Instantaneous Centers

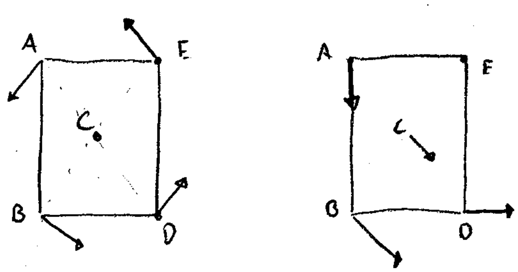
For a rigid body rotating in 2D

The instantaneous center (IC) $\boxed{M}$ is the

- Point that is "not moving"
- Point about which everything rotates

Only at this instant!

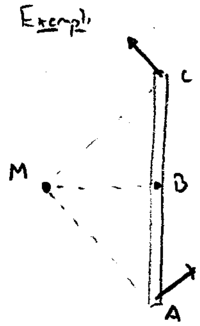
Example



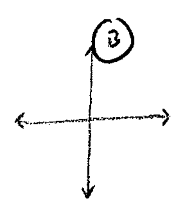
Is the instant center always inside the body? NO

Is the instant center a fixed point (i.e., can I do rotations about it?) NO

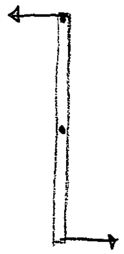
Example



Direction of $\vec{v}_B$

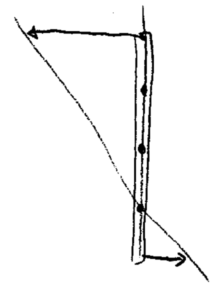


$|\vec{v}_B|$ vs $|v_C|$ Bigger / Same / Smaller



$\vec{V}_B = ?$

$\vec{\omega} =$ ↶ or ↷

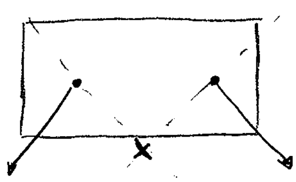


Where is M?

Similar triangles

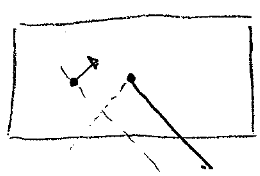
Intuition for M

- $\vec{V}_P$ perpendicular to $\vec{r}_{MP}$ for all points
- $V_P = \omega r_{MP}$ for all points
  - * Direction of rotation is consistent

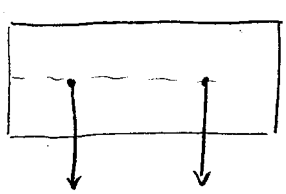


Can this happen for a rigid body?

Directions inconsistent



Magnitudes inconsistent



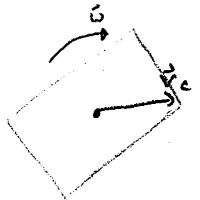
Translation

Calculating the position of M

- Use $\vec{v}_M = 0$ and RB eqn to calculate

$$\vec{v}_P = \cancel{\vec{v}_M} + \vec{\omega} \times \vec{r}_{MP} \quad \text{for all points}$$

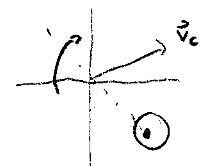
Example



$\vec{v}_c = 4\hat{i} + 2\hat{j}$ m/s

$\vec{\omega} = -2\hat{k}$ rad/s

Which quadrant?



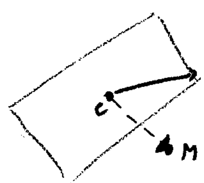
$$\vec{v}_M = \vec{v}_c + \vec{\omega} \times \vec{r}_{cM}$$

$$0 = 4\hat{i} + 2\hat{j} + (-2\hat{k}) \times (x\hat{i} + y\hat{j})$$

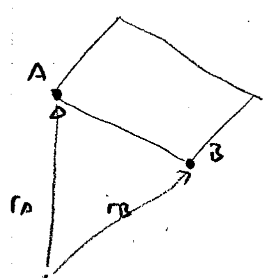
$$= 4\hat{i} + 2\hat{j} - 2x\hat{j} + 2y\hat{i}$$

$0 = 4 + 2y \qquad \boxed{y = -2}$

$0 = 2 - 2x \qquad \boxed{x = 1}$



Example



$\vec{r}_A = \hat{i} + 4\hat{j} \qquad \vec{v}_A = -3\hat{i} - 9\hat{j}$

$\vec{r}_B = 3\hat{i} + 3\hat{j} \qquad \vec{v}_B = -3\hat{j}$

Where is M?

Approach 1 : Use $\vec{v}_A$, $\vec{v}_B$, $\vec{r}_{AB}$ to find $\vec{\omega}$

Use either point to find $\vec{r}_M$

$\vec{v}_B = \vec{v}_A + \vec{\omega} \times r_{AB}$ $\qquad$ $\vec{r}_{AB} = \vec{r}_B - \vec{r}_A = 2\hat{i} - \hat{j}$

$$-3\hat{j} = -3\hat{i} - 9\hat{j} + (\omega\hat{k}) \times (2\hat{i} - \hat{j})$$

$$= -3\hat{i} - 9\hat{j} + 2\omega\hat{j} + \omega\hat{i}$$

$$0 = -3 + \omega \qquad -3 = -9 + 2\omega \quad] \quad \omega = +3$$

$v_M = v_A + \omega \times r_{AM}$ $\qquad$ $r_{AM} = r_M - r_A = (x-1)\hat{i} + (y-4)\hat{j}$

$$0 = -3\hat{i} - 9\hat{j} + (3\hat{k}) \times ((x-1)\hat{i} + (y-4)\hat{j})$$

$$3(x-1)\hat{j} - 3(y-4)\hat{i}$$

$$0 = -3 - 3(y-4) \qquad 3 = -3y + 12 \qquad 3y = 9 \qquad \boxed{y = 3}$$

$$0 = -9 + 3(x-1) \qquad 9 = 3x - 3 \qquad 12 = 3x \qquad \boxed{x = 4}$$

Approach 2 $\qquad r_{AM} = r_M - r_A = (x-1)\hat{i} + (y-4)\hat{j}$

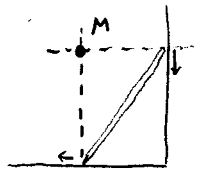
$\qquad\qquad r_{BM} = r_M - r_B = (x-3)\hat{i} + (y-3)\hat{j}$

$v_M = v_A + \omega \times r_{AM}$ $\qquad$ $0 = -3\hat{i} - 9\hat{j} + (\omega\hat{k}) \times ((x-1)\hat{i} + (y-4)\hat{j})$

$v_M = v_B + \omega \times r_{BM}$ $\qquad$ $0 = -3\hat{j} + (\omega\hat{k}) \times ((x-3)\hat{i} + (y-3)\hat{j})$

How many unknowns? How many eqns?

Example: Ladder sliding on wall/floor



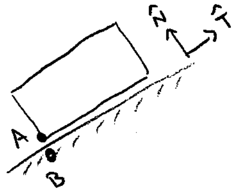
How does M move?

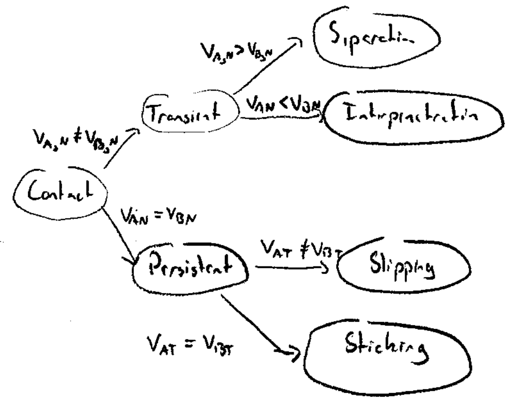
Down and to right

* Position of instantaneous center is not always fixed in time

" $\vec{v}_M = 0$ but position of M changes in time "

Rigid bodies in contact

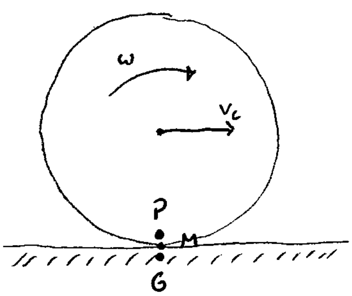




$\vec{V}_A = \vec{V}_B$

Rolling motion

Under normal driving conditions, tire/road contact is stick slip



P point on wheel touching the ground

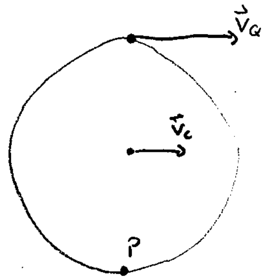
G point on ground touching wheel

M location in space of contact

V_G > 0, $\boxed{= 0}$, < 0

V_P > 0, $= 0$, < 0

$\frac{\partial}{\partial t} \vec{r}_M = \vec{v}_M$ $= 0$, $\boxed{\neq 0}$



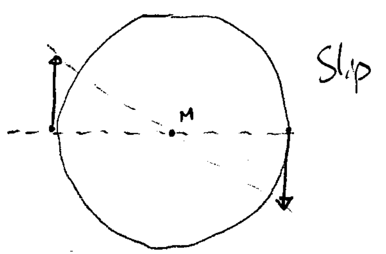
$\vec{v}_p = 0$ (sticking)

See Friday

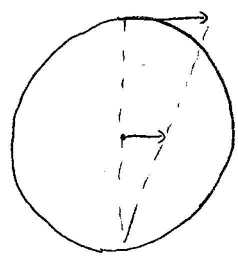
$\vec{v}_c = \vec{v}_p + \omega \hat{k} \times (r \hat{j})$

$\boxed{\vec{v}_c = -\omega r \hat{i}}$ $\vec{v}_c \rightarrow$ iff $\vec{\omega} \curvearrowright$

$\boxed{\text{Ex}}$ Slip or stick? Slip or stick?

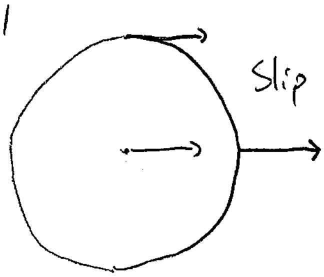


Slip

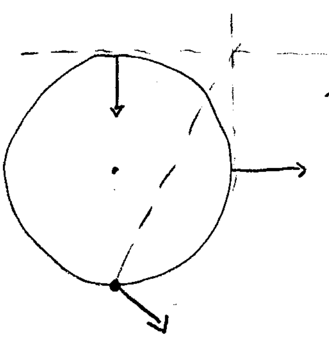


Stick

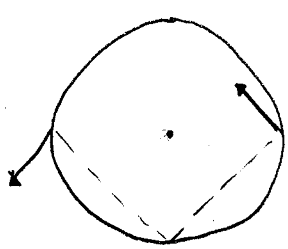
$\boxed{\text{Ex}}$



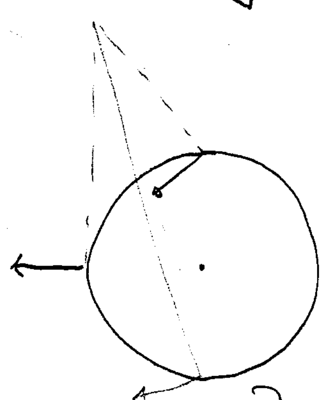
Slip



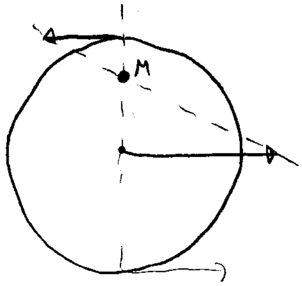
Interpenetration



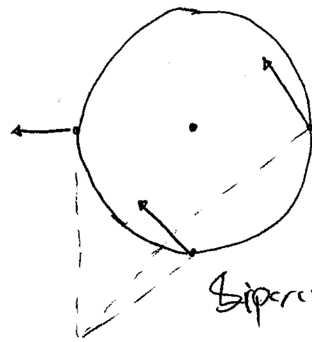
Stick



Penetration



Slip



Separation

|Recap|

Rolling without slipping : $\vec{V}_c = -\omega r \hat{\imath}$

$$\Big\downarrow \frac{\partial}{\partial t}$$

$\vec{a}_c = -\alpha r \hat{\imath}$

Sci Friday

$\vec{V}_p = 0$ $\quad a_p = \,?\, = \vec{a}_c + \alpha \hat{k} \times (-r\hat{\jmath}) + \omega^2 r \hat{\jmath}$

$= \cancel{-\alpha r \hat{\imath}} + \cancel{\alpha r \hat{\imath}} + \omega^2 r \hat{\jmath}$

$$\boxed{a_p = \omega^2 r \hat{\jmath}}$$