

Announcements : Final Exam Plan.

Wed, May 13, 1:30-4:30

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- 20 A. MC paper - Midterm
- 80 B. CTF - with retry for Partial credit - whole exam week.

Today : - Finish rod-pendulum example.

- Car dynamics

- Moments of inertia

Recap: Rigid Body Kinetics

$$\sum \vec{F} = m\vec{a}_c$$

$$\sum M_{cz} = I_{cz} \alpha_z$$

OR

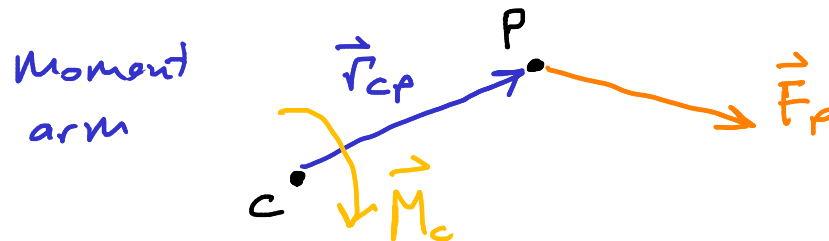
$$\sum M_{Oz} = I_{Oz} \alpha_z$$

$O = \text{fixed point}$

I_{cz} = moment of inertia about z -axis through C
(resistance to rotation)

$$\vec{M}_c = \vec{r}_{cp} \times \vec{F}_p$$

moment about C
due to force at P



Recap: Kinetics method

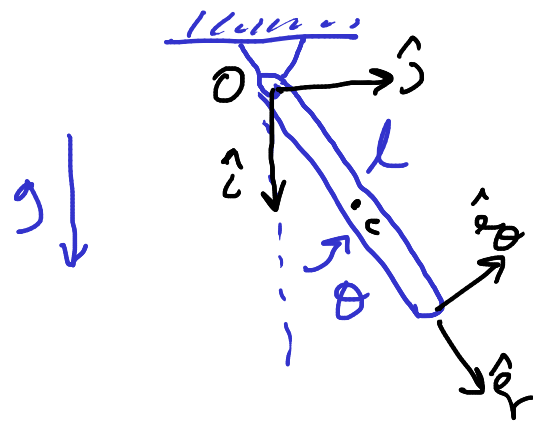
① FBD

② Kinematics: find $\vec{a}/\vec{\alpha}$ and additional constraints

③ Newton: $F = ma$ (also $M = I\alpha$)

④ Algebra: solve for F/M or a/α

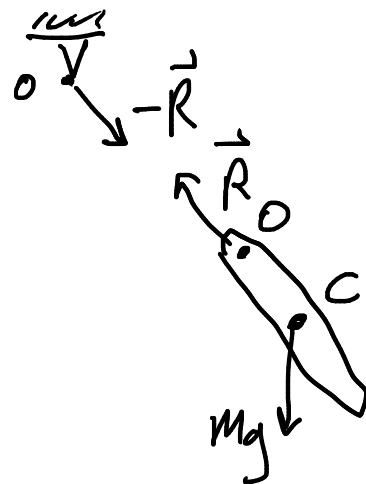
Ex Rigid-rod pendulum



Rigid rod, length l , mass m , gravity g

Find: ang. acc. as function of $\theta, \dot{\theta}$

① FBD



② Kinematics

$$\alpha_z = \ddot{\theta}$$

$$\vec{a}_c = -\frac{l}{2} \ddot{\theta} \hat{e}_r + \frac{l}{2} \ddot{\theta} \hat{e}_\theta$$

③ Newton about C

$$\sum \vec{F} = m\vec{a}_c$$

$$\vec{R} + mg\hat{c} = -m\frac{l}{2}\ddot{\theta}^2\hat{e}_r + m\frac{l}{2}\ddot{\theta}\hat{e}_\theta \quad (A)$$

$$\nwarrow R_r\hat{e}_r + R_\theta\hat{e}_\theta$$

$$\sum M_{cz} = I_{cz} \alpha_z \quad I_{cz} = \frac{1}{12}ml^2$$

$$\vec{M}_c = \vec{r}_{co} \times \vec{R}$$

$$= \left(-\frac{l}{2}\hat{e}_r\right) \times (R_r\hat{e}_r + R_\theta\hat{e}_\theta)$$

$$= -\frac{l}{2}R_\theta\hat{k}$$

$$-\frac{l}{2}R_\theta = \frac{1}{12}ml^2\ddot{\theta}$$

(B)

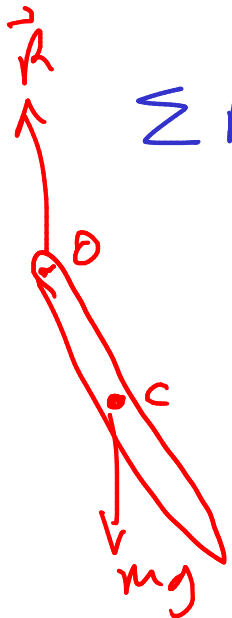
④ Algebra

$$(A)_\theta \Rightarrow R_\theta - mg\sin\theta = m\frac{l}{2}\ddot{\theta} \quad (C)$$

$$(B) \Rightarrow -\frac{l}{2}R_\theta = \frac{1}{12}ml^2\ddot{\theta} \quad (D)$$

$$l(C) + 2(D) \Rightarrow -mg\sin\theta = \frac{2}{3}ml\ddot{\theta}$$

$$\boxed{\ddot{\theta} = -\frac{3}{2}\frac{g}{l}\sin\theta} \quad (E)$$

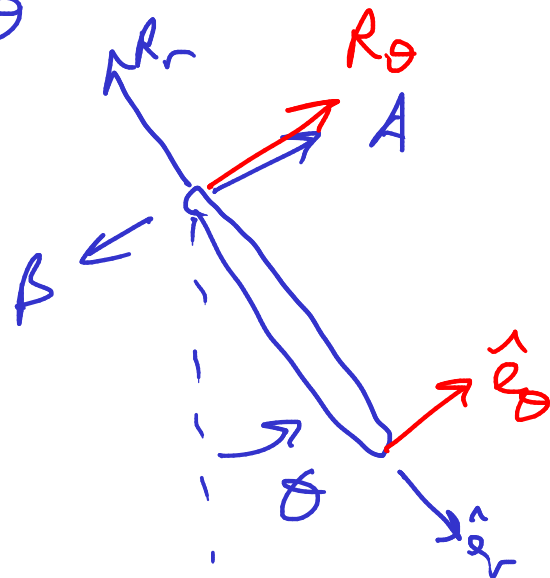


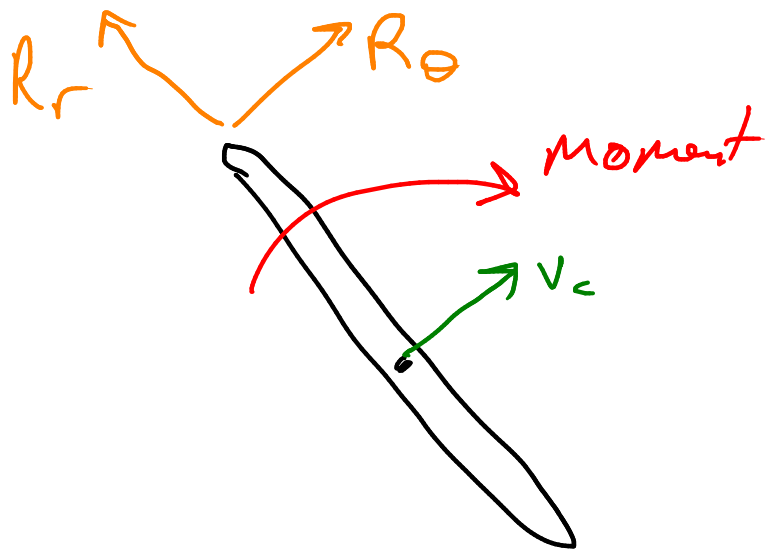
$$\vec{R} + mg\hat{c} = -m\frac{l}{2}\dot{\theta}^2\hat{e}_r + m\frac{l}{2}\ddot{\theta}\hat{e}_\theta$$

$$\ddot{\theta} = -\frac{3}{2}\frac{g}{l}\sin\theta$$

$$\begin{aligned}\vec{R} = & -mg(\cos\theta\hat{e}_r - \sin\theta\hat{e}_\theta) - \frac{ml}{2}\dot{\theta}^2\hat{e}_r \\ & + \frac{ml}{2}\left(-\frac{3}{2}\frac{g}{l}\sin\theta\right)\hat{e}_\theta\end{aligned}$$

$$\left\{ \begin{array}{l} R_r = -mg\cos\theta - \frac{ml}{2}\dot{\theta}^2 \\ R_\theta = \frac{1}{4}mg\sin\theta \end{array} \right.$$





swinging upwards

moment is CW

α_z is CW

ω_z is CCW

} opposite.

which dir is rod rotating

A. CW

☒ B. CCW

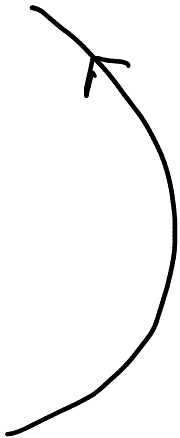
rotation rate

A. speeding up

☒ B. slowing down.

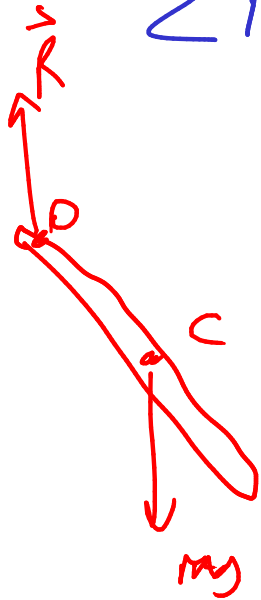
- How does the car go up on two wheels?
- How does it stay there?

top view



Alternative solution using fixed-point O

③ Newton



$$\sum M_{Oz} = I_{Oz} \alpha$$

$$I_{Oz} = \frac{1}{3} ml^2$$

$$\vec{M}_O = \vec{r}_{Oc} \times \vec{F}_g$$

$$= \left(\frac{l}{2} \hat{e}_r \right) \times (mg \hat{e})$$

$$= \left(\frac{l}{2} \hat{e}_r \right) \times (mg \cos \theta \hat{e}_r - mg \sin \theta \hat{e}_\theta)$$

$$= -mg \frac{l}{2} \sin \theta \hat{k}$$

$$-mg \frac{l}{2} \sin \theta = \frac{1}{3} ml^2 \ddot{\theta}$$

④ Algebra

$$\ddot{\theta} = -\frac{3}{2} \frac{g}{l} \sin \theta$$