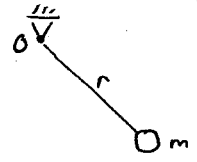
Lecture 13

Moment of Inertia Resistance to rotation

Particles

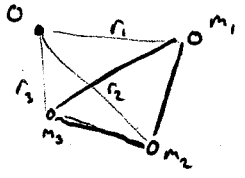


Moment for a single particle

$$I_{O,z} = m r^2$$

m: mass; r: Perpindicular distance to axis

Collections of particles



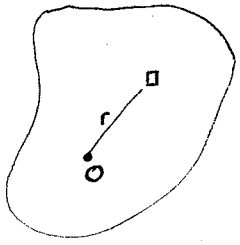
Moments <u>about the same axis</u> can be added

$$I_{O,z} = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2$$

$$= I_{1,O} + I_{2,O} + I_{3,O}$$

Rigid Bodies

Add up moments of ∂m

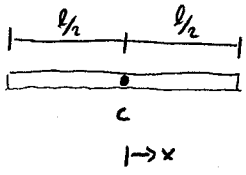


$$I_{O,z} = \iiint \rho^2 \, \partial m$$

$$= \iiint \rho \, r^2 \, \partial V$$

$$= \int\int \rho \left((x-x_0)^2 + (y-y_0)^2\right) \partial y \, \partial x$$

Exampl. Rod about center



density ρ $\left(\frac{kg}{m}\right)$ (linear density)

mass $M = \rho l$

COM at $l/2$

Establish coords. $x = 0$ at COM

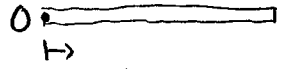
↳ Distance to center $r^2 = x^2$ for x in $\left(-\frac{l}{2}, \frac{l}{2}\right)$

$$I_{c,z} = \int_{-l/2}^{l/2} \rho x^2 \, \partial x = \frac{1}{3}\rho x^3 \Big|_{-\frac{l}{2}}^{\frac{l}{2}} = \frac{1}{3}\rho\left(\frac{l^3}{8} - \left(\frac{-l^3}{8}\right)\right) = \frac{1}{12}\rho l^3$$

$\rho l = m$

$$= \frac{1}{12} m l^2$$

Exampl. Rod about endpoint

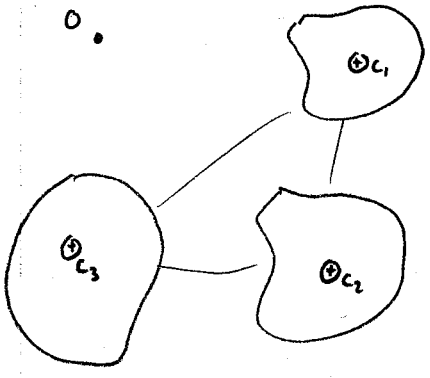


$$I_{O,z} = \int_0^l \rho x^2 \, \partial x = \frac{1}{3}\rho x^3 \Big|_0^l = \frac{1}{3}\rho l^3$$

$$= \frac{1}{3} m l^2$$

Pick $x = 0$ at O

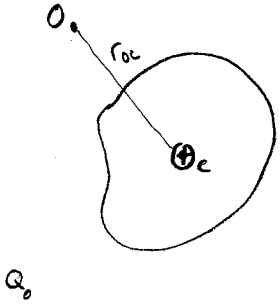
Composite Bodies



- Tables give I_c for lots of regular shapes
- NOT about same axis - can't add

$$I_O \neq I_{1,C_1} + I_{2,C_2} + I_{3,C_3}$$

Changing axis - Parallel Axis Theorem



Q_O

If $I_{C,z}$ known

$$I_{O,z} = I_{C,z} + m r_{CO}^2$$

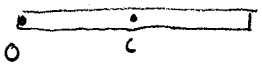
- Translate axis to O from C

ALWAYS start at C

$$I_a = I_c + m r_{ca}^2$$

$$\neq I_O + m r_{Oa}^2$$

Example: Rod about endpoint



$I_{C,z} = \frac{1}{12} m \ell^2$

- Point O is distance $\frac{\ell}{2}$ away

$$I_{O,z} = I_{C,z} + m r_{CO}^2$$

$$= \frac{1}{12} m \ell^2 + m \left(\frac{\ell}{2}\right)^2$$

$$= \frac{1}{12} m \ell^2 + \frac{1}{4} m \ell^2$$

$$= \boxed{\frac{1}{3} m \ell^2}$$

Composite bodies

1. Find moment for each body about its COM
2. Transform using parallel axis thm (common axis)
3. Add moments taken around the same axis