TAM 212 – Dynamics

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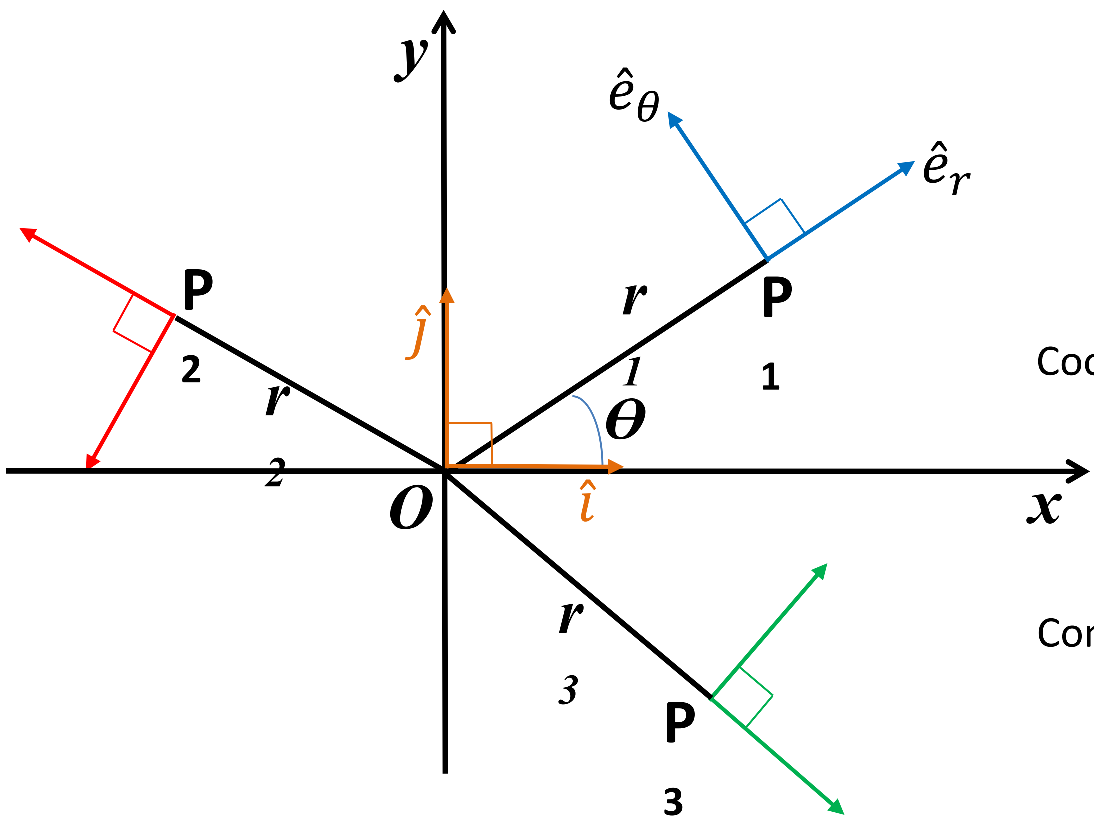
Lecture Objectives

- Calculus and vectors
  - Cartesian basis
  - Non-Cartesian basis

- Position, velocity and acceleration

Recap

- Polar coordinates and polar basis
- Polar coordinates and change of basis



Transformations

$$\hat{e}_r = \cos\theta\,\hat{\imath} + \sin\theta\,\hat{\jmath}$$
$$\hat{e}_\theta = -\sin\theta\,\hat{\imath} + \cos\theta\,\hat{\jmath}$$

$$\hat{\imath} = \cos\theta\,\hat{e}_r - \sin\theta\,\hat{e}_\theta$$
$$\hat{\jmath} = \sin\theta\,\hat{e}_r + \cos\theta\,\hat{e}_\theta$$

Coordinates of P_1 are (x,y) or (r,θ)

Components of $\overrightarrow{r_{OP_1}}$:

$$\overrightarrow{r_{OP_1}} = \overrightarrow{r_{P_1}} = x\hat{\imath} + y\hat{\jmath} = r_1\hat{e}_r$$

Calculus & vectors

Example

$\vec{v}(t) = 2t^2\hat{\imath} + 3t\hat{\jmath} \quad \mathrm{m/s} \qquad t \text{ in seconds}$

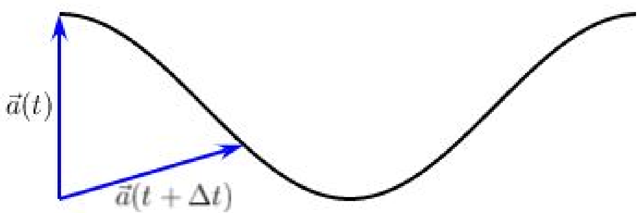
$\dot{\vec{v}}(t) =$

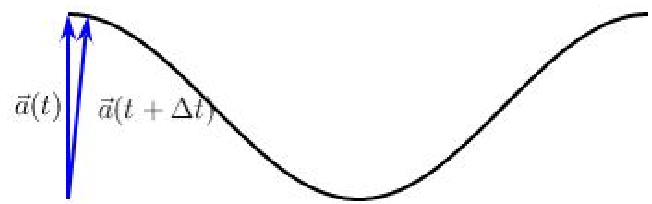
Calculus & vectors

Reference http://dynref.engr.illinois.edu/rvc.html

Vector derivative definition. #rvc-ed

$$\dot{\vec{a}}(t) = \frac{d}{dt}\vec{a}(t) = \lim_{\Delta t \to 0} \frac{\vec{a}(t + \Delta t) - \vec{a}(t)}{\Delta t}$$





$\hat{\imath}$, $\hat{\jmath}$ units are all constant (so zero derivative)

$\hat{e}_r$, $\hat{e}_\theta$ may not be constant

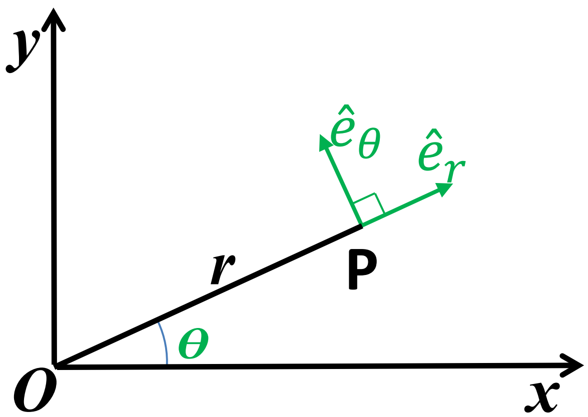
Cartesian basis: differentiate components separately

$$\vec{r} = x\hat{\imath} + y\hat{\jmath} \qquad \vec{r} = (x, y)$$

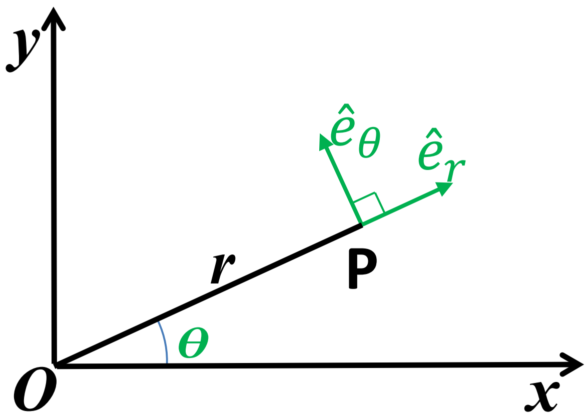
$$\dot{\vec{r}} = \dot{x}\hat{\imath} + \dot{y}\hat{\jmath} \qquad \dot{\vec{r}} = (\dot{x}, \dot{y})$$

Assume things are variable unless we know we know they're constant

Non-Cartesian derivatives

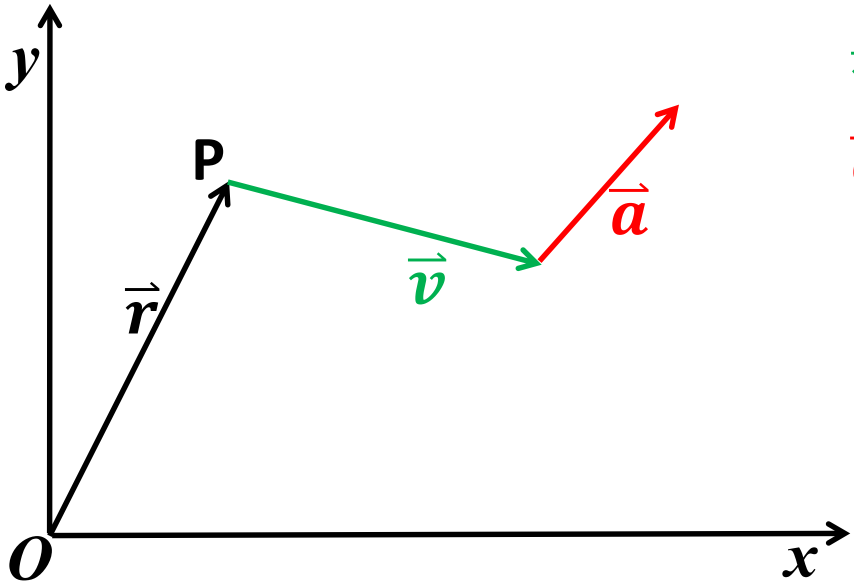


If only r changes, then



If only θ changes, then

Position, velocity, acceleration

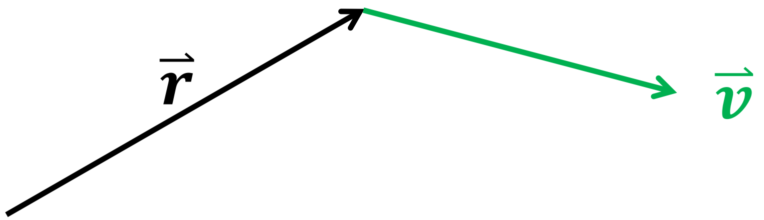


$\vec{r}$ = position

$\vec{v}$ = velocity

$\vec{a}$ = acceleration

Change in length and direction

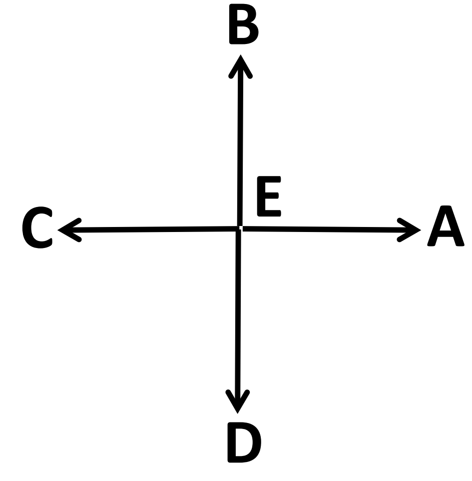


Graphical estimation of derivatives

Bounce

Stretch

Which is the direction of the derivative when it is closet to $\vec{a}(0)$



Graphical estimation of derivatives

Circle

Slider

Vertical

Derivatives of unit vectors

Unit vector $\hat{a}$ $a = \|\hat{a}\| = 1$ $a^2 = \hat{a} \cdot \hat{a} = 1$

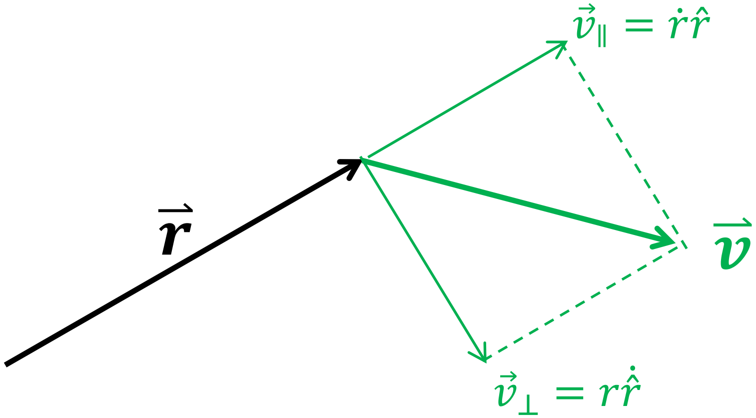
What is $\dot{a}$?
What is $\dot{\hat{a}}$?

Derivative of a general vector

$\vec{r} = r\hat{r}$

Graphical representation

$$\dot{\vec{r}} = \dot{r}\hat{r} + r\dot{\hat{r}}$$



$$\vec{v}_{\parallel} = \dot{r}\hat{r}$$

$$\vec{v}_{\perp} = r\dot{\hat{r}}$$

Example

$$\vec{r} = 6\hat{\imath} + 4\hat{\jmath}$$
$$\vec{v} = -8\hat{\imath} + 2\hat{\jmath}$$

Example

$\vec{r} = (3, -2, -4)$ m position vector of point P

$\vec{v} = (-2, -1, 1)$ m/s

$$\vec{r} \cdot \vec{v} = -8$$

$$\vec{r} \times \vec{v} = (-6, 5, -7)$$

Example

$$\dot{\boldsymbol{a}} = \left\| \dot{\vec{\boldsymbol{a}}} \right\| \;\; \textbf{?}$$

Example

If $\vec{a}$ has a fixed direction, then

$$\dot{a} = \left\|\dot{\vec{a}}\right\| \ ?$$

Example

If $\vec{a}$ has a fixed direction and is growing length, then

$$\dot{a} = \left\| \dot{\vec{a}} \right\| \text{ ?}$$

Chain rule: example (scalar)

Some instant in time

$\theta = \pi$ rad

$\dot{\theta} = 2$ rad/s

$\ddot{\theta} = -1$ rad/s^2

$r = \sin\theta$ m (always)

Example

Some instant in time

$\theta = \frac{\pi}{2}$ rad

$\dot{\theta} = -1$ rad/s

$r = \sin\theta$ m (always)