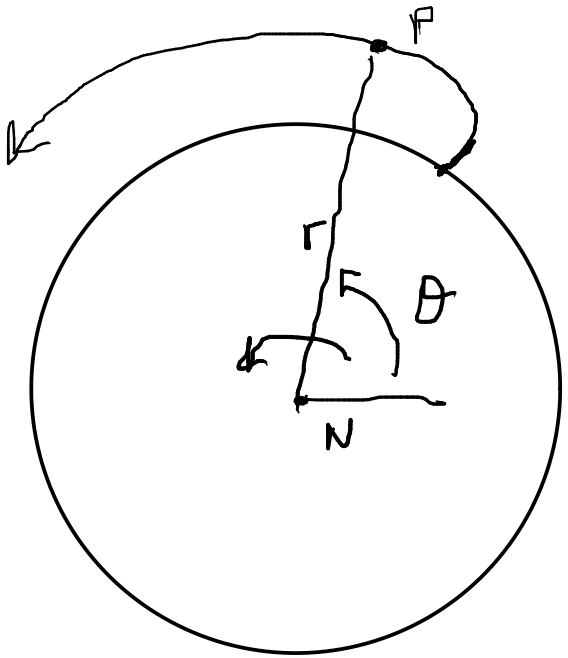
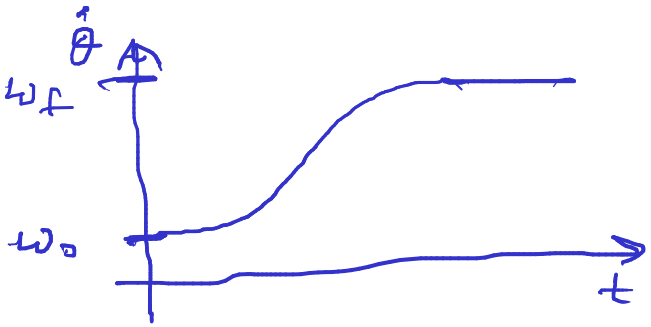
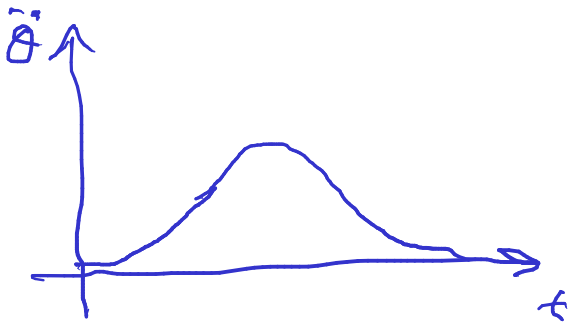
Recap — cylindrical & spherical coordinates

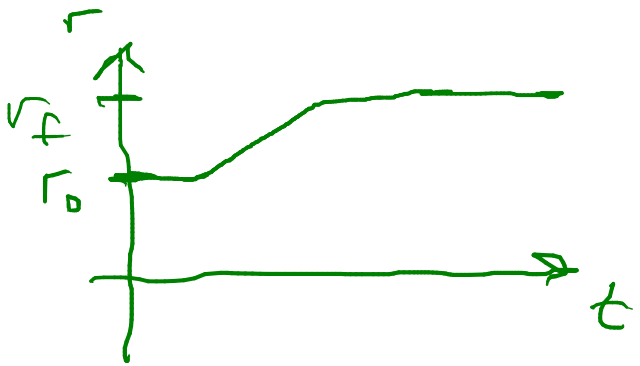
— integrating vectors : $\hat{i}, \hat{j}$ are constant

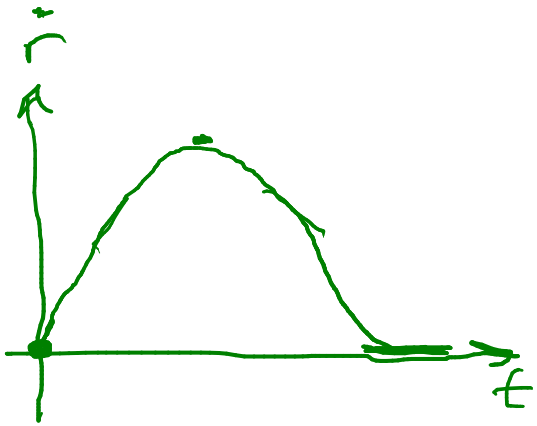
Rocket launches to orbit

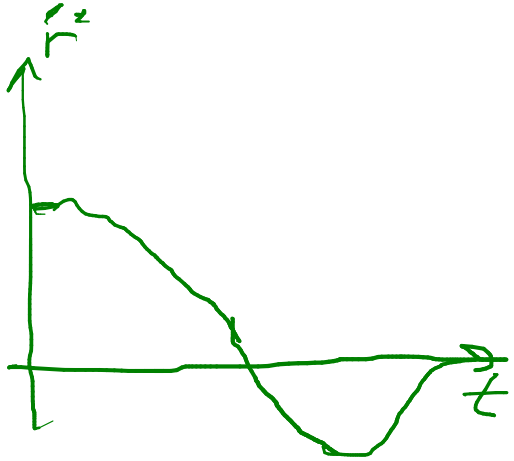








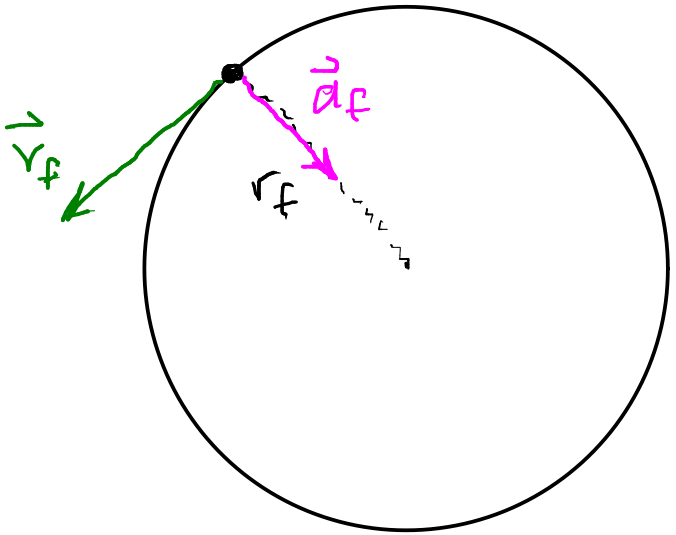




$r_0 = 6400$ km

$r_f = 6600$ km

$\omega_0 = \frac{2\pi}{24h} = 7.3 \times 10^{-5}$ rad/s



$a_f \approx$

A. 0 m/s²
B. 1 m/s²
C. 10 m/s²
D. 100 m/s²
E. 1000 m/s²

$\vec{a}_f = -r\dot{\theta}^2 \hat{e}_r$

$\omega_f \approx$

A. 10^{-7} rad/s
B. 10^{-5} rad/s
C. 10^{-3} rad/s
D. 10^{-1} rad/s
E. 10 rad/s

1.2×10^{-3}

$T_f \approx$

A. 1 min
B. 10 min
C. 1 h
D. 10 h
E. 100 h

$T = \frac{2\pi}{\omega} = 1.4h$

$v_f \approx$

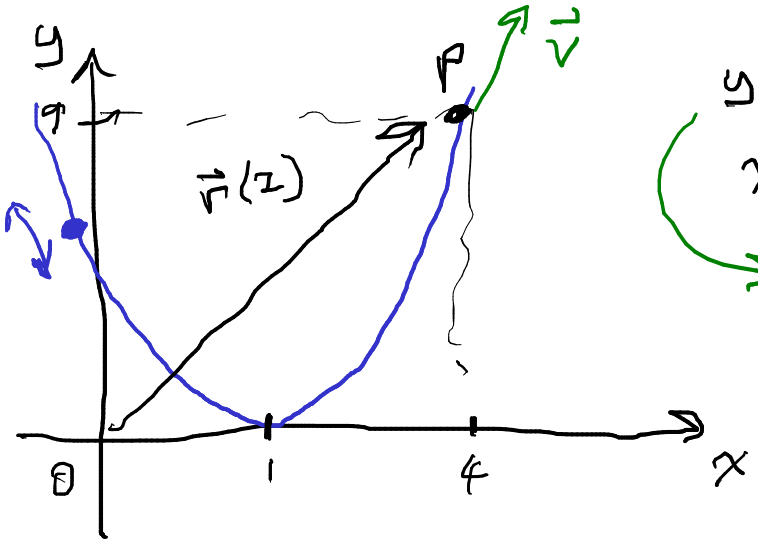
A. 1 m/s
B. 10 m/s
C. 100 m/s
D. 1 km/s
E. 10 km/s

$\vec{v} = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$

$v_f = r_f \omega_f = 8$ km/s $= 5$ miles/s

Today: – motion on a path (chain rule)

– track transition curves (WS 3)



$y_P(x) = (x-1)^2$

$x(t) = t^2$

$\Rightarrow y(x) = (x-1)^2$

$\dot{y} = \frac{d}{dt} y(x(t))$

At $t = 2s$, what is

$\vec{v}(2) =$

A. $2\hat{i} + 6\hat{j}$ m/s

B. $4\hat{i} + 24\hat{j}$ m/s

C. $2\hat{i} + 18\hat{j}$ m/s

D. $4\hat{i} + 26\hat{j}$ m/s

E. $6\hat{i} + 20\hat{j}$ m/s

$\vec{v} = \dot{x}\hat{i} + \dot{y}\hat{j}$

2 fcns $\rightarrow y_P(x)$

$\rightarrow y(t) = y_P(x(t))$

$\dot{y} = y' \dot{x}$

$\dot{y} = \frac{dy}{dt} = \frac{\partial y_P}{\partial x}(x(t)) \frac{\partial x}{\partial t}(t) = y_P' \dot{x}$

$\dot{y}$ at $t=2s$

$x(t)=4$

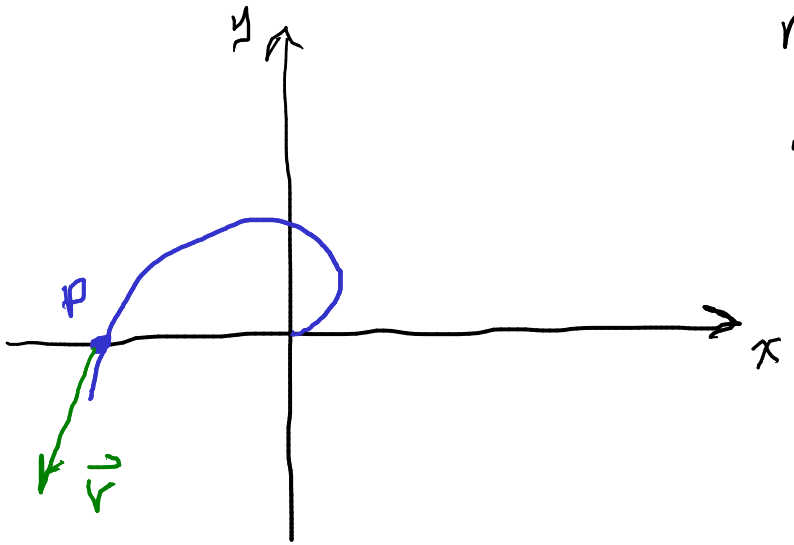
$y'(x)=2(x-1)$

$y'(4)=6$

$\dot{y}=y'\dot{x}$

$=6.4$

$=24$ m/s



$r = 3\theta^2$

$\dot{\theta} = \omega = \text{const}$

At P, what is v?

v = A. 3.7ω

B. 6.4ω

C. ~~7.1ω~~ 35.1

D. 11.6ω

E. 14.2ω

$\vec{v} = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$

$v = \sqrt{\dot{r}^2 + (r\dot{\theta})^2}$

At P: $\theta = \pi$ $\quad r = 3\pi^2$

$\dot{\theta} = \omega$ $\quad \dot{r} = ?$

$= 6\theta\omega$

$= 6\pi\omega$

$\dot{r} = \frac{\partial r}{\partial t} = 6\theta \cdot \dot{\theta}$

$\dot{r} = \frac{\partial r}{\partial \theta} \cdot \frac{\partial \theta}{\partial t}$

6θ $\quad \omega$

$\dot{r} = r' \dot{\theta}$

$$v = \sqrt{36\pi^2\omega^2 + 9\pi^4\omega^2}$$

$$= \sqrt{36\pi^2 + 9\pi^4}\ \omega = 35.1\omega$$

$$\dot{\hat{r}} = \frac{d}{dt}\frac{\vec{r}}{r} = \frac{\dot{\vec{r}}\, r - \dot{r}\vec{r}}{r^2}$$

$$= \frac{1}{r}\text{Comp}(\dot{\vec{r}}, \hat{r})$$

$$\dot{r} = \dot{\vec{r}} \cdot \hat{r}$$