TAM 212 – Dynamics

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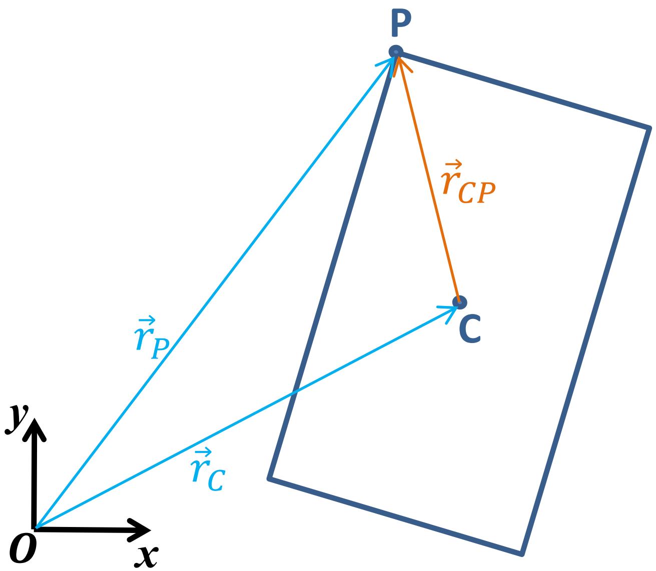
Summer 2019

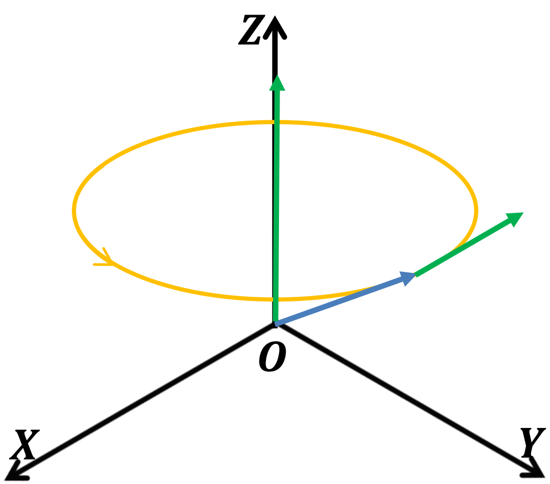
Rigid Body Acceleration



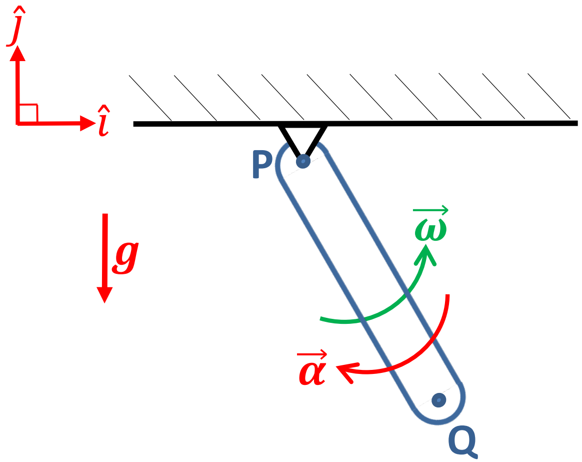
(recall: pure rotation $\dot{\vec{r}} = \vec{\omega} \times \vec{r}$)

$\vec{r}_{P/C} = \vec{r}_P - \vec{r}_C$ $\vec{r}_P = \vec{r}_C + \vec{r}_{CP}$

$\vec{v}_{P/C} = \vec{\omega} \times \vec{r}_{CP}$ $\vec{v}_P = \vec{v}_C + \vec{\omega} \times \vec{r}_{CP}$



Example: Pendulum



$\vec{r}_{PQ} = (3, -4)$ m

$\vec{\omega} = 2\hat{k}$ rad/s

$\vec{\alpha} = -\hat{k}$ rad/s²

What are $\vec{v}_Q$ and $\vec{a}_Q$?

$\vec{v}_{Q/P} = \vec{\omega} \times \vec{r}_{PQ}$

$\vec{a}_{Q/P} = \vec{\alpha} \times \vec{r}_{PQ} + \vec{\omega} \times (\vec{\omega} \times \vec{r}_{PQ})$

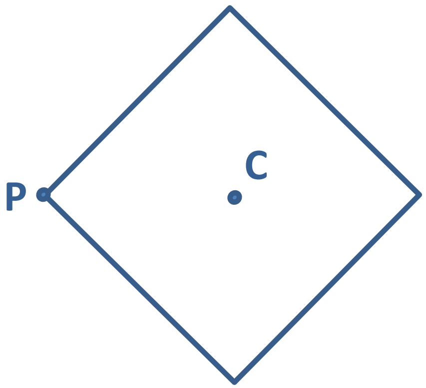
$\vec{v}_Q = (8, \_\_\_\_)$ **m/s**

A. -8 B. -6 C. 0 D. 6 E. 8

$\vec{a}_Q = (\_\_\_\_, 13)$ **m/s²**

A. -3 B. -4 C. -7 D. -12 E. -16

Example

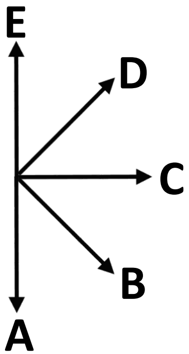


$$\vec{a}_C = 0$$

$$\omega_Z = 10 \text{ rad/s}$$

$$\alpha_Z = 10 \text{ rad/s}^2$$

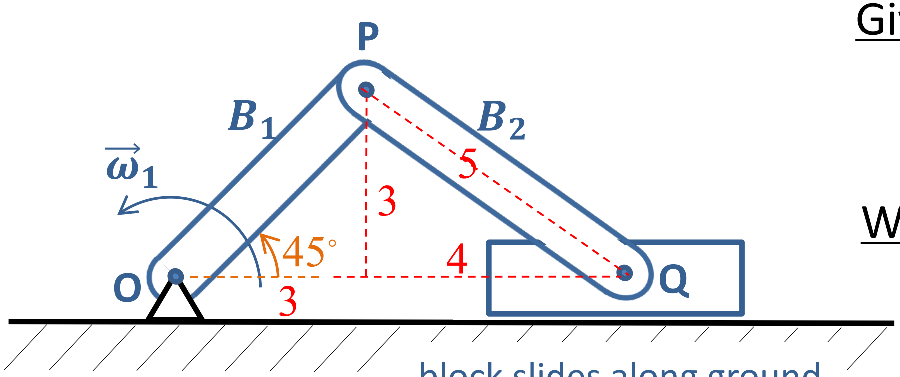
Which direction is closet to $\vec{a}_P$?



$$\vec{v}_{P/C} = \vec{\omega} \times \vec{r}_{CP}$$

$$\vec{a}_{P/C} = \vec{\alpha} \times \vec{r}_{CP} + \vec{\omega} \times (\vec{\omega} \times \vec{r}_{CP})$$

Example



Given: $OP = 3\sqrt{2}$ m
$PQ = 5$ m
$\vec{\omega}_1 = 4\hat{k}$ rad/s

We calculated: $\vec{\omega}_2 = -3\hat{k}$ rad/s
$\vec{v}_Q = (-12, 12)$ m/s
$\vec{v}_P = (-21, 0)$ m/s

Now given: $\vec{a}_P = (-54, -42)$ m/s^2

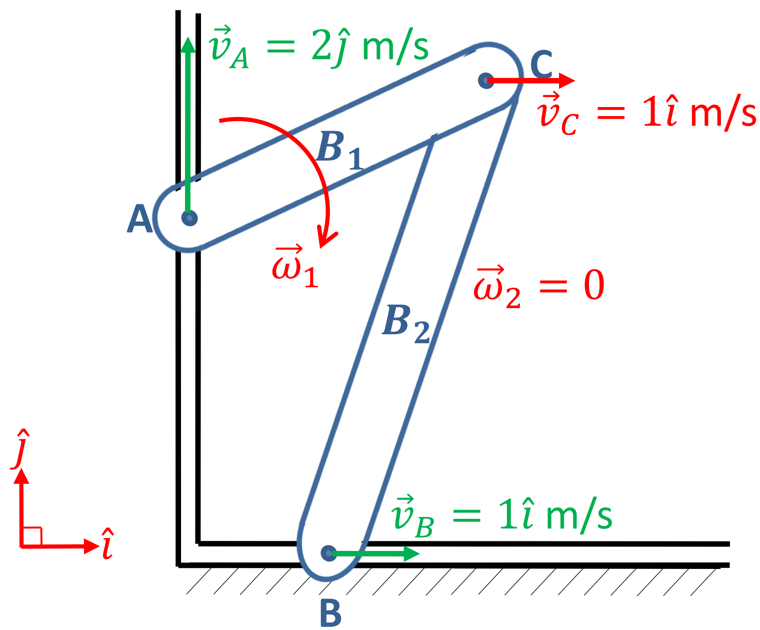
What is the acceleration of block $\vec{a}_Q$?

1. Predict What is the direction of $\vec{\alpha}_1$?

2. Calculate

3. Reflect Does this make sense?

Example



<u>Given:</u>

$\vec{r}_{AC} = (2, 1)$ m

$\vec{r}_{BC} = (1, 3)$ m

$\vec{v}_A = (0, 2)$ m/s

$\vec{v}_B = (1, 0)$ m/s

<u>We calculated:</u>

$\vec{v}_C = (1, 0)$ m/s

$\vec{\omega}_1 = -1\hat{k}$ rad/s

$\vec{\omega}_2 = 0$

<u>Now given:</u> $\vec{a}_A = 0$ $\vec{a}_B = 0$

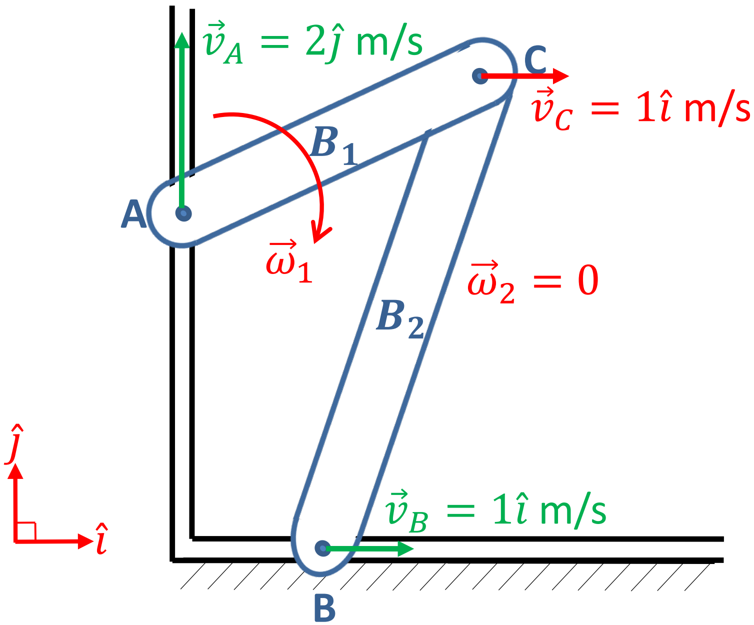
What is $\vec{\alpha}_2$?

1. <u>Predict</u>

What is the direction of $\vec{\alpha}_2$?

A. CCW
B. 0
C. CW
D. can't tell

Example



Given:

$\vec{r}_{AC} = (2, 1)$ m

$\vec{r}_{BC} = (1, 3)$ m

$\vec{v}_A = (0, 2)$ m/s

$\vec{v}_B = (1, 0)$ m/s

We calculated:

$\vec{v}_C = (1, 0)$ m/s

$\vec{\omega}_1 = -1\hat{k}$ rad/s

$\vec{\omega}_2 = 0$

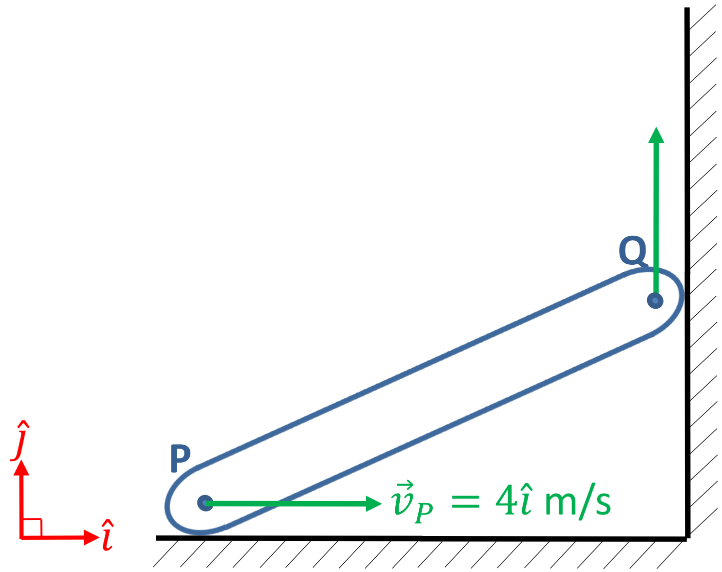
Now given: $\vec{a}_A = 0$ $\vec{a}_B = 0$

What is $\vec{\alpha}_2$?

2. Calculate

3. Reflect Does this match our prediction?

Example



$\vec{r}_{PQ} = (2, 1)$ m

$\vec{v}_P = (4, 0)$ m/s

$\vec{a}_P = (0, 0)$ m/s^2

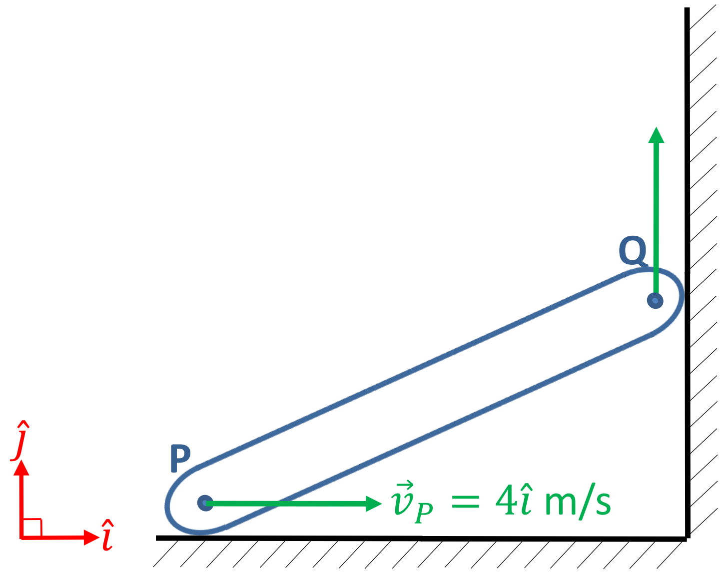
What is $\vec{v}_Q$ and $\vec{a}_Q$?

1. Predict

What is the direction of $\vec{\omega}$?

A. CCW
B. 0
C. CW
D. can't tell

Example



$\vec{r}_{PQ} = (2, 1)$ m

$\vec{v}_P = (4, 0)$ m/s

$\vec{a}_P = (0, 0)$ m/s^2

What is $\vec{v}_Q$ and $\vec{a}_Q$?

1. Predict

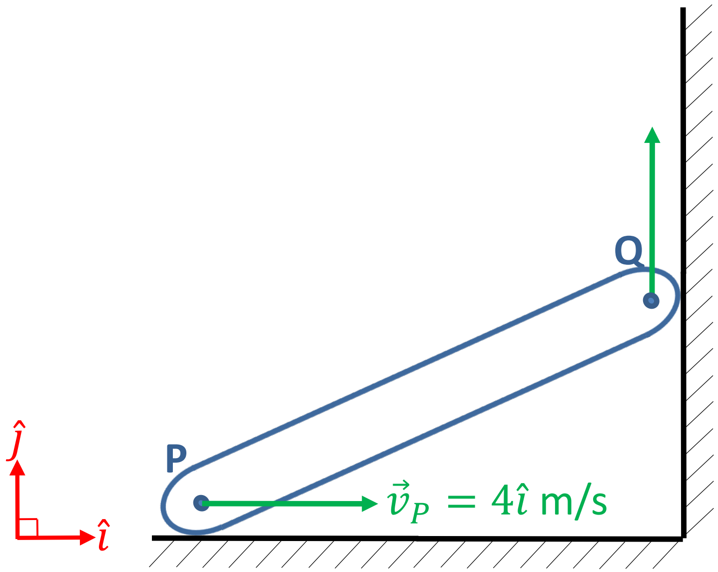
What is $\vec{v}_Q$?

A. $|v_{Qy}| > 4$ **m/s**
B. $|v_{Qy}| = 4$ **m/s**
C. $|v_{Qy}| < 4$ **m/s**

What is the direction of $\vec{\alpha}$?

A. **CCW**
B. **0**
C. **CW**
D. **can't tell**

Example



$\vec{r}_{PQ} = (2, 1)$ m

$\vec{v}_P = (4, 0)$ m/s

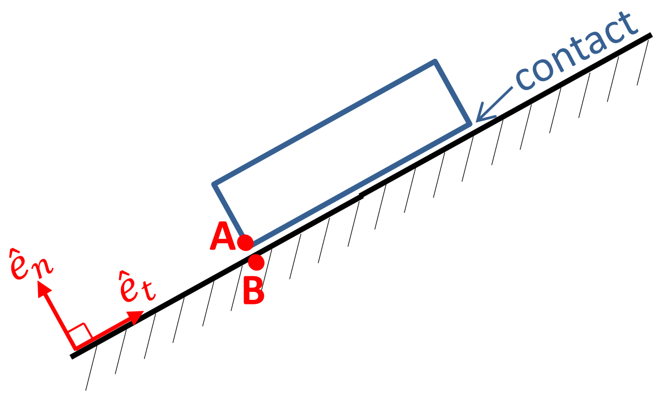
$\vec{a}_P = (0, 0)$ m/s^2

What is $\vec{v}_Q$ and $\vec{a}_Q$?

2. Calculate

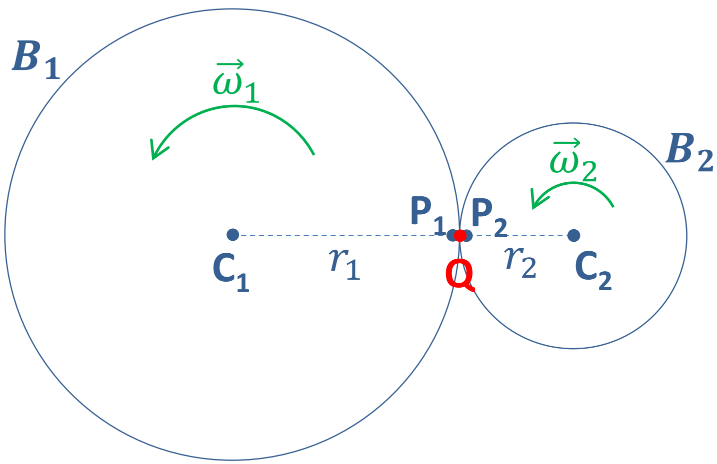
3. Reflect   Does this match our prediction?

Rigid bodies in contact



contact
- persistent
  - stick = no relative velocity at contact
    $\vec{v}_A = \vec{v}_B \qquad v_{An} = v_{Bn}$
  - slip = relative velocity at contact
    $\vec{v}_A \neq \vec{v}_B \qquad v_{An} = v_{Bn}$
- transient
  - separation $v_{An} > v_{Bn}$
  - interpenetration $v_{An} < v_{Bn}$

Gears



Meshed gears
Fixed centers

P_1 = attached to B_1

P_2 = attached to B_2

Q = the position in space of contact point

A. $\vec{v}_{P_1} = \vec{v}_Q = \vec{v}_{P_2}$

B. $\vec{v}_{P_1} = \vec{v}_Q \neq \vec{v}_{P_2}$

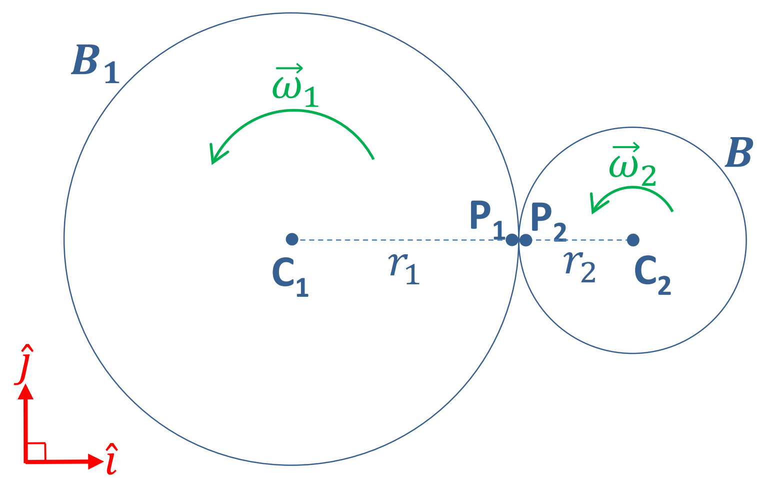
C. $\vec{v}_{P_1} = \vec{v}_{P_2} \neq \vec{v}_Q$

D. $\vec{v}_Q = \vec{v}_{P_2} \neq \vec{v}_{P_1}$

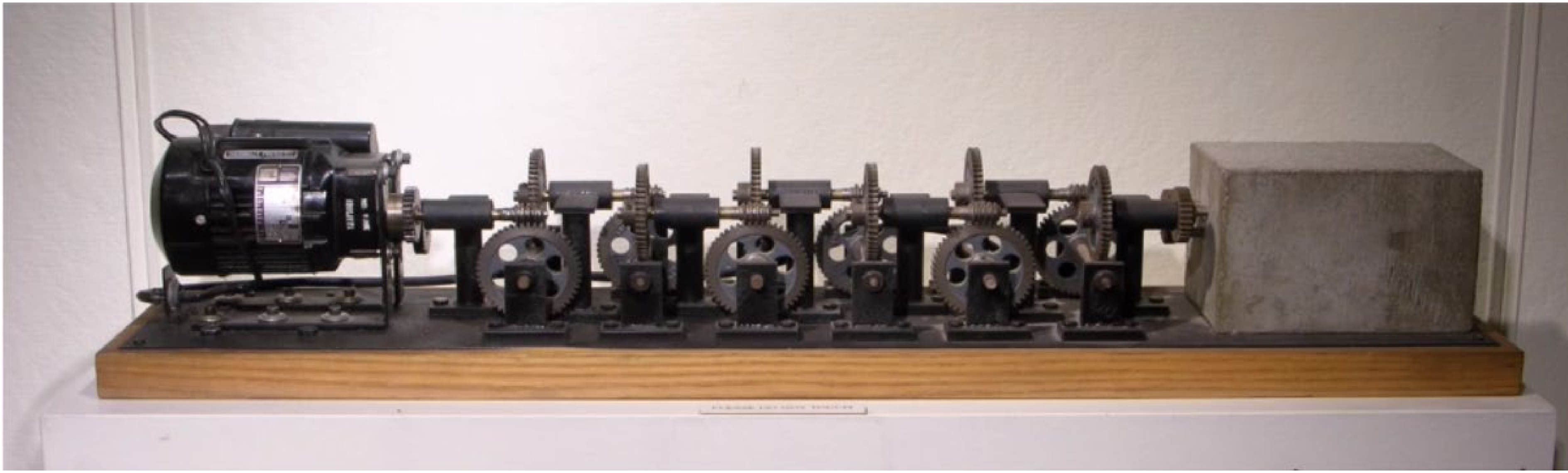
E. all different

Standard Sign Conventions

$$\vec{v}_{P_1} = \vec{v}_{C_1} + \vec{\omega}_1 \times \vec{r}_{C_1 P_1}$$



"Machine with Concrete" by Arthur Ganson



Motor on the left turns at 200 rpm.
Sequence of metal gears, rigidly connected.
Last gear on the right is embedded in a concrete block.

What happen?

A. Concrete breaks
B. motor breaks
C. gears break
D. nothing

worm gear