TAM 212 – Dynamics

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Recap

- Tangential-normal to Cartesian mapping.
- Particle Kinetics, numerical Integration.

Today

- Rigid Body Motion

Solving dynamics problems:

1. System diagram ← coordinates, measurements
2. Free body diagrams (FBD) ← real forces only (no centripetal “force”)
3. Kinematics ← acceleration in appropriate coordinates
4. Kinetics ← $\Sigma\vec{F} = m\vec{a}$ or in each component
5. Algebra

Tangential-Normal Basis Formulation

$$\vec{r} = 2t^2\hat{\imath} + (-3t - 3)\hat{\jmath}\ \mathrm{m}$$

What is the tangential basis vector $\hat{e}_t$ at $t = 2\ \mathrm{s}$?

What is the normal basis vector $\hat{e}_n$ at $t = 2\ s$?

Tangential-Normal Basis Formulation

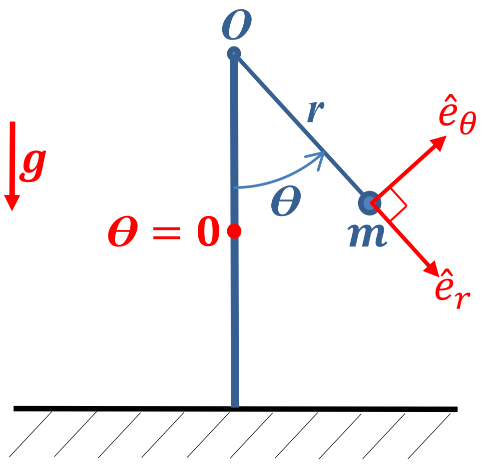
$$\vec{r} = 2t^2\hat{\imath} + (-3t - 3)\hat{\jmath}\ \mathrm{m}$$

What is the radius of curvature at $t = 2\ s$?

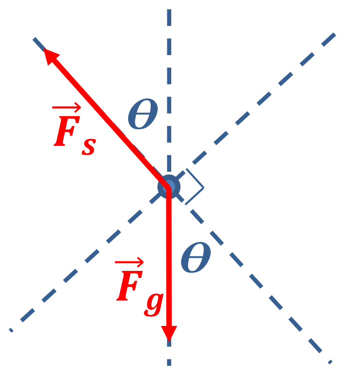
What are the Cartesian coordinates of the center of the osculating circle at $t = 2\ s$?

Particle Kinematics

1. System diagram



2. FBD



$$F_{sr} = -T$$

$$F_{s\theta} = 0$$

$$F_{gr} = mg\cos\theta$$

$$F_{g\theta} = -mg\sin\theta$$

3. Kinematics

$$a_r = -r\dot{\theta}^2$$

$$a_\theta = r\ddot{\theta}$$

4. Kinetics

$$\Sigma F_r = ma_r\text{: } -\mathrm{T} + \mathrm{mg}\cos\theta = -mr\dot{\theta}^2$$

$$\Sigma F_\theta = ma_\theta\text{: } 0 - \mathrm{mg}\sin\theta = mr\ddot{\theta}$$

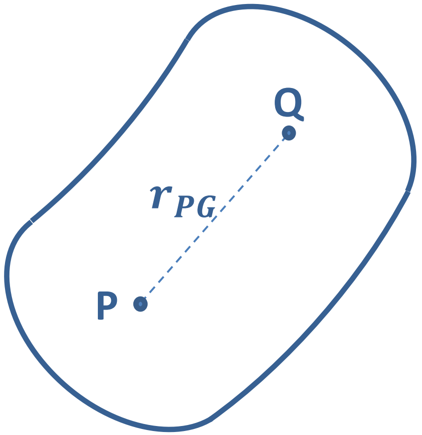
5. Algebra

$$\ddot{\theta} = -\frac{g}{r}\sin\theta = \alpha$$

$$T = mg\cos\theta + mr\dot{\theta}^2$$

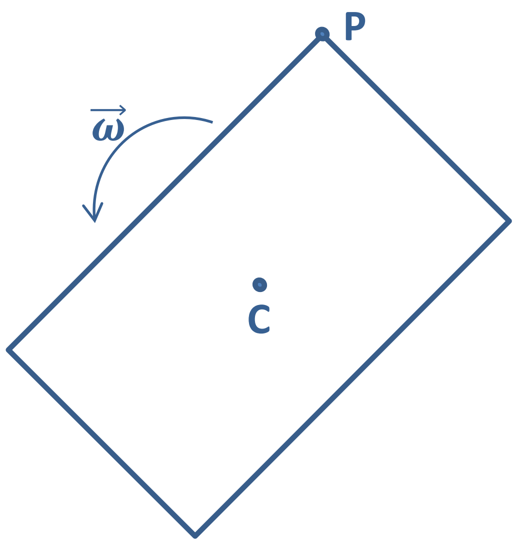
Rigid Bodies

Definition: A rigid body has constant distance between any two points on the body.



$r_{PG} =$ constant

Rotating Rigid Bodies

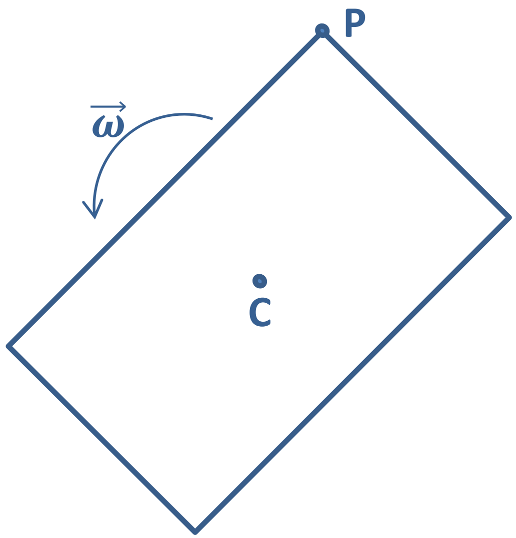


Angular velocity vectors $\vec{\omega}_C$, $\vec{\omega}_P$

$$\vec{\omega}_C = \omega_{CZ}\hat{k}$$

$$\vec{\omega}_P = \omega_{PZ}\hat{k}$$

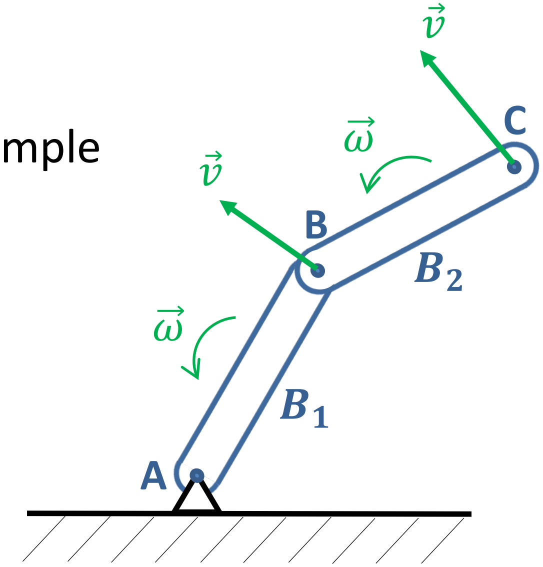
Rotating Rigid Bodies



Angular velocity $\vec{\omega}$ belongs to the whole rigid body.

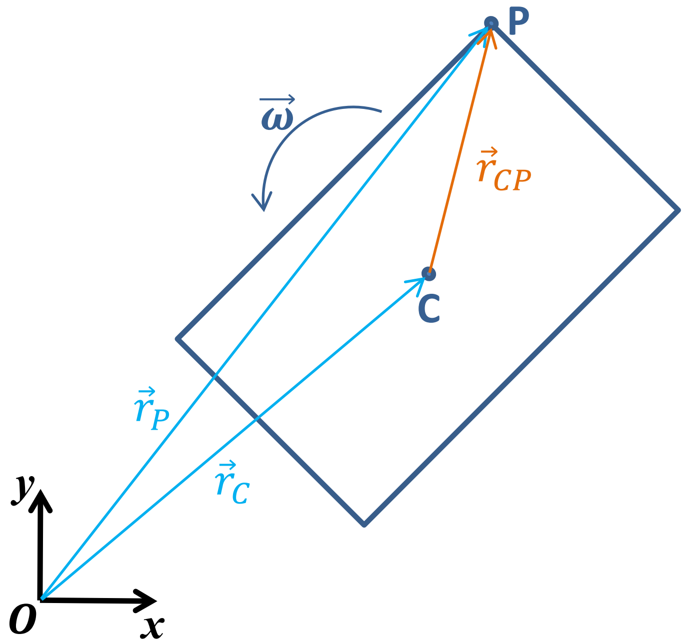
Positions and velocities belong to points.

Example



How should we label velocities $\vec{v}$ and angular velocities $\vec{\omega}$?

Rigid body velocity formula (relative velocity)



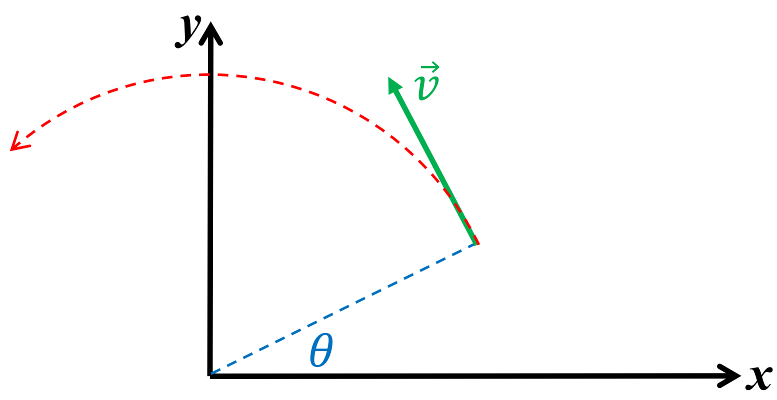
$\vec{v}_{P/C}$ = velocity of P relative to C

What direction and magnitude does $\vec{v}_{P/C}$ have?

$\vec{v}_{P/C} =$

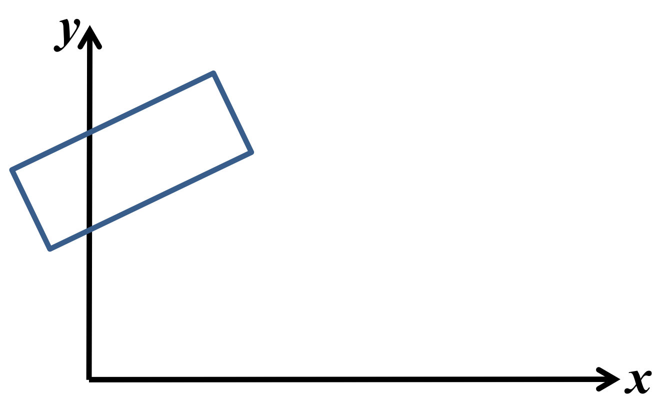
Two different usages of “angular velocity”

1. Angular velocity of a point about the origin

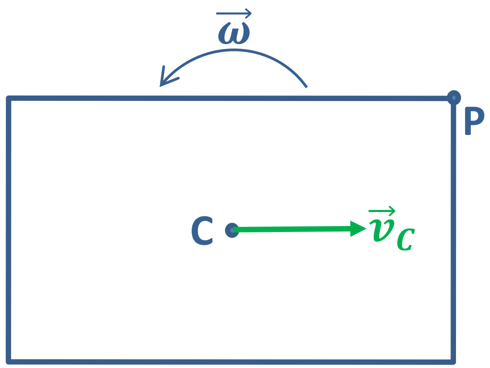


$$\omega = \dot{\theta}$$

2. Angular velocity of a rigid body



Example

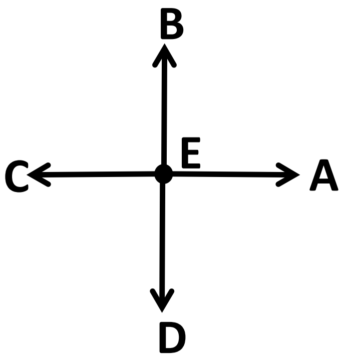


$\vec{v}_C = 3\hat{\imath}$ m/s

$\vec{\omega} = 2\hat{k}$ rad/s

$\vec{r}_{CP} = (2\hat{\imath} + \hat{\jmath})$ m

What is $\vec{v}_P$?

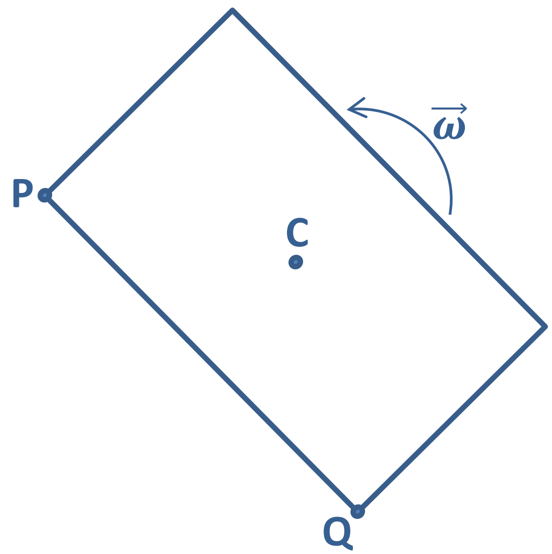


1. Predict **Which direction is closest to $\vec{v}_P$?**

2. Calculate

3. Reflect Does this make sense?

Example

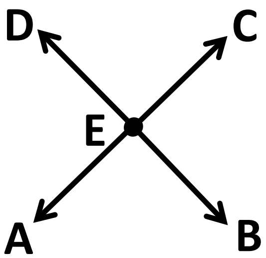


$\vec{v}_P = -3\hat{j}$ m/s

$\vec{\omega} = \hat{k}$ rad/s

$\vec{r}_{PQ} = (2\hat{\imath} - 2\hat{\jmath})$ m

What is $\vec{v}_Q$?

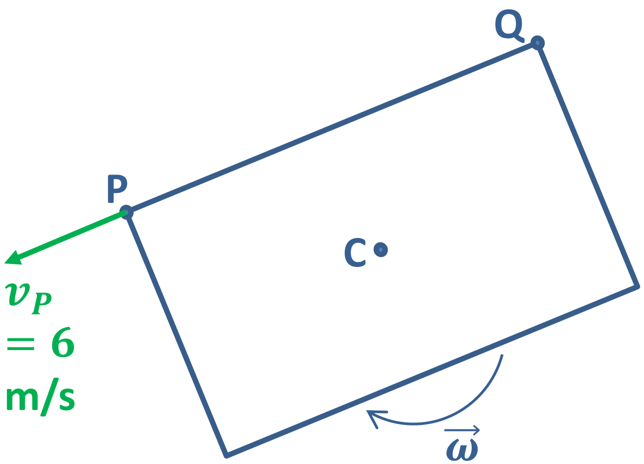


1. <u>Predict</u> **Which direction is closest to $\vec{v}_Q$?**

2. <u>Calculate</u>

3. <u>Reflect</u> Does this make sense?

Example

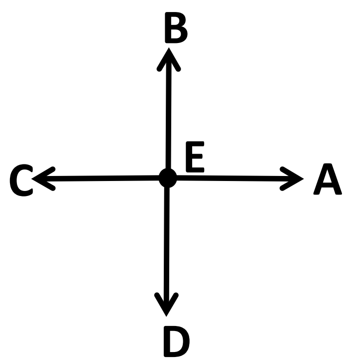


$v_P = 6$ m/s

$\vec{\omega} = -3\hat{k}$ rad/s

$r_{PQ} = 2$ m

What is the speed v_Q ?

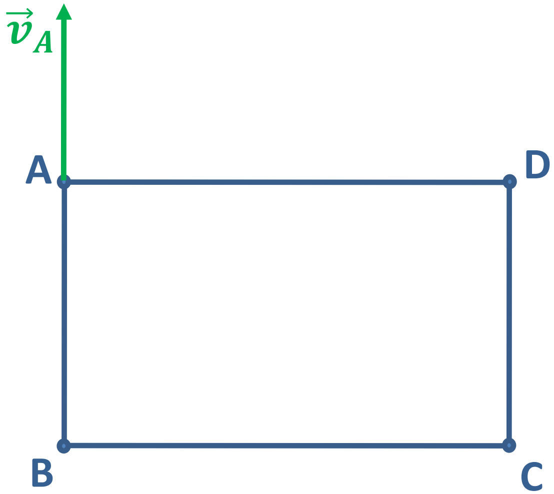


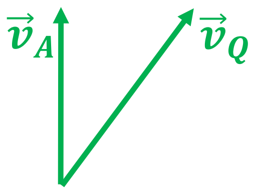
1. Predict **(a) Which direction is closest to $\vec{v}_Q$?**

2. Calculate

3. Reflect Does this make sense?

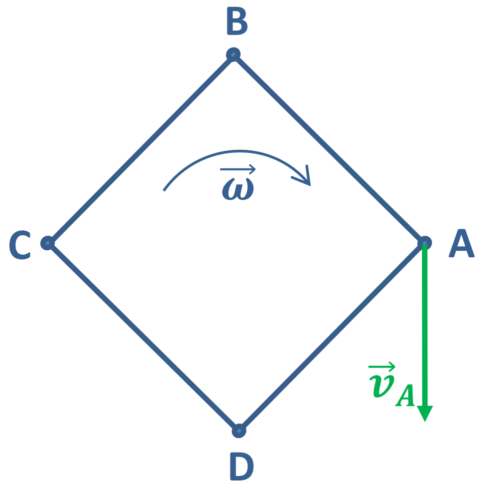
Example

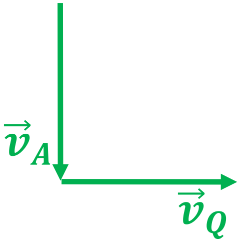




Which point is Q?

Example





Which point is Q?