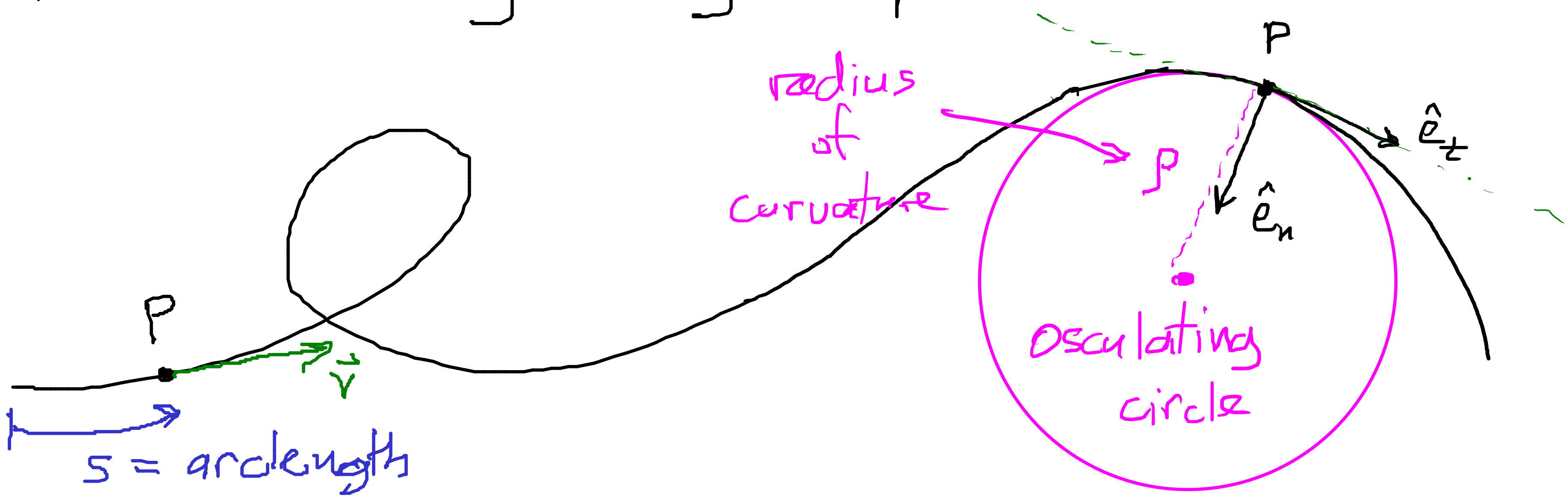
Recap

	Cartesian	Polar	Tang/Norm
position P	x, y	r, θ	s
position vec.	$\vec{r} = x\hat{i} + y\hat{j}$	$\vec{r} = r\hat{e}_r$	—
velocity	$\vec{v} = \dot{x}\hat{i} + \dot{y}\hat{j}$	$\vec{v} = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$	$\vec{v} = \dot{s}\hat{e}_t$
acceleration	$\vec{a} = \ddot{x}\hat{i} + \ddot{y}\hat{j}$	$\vec{a} = (\ddot{r} - r\dot{\theta}^2)\hat{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{e}_\theta$	$\vec{a} = \ddot{s}\hat{e}_t + \frac{\dot{s}^2}{\rho}\hat{e}_n$

Particle P moving along a path



Velocity $\vec{v}$ is always tangent to the path

$\hat{e}_t$ = tangential basis vector — tangential in direction of motion

$\hat{e}_n$ = normal basis vector — orthogonal, inwards direction

s = arclength

$\dot{s}$ = speed

$\hat{e}_t = \hat{v}$

$\dot{s} = v$

speed

$\vec{v} = \dot{s}\,\hat{e}_t$

curvature of path

$\vec{a} = \ddot{s}\,\hat{e}_t + \dot{s}^2 K\,\hat{e}_n$

$K = \frac{1}{\rho}$

rate of change of speed

Today:

- phone accelerometer / gyros
- track design
- T/N in 3D
- path examples
- animations comparing coordinate systems
- introduce CAs

Tangential / Normal basis in 3D

In 3D: $\hat{e}_n$ should be in the remaining acceleration direction.

$$\vec{a} = \_\_\_ \, \hat{e}_t + \_\_\_ \, \hat{e}_n$$

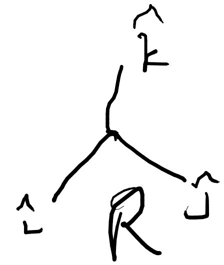
$$\Longrightarrow \quad \text{span}\{\hat{e}_t, \hat{e}_n\} = \text{span}\{\vec{v}, \vec{a}\}$$

definition: $\vec{e}_n = \dot{\hat{e}}_t \Rightarrow \hat{e}_n = \dfrac{\dot{\hat{e}}_t}{\|\dot{\hat{e}}_t\|}$

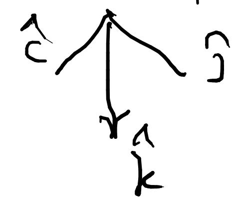
3rd basis vector: binormal $\hat{e}_b = \hat{e}_t \times \hat{e}_n$

3D T/N basis $\hat{e}_t, \hat{e}_n, \hat{e}_b$

$\hat{\imath}, \hat{\jmath}, \hat{k}$



R



L

Ex

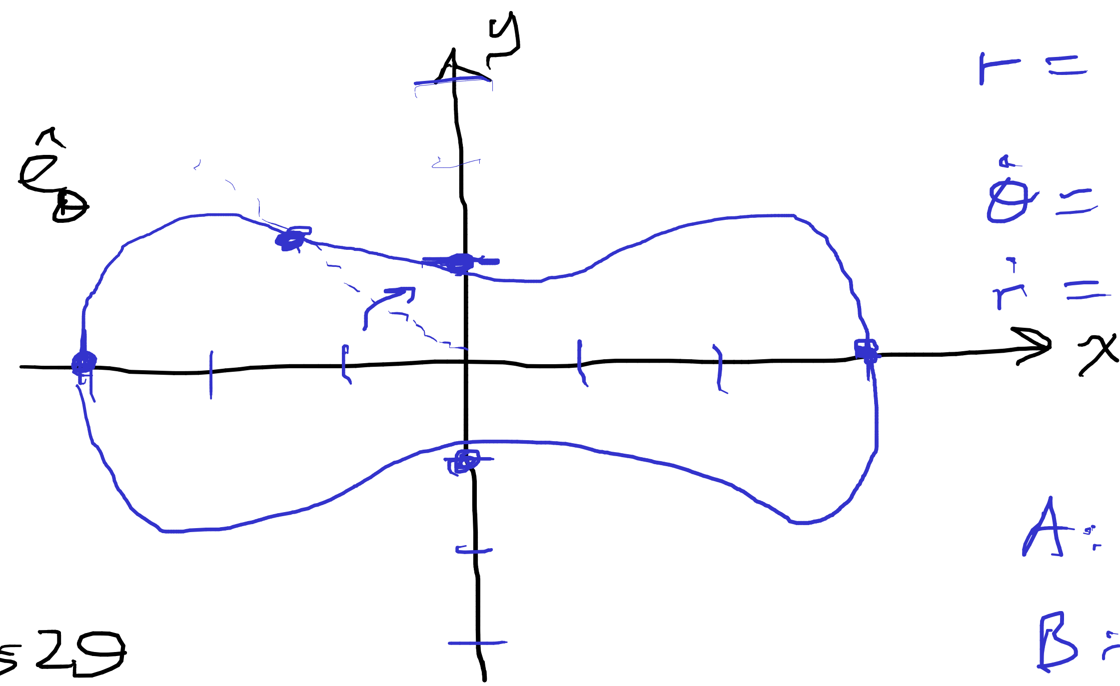
$r = 2 + \cos 2\theta$ m

At an instant, $\theta = \frac{3\pi}{4}$, $\dot{\theta} = -2$ rad/s

$\ddot{\theta} = -2$ rad/s^2

(a) what is the path?

$\vec{a} = \_\_\_\ \hat{e}_r + \_\_\_\ \hat{e}_\theta$



$\theta = \frac{3\pi}{4}$

$r = 2$

$\dot{\theta} = -2$

$\dot{r} =$

A: 2 m/s

B: 4

C: -2

D: -4

E: 0

$r(\theta) = 2 + \cos 2\theta$

$r'(\theta) = -2\sin 2\theta$

$r'(\frac{3\pi}{4}) = 2$

$r = r(\theta)$

$\dot{r} = r'(\theta)\,\dot{\theta}$

$$\vec{V} = \underline{\dot{r}}\ \hat{e}_r + \underline{r\dot{\theta}}\ \hat{e}_\theta$$

A. 2
B. 4
C. -2
D. -4
E. 0

A. 2
B. 4
C. -2
D. -4
E. 0

$\ddot{r} =$ A. 4 m/s^2
B. 2
C. 0
D. -2
E. -4

$$r'(\theta) = -2\sin 2\theta \to 2$$

$$r''(\theta) = -4\cos 2\theta \to 0$$

$$\dot{r} = r'(\theta)\dot{\theta}$$

$$\ddot{r} = \underbrace{r''(\theta)}_{0}\dot{\theta}^2 + \underbrace{r'(\theta)}_{2}\underbrace{\ddot{\theta}}_{-2}$$

$$= -4\ m/s^2$$