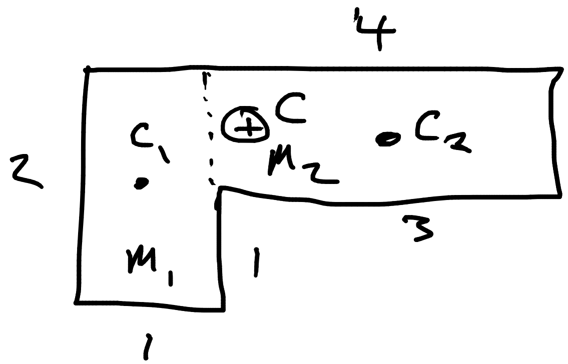
Centers of Mass

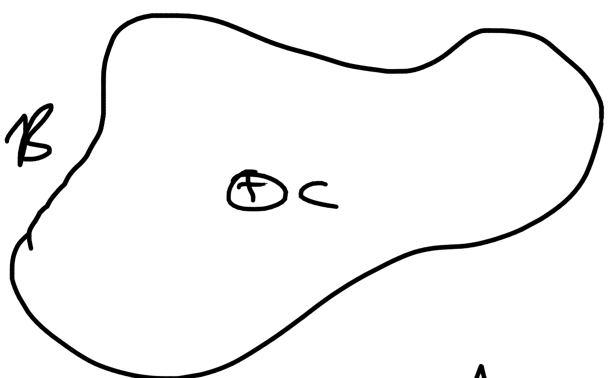


$$\rho = \text{constant}$$

$$m_1 = 2\rho$$

$$m_2 = 3\rho$$

$$\vec{r}_c = \frac{1}{m_1+m_2}\left(m_1 \vec{r}_{c_1} + m_2 \vec{r}_{c_2}\right) = \frac{1}{m}\sum_i m_i \vec{r}_i$$



$$m = \iiint_B \rho \, dV$$

$$\vec{r}_c = \frac{1}{m}\iiint_B \rho \, \vec{r} \, dV$$

$$[\rho] = kg^{\bigcirc} m^{\bigcirc}$$

A. -2
B. -1
C. 0
D. 1
E. 2

A. -3 vol.
B. -2 area
C. 0
D. 2
E. 3

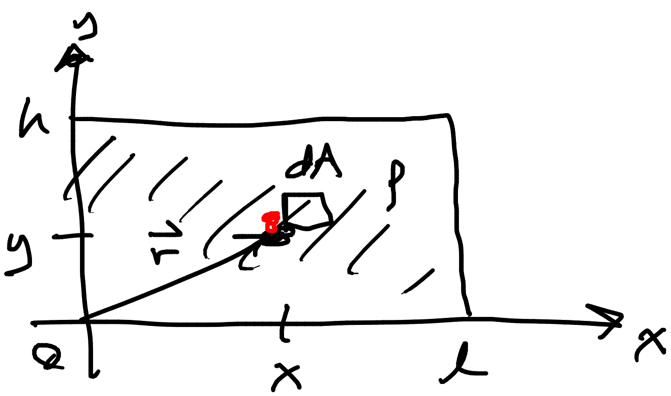
$$[\rho] = \text{kg per } \_\_\_$$

or 2D. $$\vec{r}_c = \frac{1}{m}\iint_{\mathcal{B}} \rho \vec{r}\, dA \qquad [\rho] = kg/m^2$$

$$m = \frac{1}{\cancel{kg}}\frac{\cancel{kg}}{\cancel{m^2}}\, m\, \cancel{m^2}$$

or 1D. $$\vec{r}_c = \frac{1}{m}\int_{\mathcal{B}} \rho\, \vec{r}_c\, dl \qquad [\rho] = kg/m$$

Ex



$$m = \iint_{\mathcal{B}} \rho\, dA = \int_0^h\int_0^l \rho(x,y)\, dx\, dy$$

$$= \rho l h \qquad \rho = \text{const}$$

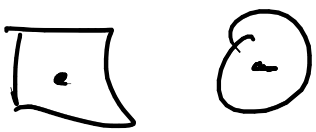
$$\vec{r}_c = \frac{1}{\rho l h}\int_0^h\int_0^l \cancel{\rho(x,y)}(x\hat{\imath} + y\hat{\jmath})\, dx\, dy$$

$$\vec{r}_c = \frac{1}{lh}\left(\int_0^h\int_0^l x\, dx\, dy\, \hat{\imath} + \int_0^h\int_0^l y\, dx\, dy\, \hat{\jmath}\right)$$

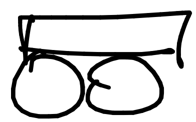
$$\vec{r}_c = \frac{1}{lh}\left(\frac{1}{2}l^2 h\hat{i} + \frac{1}{2} l h^2 \hat{j}\right)$$

$$= \frac{1}{2} l \hat{i} + \frac{1}{2} h \hat{j}$$

simple shapes



combinations of simple shapes



decompose into simple shapes

complex shapes

integrate.

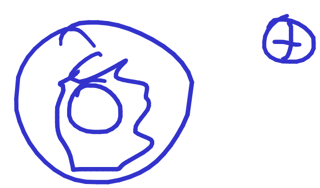
or

numerically integrate.

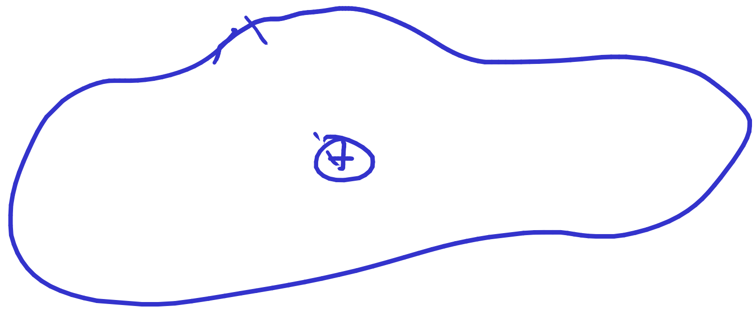
Q: COM of a RB is always inside the body.

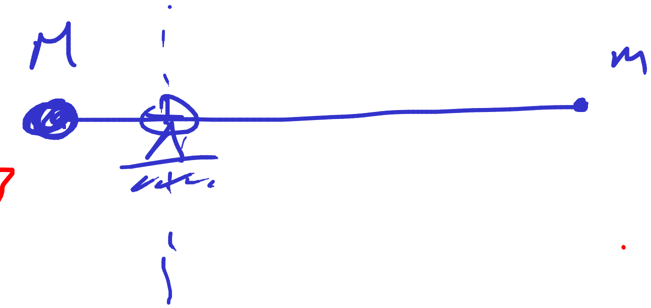
A. true

B. false.



can't happen.




M
m