Lecture 18

Friction (Coulomb or dry friction)

Two cases (an assumption and a check)

① Stick

Assumption: No relative motion $v_{px} = a_{px} = 0$

Requirement/Check: $|F| \leq \mu |N|$

② Slip

Assumption: $|F| = \mu |N|$

Requirement/Check: $\vec{F}$ opposes $\vec{v}$ (or $\vec{a}$ if $\vec{v} = 0$)

1.5 Critical transition

$v_x = a_x = 0$ and $|F| = \mu |N|$

Approach

Known case (stick/slip)

① Apply corresponding assumptions
② Compute unknown quantities

Unknown case

① Select a case
- Stick
- Slip left
- Slip right

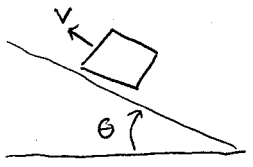
② Make appropriate assumption

③ Solve for motion

④ CHECK ASSUMPTION

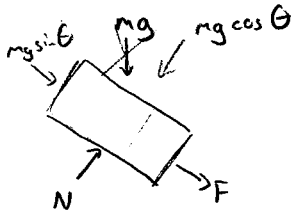
Example

Block sliding uphill, $v = 9$ m/s $\theta = 30°$ $\mu = 0.25$ $m = 8$ kg



Case: KNOWN - slip

FBD



$ma_x = +mg\sin\theta + F$

~~$ma_y = N - mg\cos\theta$~~ θ

$F = \mu N$

3 eqns
3 unknowns

$N = mg\cos\theta = (8)(10)\left(\frac{\sqrt{3}}{2}\right)$

$F = \frac{1}{4}(40\sqrt{3}) = 10\sqrt{3}$

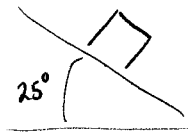
$ma_x = (8)(10)\left(\frac{1}{2}\right) + 10\sqrt{3} = \boxed{40 + 10\sqrt{3}}$

$m = 8\ kg$ $v = 5\ m/s$ $\theta = 25°$ $\mu = 0.25$

What direction is F?

What direction is a_x?

Block at rest $m = 9\ kg$ $\theta = 25$

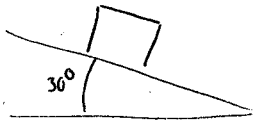


What is the minimum μ s.t. this sticks?

Transition

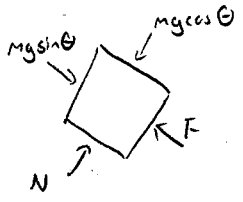
$F = \mu N$ and $a_x = v_x = 0$

Block at rest $m = 3$ $\theta = 30°$ $\mu = 0.5$



Slip or stick?

(I) Stick



Assum: $a_x = v_x = 0$

$N = mg\cos\theta$ and $F = mg\sin\theta$

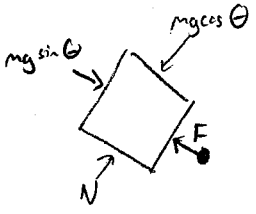
$= 3(10)\left(\frac{\sqrt{3}}{2}\right)$ $\quad F = 3(10)\left(\frac{1}{2}\right)$

$= 15\sqrt{3}$ $\quad = 15$

$F = 15 \not{P}\,(0.5)\,15\sqrt{3} = \mu N$

Assumption is <u>wrong</u>

(II) Slip (downhill)

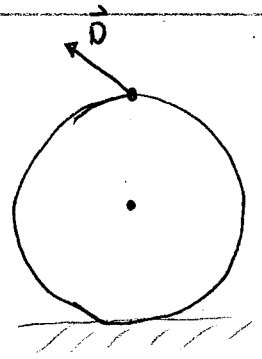


$mg\cos\theta = N = 15\sqrt{3}$ $\quad F = \mu N = \frac{15\sqrt{3}}{2}$

$mg\sin\theta - F = ma_x$

$15 - \frac{15\sqrt{3}}{2} > 0$ $\quad a_x > 0$

F oppose a_x ✓



$\vec{D} = -44\hat{\imath} + 34\hat{\jmath}$

$m = 7\ kg \quad r = 4\ m$

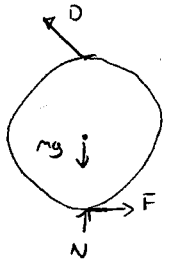
$\mu = 0.25$

Find α

① Does it lift off? $\quad \Sigma F_{\hat{\jmath}} : 34\hat{\jmath} - 7(10) < 0$ ✓

So persistent contact

Assume Stick



Assume Stick

$\vec{a}_c = -\alpha r_c$

$\Sigma F: \quad N + D_y - mg = 0$

$D_x + F = ma_c$

$D_x r + Fr = \left(\frac{1}{2}mr^2\right)\alpha$

Unknowns: N, F, a_x, ~~α~~

$a_c = -\alpha r$

$N = 34.6\hat{\jmath}$

$F = -14.7\hat{\imath}$

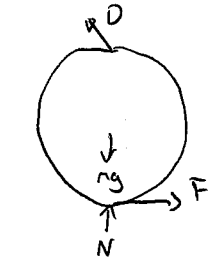
$a_c = -8.38\hat{\imath}$

$\alpha = 2.1\ \hat{k}$

$\mu N = (.25)(34.6) < 14.7$

Does this happen? NO

Assume Slip lift



$F = \mu N$

$N + D_y - mg = 0$

$D_x + F = ma_c$

$D_x r + Fr = \left(\frac{1}{2}mr^2\right)\alpha$

Unknowns: N, F, a_x, α

$F = 8.65\hat{\imath}$

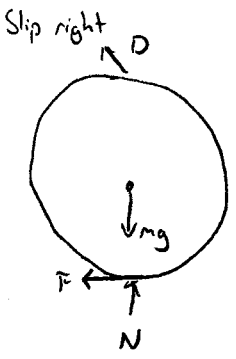
$N = 34.6\hat{\jmath}$

$a_c = -5.05\hat{\imath}$

$\alpha = 3.76$

$a_Q = 9.91\hat{\imath}$

Does not oppose F



$F = \mu N$

$-mg + N + D_y = 0$

$D_x \ominus F = m a_c$

$D_x r \ominus F r = \left(\frac{1}{2} m r^2\right) \alpha$

$F = -8.65\,\hat{i}$

$N = 34.6\,\hat{j}$

$\vec{a}_c = -7.52\,\hat{i}$

$\alpha = 2.52\,k$

$\downarrow\downarrow$

$a_a = 2.68\,\hat{i}$ opposes $\vec{F}$