-r_2 \hat{\imath})$$

$$= \omega_1 r_1 \hat{\jmath} \qquad = -\omega_2 r_2 \hat{\jmath}$$

$$\omega_1 r_1 = -\omega_2 r_2 \quad \text{or} \quad \frac{\omega_1}{\omega_2} = -\frac{r_2}{r_1}$$

Beware <u>non-standard</u> choices that make $\omega_i > 0 \quad \forall i$

Example) Three gears



$r_1 = 3$ m $\qquad \omega_1 = 1$ rad/s $\qquad$ Direction of ω_3?

$r_2 = 4$ m $\qquad \omega_3 > \omega_1 \quad < \omega_1 \quad = \omega_1 \quad ?$

$\omega_3 = ?$

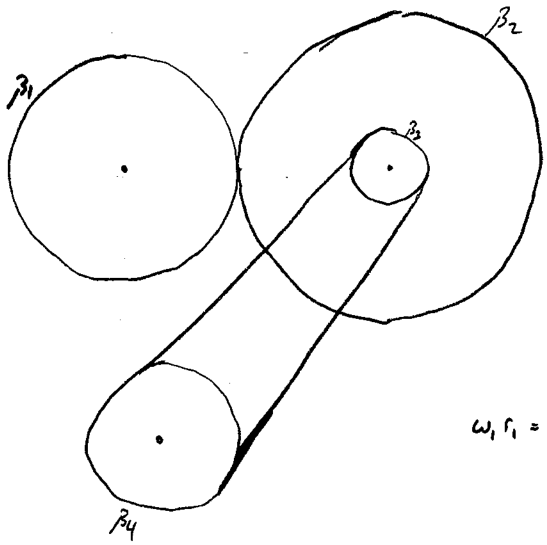
$r_3 = 2$ m

$\omega_1 r_1 = -\omega_2 r_2$ and $\omega_2 r_2 = -\omega_3 r_3$

$$\omega_1 r_1 = \omega_3 r_3$$

$$\omega_3 = \omega_1 \frac{r_1}{r_3} = (1)\frac{(3)}{(2)} = \frac{3}{2} \ \frac{\text{rad}}{\text{s}}$$

Example Many gears



Relationships

$$\omega_1 r_1 = -\omega_2 r_2$$

$$* \ \omega_2 = \omega_3$$

$$\omega_3 r_3 = \pm \omega_4 r_4$$

$$\omega_1 r_1 = -\omega_2 r_2 = -\omega_3 r_2$$

$$= -\frac{\omega_3 r_3 r_2}{r_3} = -\frac{\omega_4 r_4 r_2}{r_3}$$

So $\omega_4 = -\omega_1 \frac{r_1 r_3}{r_2 r_4}$

Work gear to gear; write <u>many</u> <u>simple</u> relationships

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<u>Accelerations</u>

Equal velocities (in tangential direction)

$\downarrow$ $\searrow$

Equal accelerations (tangential) $\boxed{Y}$ Equal normal acceleration? $\boxed{NO}$

Differentiate

$$\omega_1 r_1 = \omega_2 r_2$$

$$\Downarrow \frac{\partial}{\partial t} \qquad (\dot{r}_1 = \dot{r}_2 = 0)$$

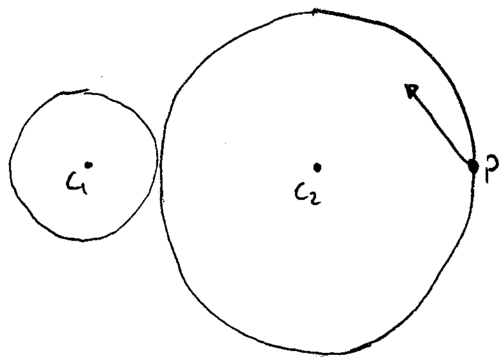
$$\alpha_1 r_1 = \alpha_2 r_2$$

Equal Not equal!

$$\vec{a}_{p_1} = \alpha_1 r_1 \hat{j} - \omega_1^2 r_1 \hat{i}$$

$$\vec{a}_{p_2} = -\alpha_2 r_2 \hat{j} + \omega_2^2 r_2 \hat{i}$$

Example



$r_1 = 2\ m$

$r_2 = 4\ m$

$\omega_1 = 2\ rad/s$

$\alpha_1 = -4\ rad/s$

What is $\vec{a}_p$?

! $r_1 \omega_1 = -r_2 \omega_2$

$\omega_2 = -\frac{r_1}{r_2}\omega_2 = -\frac{2}{4}(2) = -1\ rad/s$

$r_1 \alpha_1 = -r_2 \alpha_2$

$\alpha_2 = -\alpha_1 \frac{r_1}{r_2} = -(-4)\left(\frac{2}{4}\right) = 2\ rad/s^2$

$\vec{a}_p = \vec{a}_{c_2} + \alpha_2 \hat{k} \times (r_2 \hat{\imath}) - \omega_2^2 r_2 \hat{\imath}$

$= 0 + (\alpha_2 r_2)\hat{\jmath} - \omega_2^2 r_2 \hat{\imath}$

$= \boxed{-4\hat{\imath} + 8\hat{\jmath}\ m/s^2}$

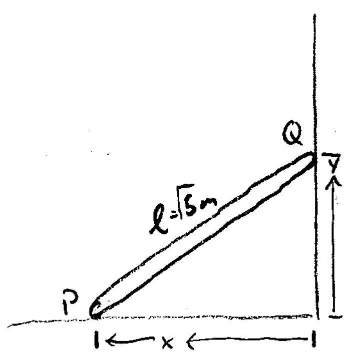
Example   Absolute motion analysis

A third way to incorporate constraints   "shape of the problem"

Good for
- Constant-length connections
- Fixed relationships (e.g. right triangles)

- Use implicit differentiation to learn about coordinates

Example   Ladder against wall



$\ell = \sqrt{5}\ m$

$\vec{r}_{PQ} = (2, 1)\ m$

$\vec{v}_P = (4, 0)$

$\vec{a}_P = (0, 0)$

$v_Q \hat{j} = v_P \hat{i} + \omega \hat{k} \times (2\hat{i} + \hat{j})$

Since $x^2 + y^2 = \ell^2 = 5$

$$2x\dot{x} + 2y\dot{y} = 0$$

$$\dot{y} = \frac{x}{y}\dot{x} = \frac{2}{1}(4) = 8\ m/s$$