Lecture 14

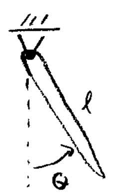
Announcements: RD Project report due Fri

Recap: Rigid body kinetics

For each rigid body: $\sum \vec{F} = m\vec{a}_c$ and $\sum \vec{M}_c = I_c \vec{\alpha}$ OR $\sum \vec{M}_o = I_o \alpha$

Recall $\vec{M}_c = \vec{r}_c \times \vec{F}$ moment about c due to $\vec{F}$

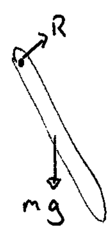
<u>Example</u> Rod pendulum



Find $\ddot{\theta}$ as function of $\theta, \dot{\theta}$

Given m, ℓ, g

$\boxed{\text{FBD}}$



Approach $\vec{a}_c$ and $\vec{\alpha}$

What basis to express $\vec{a}_c$ in? Cartesian
Polar

Forces

Reaction $\vec R = R_x\hat i + R_y\hat j = R_r\hat e_r + R_\theta\hat e_\theta$

Gravity $\vec F_G = -mg\hat j$

mg 

$$-mg\hat j = \underset{-}{\oplus}\, mg\cos\theta\,\hat e_r\ \overset{+}{\ominus}\ mg\sin\theta\,\hat e_\theta$$

Kinematics

$$\vec a_c = (\cancel{\ddot r}^{\,0} - r\dot\theta^2)\,\hat e_r + (\cancel{2\dot r\dot\theta}^{\,0} + r\ddot\theta)\,\hat e_\theta$$

$$\vec a_c = -r\dot\theta^2\,\hat e_r + r\ddot\theta\,\hat e_\theta$$

$$= -\frac{l}{2}\dot\theta^2\,\hat e_r + \frac{l}{2}\ddot\theta\,\hat e_\theta$$

$$\Sigma\vec F_a = (R + mg\cos\theta)\,\hat e_r + (R_\theta - mg\sin\theta)\,\hat e_\theta = m\left(-\frac{l}{2}\dot\theta^2\,\hat e_r + \frac{l}{2}\ddot\theta\,\hat e_\theta\right)$$

Moments

$$M_G = \cancel{r}^{\,0}\times F_G = 0$$

$$M_R = \left(-\frac{l}{2}\hat e_r\right)\times\left(R_r\hat e_r + R_\theta\hat e_\theta\right)$$

$$= -\frac{l}{2}\,\hat e_r\times\hat e_\theta$$

$$= \boxed{-\frac{l}{2}\hat k}$$

$$M_c = -\frac{l}{2} = \left(\frac{1}{12}ml^2\right)\ddot\theta$$

$$R_R + mg\cos\theta = -\frac{lm}{2}\dot\theta^2 \qquad \Sigma F\cdot\hat e_r$$

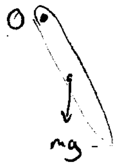
$$R_\theta - mg\sin\theta = \frac{l}{2}m\ddot\theta \qquad \Sigma F\cdot\hat e_\theta$$

$$-\frac{l}{2}R_\theta = \frac{1}{12}ml^2\ddot\theta$$

$$-mgl\sin\theta = \frac{2}{3}ml^2\ddot\theta$$

$$\boxed{\ddot\theta = -\frac{3g}{2l}\sin\theta}$$

Alternate approach: Fixed point O



$$M_R = 0$$

$$M_g = \left(\frac{l}{2}\hat{e}_r\right) \times \left(-\cancel{mg\hat{j}}\right) \quad (mg\,\hat{i})$$

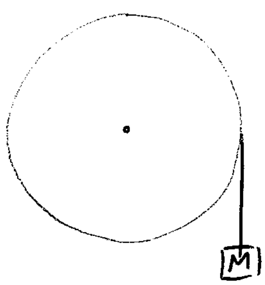
$$= \left(\frac{l}{2}\hat{e}_r\right) \times \left(mg\cos\Theta\,\hat{e}_r - mg\sin\Theta\,\hat{e}_\Theta\right)$$

$$= -mg\frac{l}{2}\sin\Theta\,\hat{k}$$

$$I_O = \frac{1}{3}ml^2$$

So $\boxed{-mg\frac{l}{2}\sin\Theta = \frac{1}{3}ml^2\ddot{\Theta}}$

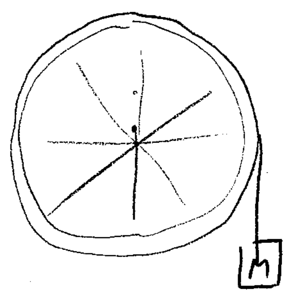
<u>More Examples</u>



Disk

mass m

radius r



Wheel

mass m

radius r

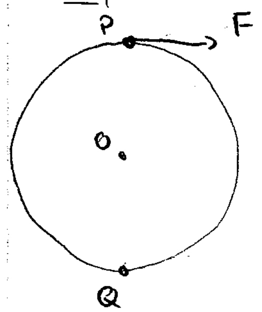
Which hits the ground first?

↳ Which accelerates faster?

$I = \frac{1}{2}mR^2$

$I = mR^2$

Example



Disk at rest

F applied

P accelerates L O R

O accelerates L O R

Q accelerates L O R

$\sum \vec{F} = m\vec{a}_c$ $\quad F = m\vec{a}_c$ $\quad$ F to right, $\vec{a}_c$ to right

$\sum M = I_c \alpha$ $\quad r\hat{j} \times F\hat{i} = -rF\hat{k}$

$$-RF = \left(\tfrac{1}{2}mR^2\right)\alpha \quad \Rightarrow \quad \alpha = \frac{-2F}{mR} < 0$$

$$\vec{a}_p = a_0 + \alpha \times r \; \cancel{-\omega^2 r} \quad 0 \text{ at rest}$$

$$= \frac{F}{m} + \left(-\frac{2F}{mR}\hat{k}\right) \times (R\hat{j})$$

$$= \frac{F}{m} + \frac{2F}{m} > 0$$

P accelerates right

$$\vec{a}_Q = \left(\frac{F}{m}\right) + \left(-\frac{2F}{mR}\hat{k}\right) \times (-R\hat{j})$$

$$= \frac{F}{m} - \frac{2F}{m} = -\frac{F}{m} < 0$$

Q accelerates left