$\boxed{\text{Lecture 16}}$

Launcher competition details

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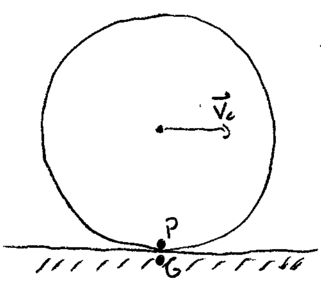
$\boxed{\text{Recall}}$ Rolling motion (no slipping)




$\vec{V}_G = 0$

$\vec{V}_P = \vec{V}_G$ (no slip) $= 0$

$\vec{a}_P = 0$ ? NO

Velocity of center of mass: $\vec{V}_c = \vec{V}_p + \vec{\omega} \times \vec{r}_{pc}$

$$= 0 + (\omega \hat{k}) \times (r \hat{j})$$

$$= \boxed{-\omega r \hat{i}}$$ When no-slip

Acceleration: $\frac{\partial}{\partial t} \vec{V}_c = \vec{a}_c = -\dot{\omega} r \hat{i} - \omega \cancelto{0}{\dot{r}} \hat{i} - \omega r \cancelto{0}{\dot{\hat{i}}}$

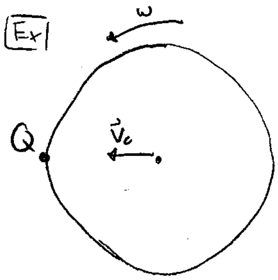
$$= \boxed{-\alpha r \hat{i}}$$

$$\vec{a}_p = \vec{a}_c + \vec{\alpha} \times \vec{r}_{cp} - \omega^2 \vec{r}_{cp}$$

$$= -\alpha r \hat{i} + (\alpha \hat{k}) \times (-r \hat{j}) - \omega^2 (-r \hat{j})$$

$$= \boxed{\omega^2 r \hat{j}}$$

Ex




Car driving left at $v = 4\,m/s$

slowing down at $2\,m/s^2$

$r = 0.5\,m$

What is $\vec{v}_Q$?

$\vec{v}_c = -4\hat{\imath}$ $\quad \vec{v}_Q = \vec{v}_c + \omega k \times (-r\hat{\imath}) = -4\hat{\imath} - (0.5)(8)\hat{\jmath} = -4\hat{\imath} - 4\hat{\jmath}$

$\vec{v}_p = 0 = \vec{v}_c + \omega\hat{k} \times (-r\hat{\jmath}) = -4\hat{\imath} + \omega(0.5)\hat{\imath} \Rightarrow \omega = 8\ ^{rad}/_{s}$

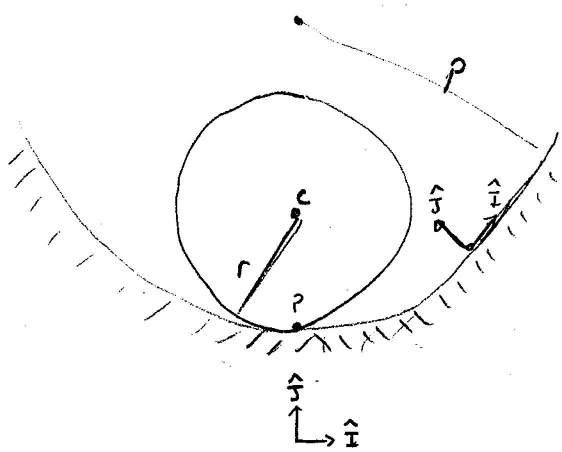
$a_Q = ?$ $\quad \vec{a}_c = 2\hat{\imath} = -\alpha r\hat{\imath} = -\alpha(0.5) \Rightarrow \alpha = -4\ ^{rad}/_{s^2}$

$\vec{a}_Q = 2\hat{\imath} + (-4\hat{k}) \times (-r\hat{\imath}) + \omega^2 r\hat{\imath}$

$2\hat{\imath} + 4(0.5)\hat{\jmath} + (8)^2(0.5)\hat{\imath}$

$= 34\hat{\imath} + 2\hat{\jmath}$

Rolling on curved surfaces




$\boxed{R := \rho - r}$

$$\vec{V}_c = \cancel{\vec{V}_p}^{\,0} + \vec{\omega} \times \vec{r}_{pc} = 0 + \omega\hat{k} \times r\hat{J} = \boxed{-\omega r \hat{I}}$$

$$\vec{a}_c = \dot{\vec{V}}_c = -\dot{\omega} r \hat{I} - \omega \cancel{\dot{r}}^{\,0} \hat{I} - \omega r \dot{\hat{I}}$$

What is $\dot{\hat{I}}$

$$= -\alpha r \hat{I}$$

For point C

At C $\quad \hat{I} = \hat{e}_t$

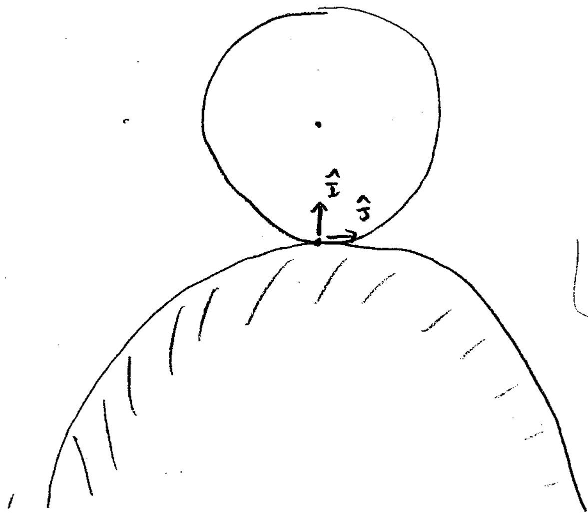
$$\dot{\hat{I}} = \dot{\hat{e}}_t = v_c K \hat{e}_n = v_c K \hat{J}$$

$$= \frac{v_c}{R} \hat{J}$$

$$\vec{a}_c = -\alpha r \hat{I} - \omega r \left(\frac{v_c}{R} \hat{J}\right)$$

$\boxed{-\omega r = v_c}$

$$\boxed{= -\alpha r \hat{I} + \frac{v_c^2}{R} \hat{J}}$$




Same derivation ; $R := r + \rho$

$$\vec{v}_c = -\omega r \ \hat{I}$$

$$\boxed{\vec{a}_c = -\alpha r \hat{I} - \frac{v_c^2}{R} \hat{J}}$$