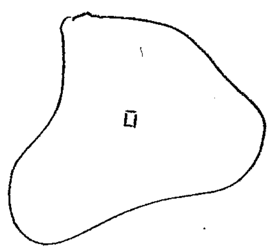
Lecture 17

Energy and Work

Another way of discribing state of system



$E = T + V$

T → kinetic

V → potential

Kinetic energy

Point mass : $\boxed{T = \frac{1}{2} m v^2}$

General body : $\iiint \frac{1}{2} \rho v^2 \, \partial V$

For Rigid Bodies $T = \frac{1}{2} m v_c^2 + \frac{1}{2} I_c \omega^2$ (because math)

- Center of mass always works

OR $T = \frac{1}{2} I_O \omega^2$ if O is fixed

OR $T = \frac{1}{2} I_M \omega^2$ if M is instant center

Potential energy $V = m g h_c$ always height of COM

Work - Energy Principle

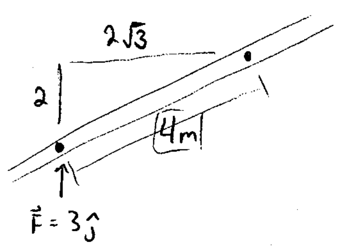
Work done on the system changes total energy

$$W = \Delta E = E_f - E_i$$

Work done by a force $\vec{F}$

$$W = \underbrace{\int_{r_0}^{r_1} \vec{F} \cdot \partial \vec{r}}_{\text{Incremental work}} = \underbrace{\int_{t_0}^{t_1} \vec{F} \cdot \vec{v} \, \partial t}_{\text{Power}}$$

Example Constant force



What is the work done by $\vec{F}$?

$$W = \int_{r_0}^{r_1} \vec{F} \cdot \partial \vec{r}$$

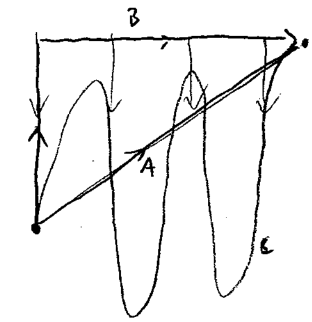
$$= \vec{F} \cdot \int_{r_0}^{r_1} \partial \vec{r}$$

$$= \vec{F} \cdot (\vec{r}_f - \vec{r}_0) = \vec{F} \cdot \Delta \vec{r}$$

$$W = (3\hat{j}) \cdot (2\sqrt{3}\hat{i} + 2\hat{j})$$

$$= \boxed{6 \text{ J}}$$

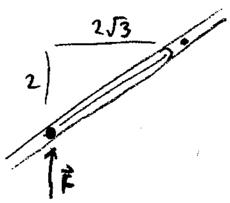
Example Riding a bike in the wind



Work done by wind on each path

$A \stackrel{?}{=} B \stackrel{?}{=} C$

Example - Bead in slot



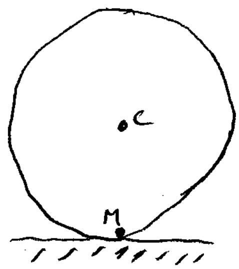
Constraint force R

How much work?

$$\int R \cdot d\vec{r} \qquad \vec{R} \perp d\vec{r} \text{ always!}$$

$\boxed{\text{Constraint forces do not do work}}$

Energy of disk



Remember $T = \frac{1}{2} m v_c^2 + \frac{1}{2} I_c \omega^2$

$V = m g h_c$

Rolling w/out slip

$$T = \frac{1}{2} m v_c^2 + \frac{1}{2}\left(\frac{1}{2} m r^2\right)\omega^2 \qquad / \; |v_c| = |\omega r|$$

$$= \frac{1}{2} m (\omega r)^2 + \frac{1}{2}\left(\frac{1}{2} m r^2\right)\omega^2$$

$$= \boxed{\frac{3}{4} m r^2 \omega^2}$$

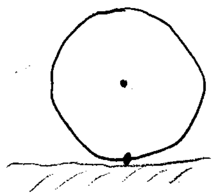
OR

Instant Cntr at ground

$$I_M = I_c + m r^2 = \frac{1}{2} m r^2 + m r^2 = \frac{3}{2} m r^2$$

$$T = \frac{1}{2} I_M \omega^2 = \frac{1}{2}\left(\frac{3}{2} m r^2\right)\omega^2 = \boxed{\frac{3}{4} m r^2 \omega^2}$$

Work done by friction



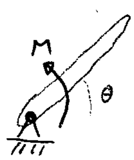
Where does F act?

Which way does F act?

How much work is it doing?

Is W_F > 0 if not slipping
$= 0$
< 0
depends

Work done by a moment



$$W = \underbrace{\int_{\Theta_0}^{\Theta_1} M \, \partial\Theta}_{\text{Inc. Work}} = \int_{t_0}^{t_1} \underbrace{M \dot{\Theta}}_{\text{Power}} \, \partial t$$

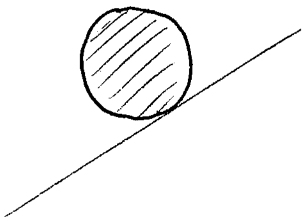
<u>Example</u>

Starts at $\Theta = \frac{\pi}{4}$ falls to horizontal

Friction moment 2 Nm

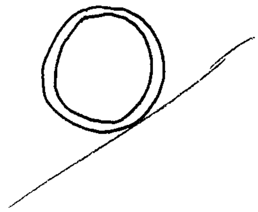
$$W = (2 \text{ Nm})\left(-\frac{\pi}{4}\right) \ominus \boxed{-2 < W < 0}$$

Example



Solid cylinder

$I_c = \frac{1}{2} m r^2$

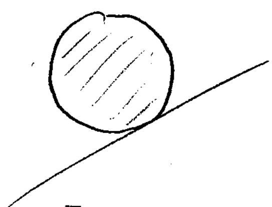


Hoop

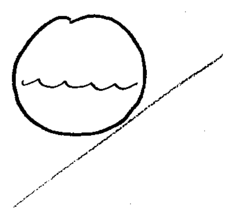
$I_c = m r^2$

If both have the same mass, which accelerates faster?

Example Can of coke



Frozen



Liquid

Which accelerates faster?