TAM 212 – Dynamics

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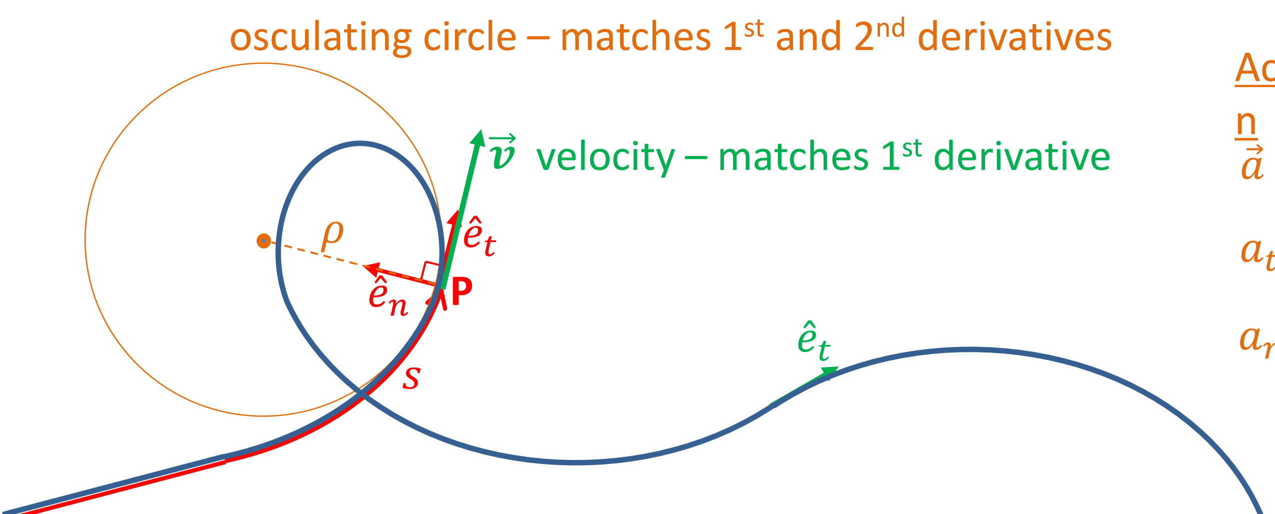
Recap

- Tangential-normal acceleration

Today

- Tangential-normal basis examples
- Tangential-normal basis in 3D



$\rho = \dfrac{1}{\kappa}$ $\begin{cases} \rho = \text{ radius of curvature} \\ \kappa = \text{ curvature} \end{cases}$

$s =$ arc length

$\dot{s} = v =$ speed

$\ddot{s} =$ rate of change of speed

<u>Velocity</u>

$$\vec{v} = \dot{s}\hat{e}_t = v_t\hat{e}_t + v_n\hat{e}_n$$

$$v_t = \dot{s} \qquad v_n = 0$$

<u>Acceleration</u>

$$\vec{a} = \ddot{s}\hat{e}_t + \frac{\dot{s}^2}{\rho}\hat{e}_n = a_t\hat{e}_t + a_n\hat{e}_n$$

$$a_t = \ddot{s}$$

$$a_n = \frac{\dot{s}^2}{\rho} = \text{ centripetal acceleration}$$

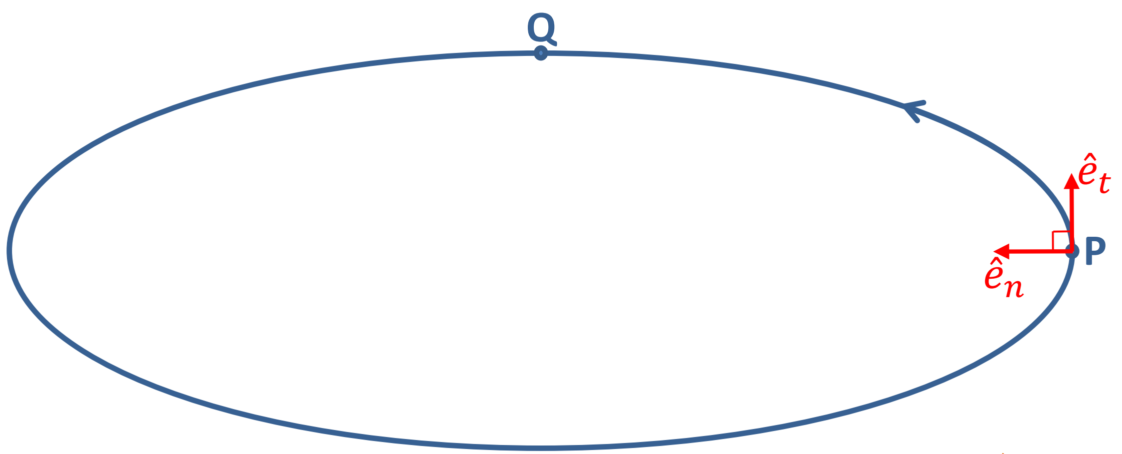
Dashboard of a car

Which meters will indicate acceleration?

Example

Oval racetrack

Car travels CCW at constant speed

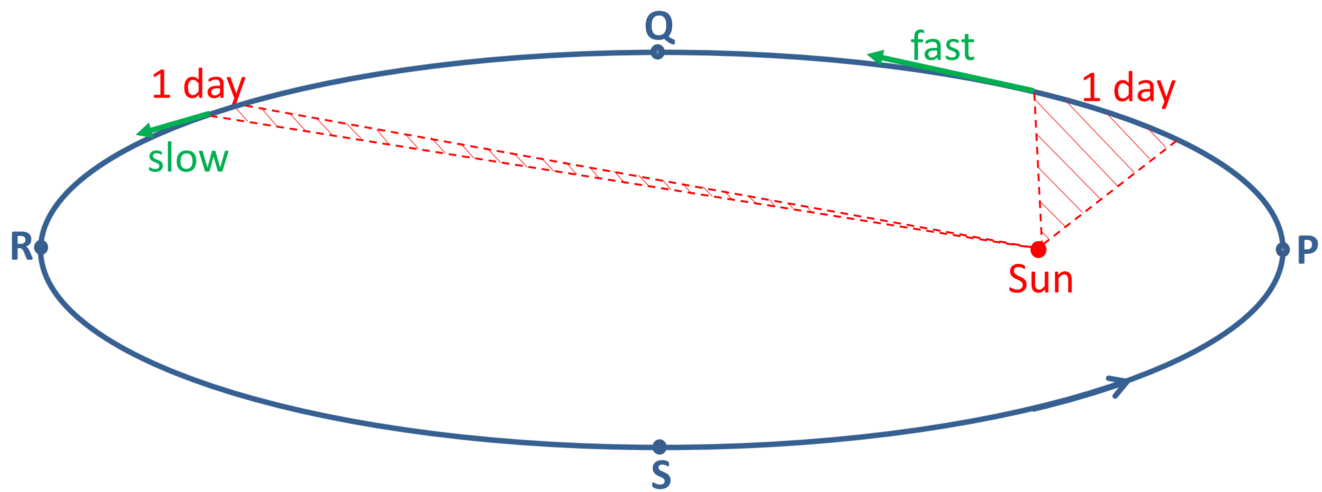


$$\vec{a} = \ddot{s}\hat{e}_t + \frac{\dot{s}^2}{\rho}\hat{e}_n$$

Example

Comet travels CCW around an elliptical orbit.

Kepler's second law: A line joining the sun and comet sweeps equal areas in equal time.



$$\vec{v} = \dot{s}\hat{e}_t$$

$$\vec{a} = \ddot{s}\hat{e}_t + \frac{\dot{s}^2}{\rho}\hat{e}_n$$

$$= a_t\hat{e}_t + a_n\hat{e}_n$$

Tangential-Normal basis in 3D

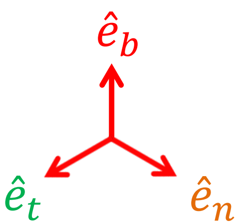
$\hat{e}_t = \hat{v}$ tangential

$=$

$$\hat{e}_n = \frac{\dot{\hat{e}}_t}{\|\dot{\hat{e}}_t\|} = \text{normal}$$

$$\hat{e}_b = \hat{e}_t \times \hat{e}_n = \text{binormal}$$

Can also find $\hat{e}_n$ from $\vec{a}$



Example

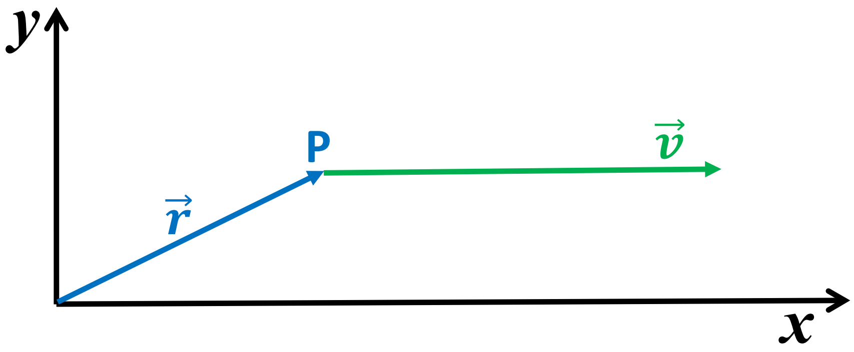
Car is speeding up and has:

$$\vec{r} = 4\hat{\imath} + 2\hat{\jmath} \ \text{ m}$$

$$\vec{v} = 6\hat{\imath} \ \text{ m/s}$$

$$a = 5 \ \text{ m/s}^2$$

$$\rho = 12 \ \text{ m}$$



$\dot{v} =$ A. -4 m/s^2 B. -2 m/s^2 C. 0 m/s^2 D. 2 m/s^2 E. 4 m/s^2

$$v = \dot{s}$$

$$\dot{v} = \ddot{s}$$

$$\vec{a} = \ddot{s}\hat{e}_t + \frac{\dot{s}^2}{\rho}\hat{e}_n$$

Particle Kinetics

Kinematics ⟶ $\vec{r}, \vec{v}, \vec{a}$ (no forces)

Kinetics ⟶ $\vec{F} = m\vec{a}$, forces, moments

Statics

$$\sum \vec{F} = 0$$

Kinematics

$$\vec{a}$$

↘ ↙

Kinetics

$$\sum \vec{F} = m\vec{a}$$

Classical Mechanics: "All models are wrong. Some models are useful." - George Box

Classical Mechanics: $\sum \vec{F} = m\vec{a}$

Method of assumed forces: $\vec{F} \rightarrow \vec{a}$ (simulation)

Method of assumed motion: $\vec{a} \rightarrow \vec{F}$ (measurement)

Example: Assumed Forces: Cannon