Announcements

- HW9 / R9 due Mon, May 4
- HW10 not for credit
- R10 canceled
- Final exam schedule

Dry / Coulomb Friction (2D) — multiple cases:

Ⓐ Slip

1. relative motion
   $v \neq 0$ or $a \neq 0$
2. $F = \mu N$
3. $\hat{F}$ opposes motion
   ($\hat{v}$ or $\hat{a}$)

Ⓑ Stick

1. no relative motion
   $v = 0$ and $a = 0$
2. $F \leq \mu N$
   (magnitudes)

Ⓒ Transition

1. no relative motion
   $v = 0$ and $a = 0$
2. critical friction force
   $F = \mu N$

Solution Procedure (known case)

- determine case (stick, transition, slip left, slip right)
- FBD, equations depending on case, solve

Solution Procedure (unknown case)

contact point

- Try stick: assume $v=0$, $a=0$, N, F

  assume motion of contact.

  solve for motion, N, F

  check $|F| \le \mu |N|$

- Try slip left: assume $F=\mu N$ to the right

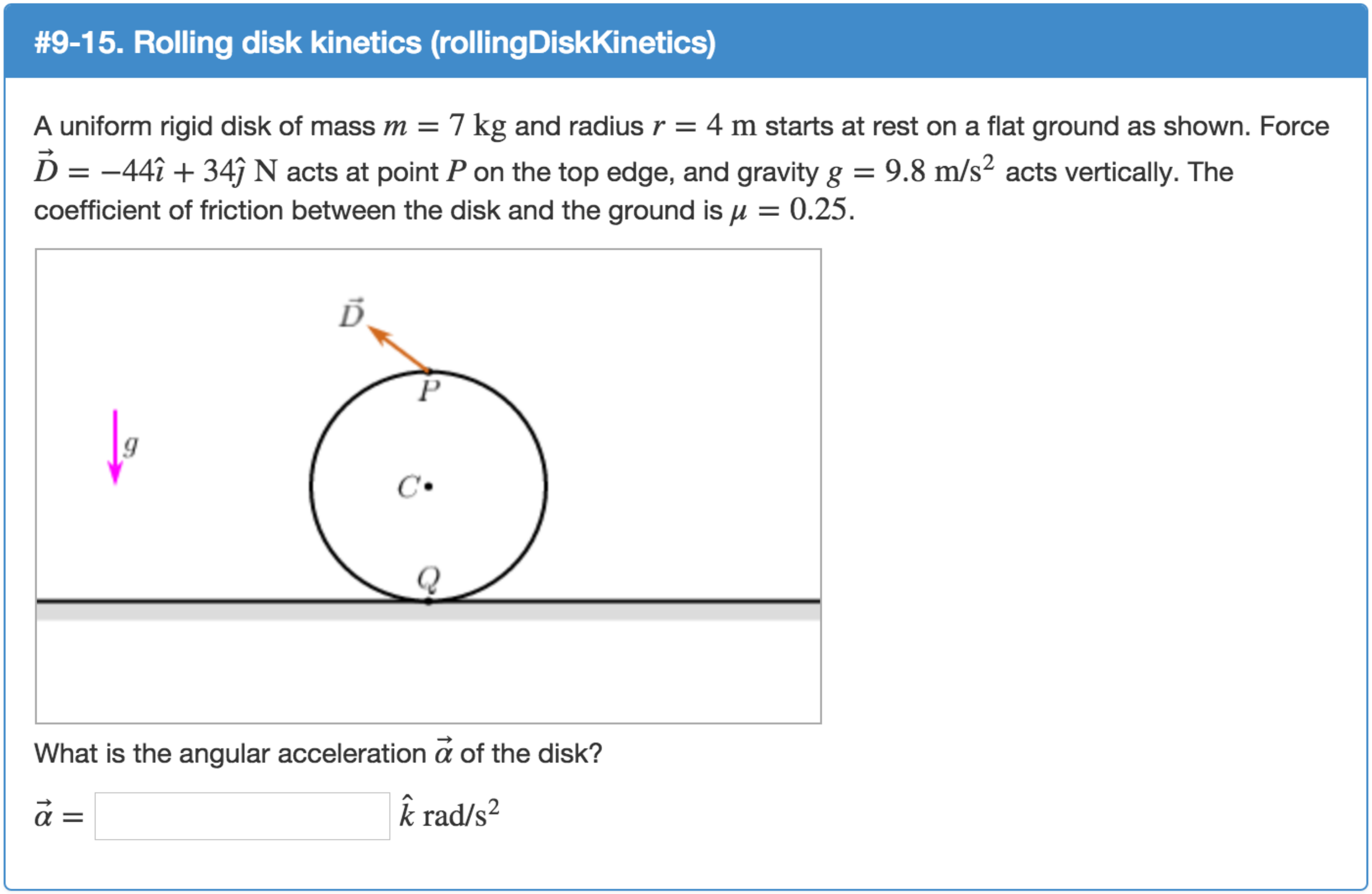
  assume friction force

  solve for motion of contact point and N, F

  check $v \neq 0$ or $a \neq 0$ and $\hat{F}$ opposes motion ($\hat{v}$ or $\hat{a}$)

- Try slip right: assume $F=\mu N$ to the left

  then same as slip left.



#9-15. Rolling disk kinetics (rollingDiskKinetics)

A uniform rigid disk of mass $m = 7\ \mathrm{kg}$ and radius $r = 4\ \mathrm{m}$ starts at rest on a flat ground as shown. Force $\vec{D} = -44\hat{\imath} + 34\hat{\jmath}\ \mathrm{N}$ acts at point P on the top edge, and gravity $g = 9.8\ \mathrm{m/s^2}$ acts vertically. The coefficient of friction between the disk and the ground is $\mu = 0.25$.

What is the angular acceleration $\vec{\alpha}$ of the disk?

$\vec{\alpha} =$ ☐ $\hat{k}\ \mathrm{rad/s^2}$

Stick

Which var do we NOT solve for?

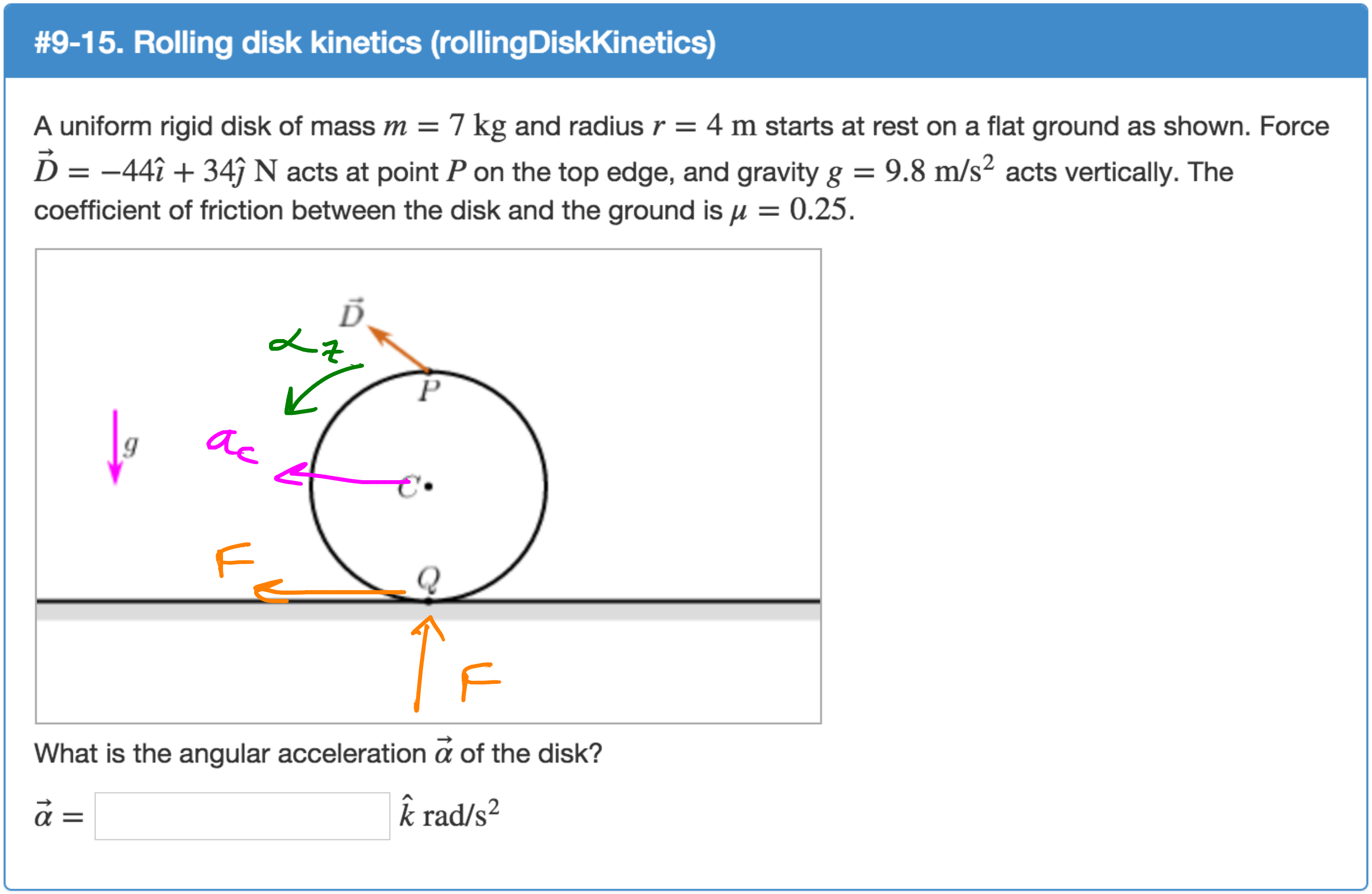
- A. a_{cx}
- [✓ B.] a_{Qx}
- C. α_z
- D. F
- E. N

Which eqn do we NOT use?

- A. $\sum \vec{F} = m\vec{a}_c$ ✓
- [✓ B.] $\sum M_{Qz} = I_{Qz}\alpha_z$ Q not fixed
- C. $\sum M_{cz} = I_{cz}\alpha_z$ ✓
- D. $\vec{a}_Q = 0$ ✓
- [✓ E.] $F = \mu N$

What do we check?

- [✓ A.] $|F| \leq \mu |N|$
- B. $\hat{F}$ opposes $\hat{v}$ or $\hat{a}$
- C. $a_{cx} = 0$
- D. $a_{Qx} = 0$
- E. $|F| = \mu |N|$



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What is the angular acceleration $\vec{\alpha}$ of the disk?

$\vec{\alpha} =$ ☐ $\hat{k}\ \mathrm{rad/s^2}$

$F \leq \mu N$

$|-14| \leq |8.5|$

Is this valid?

A. Yes

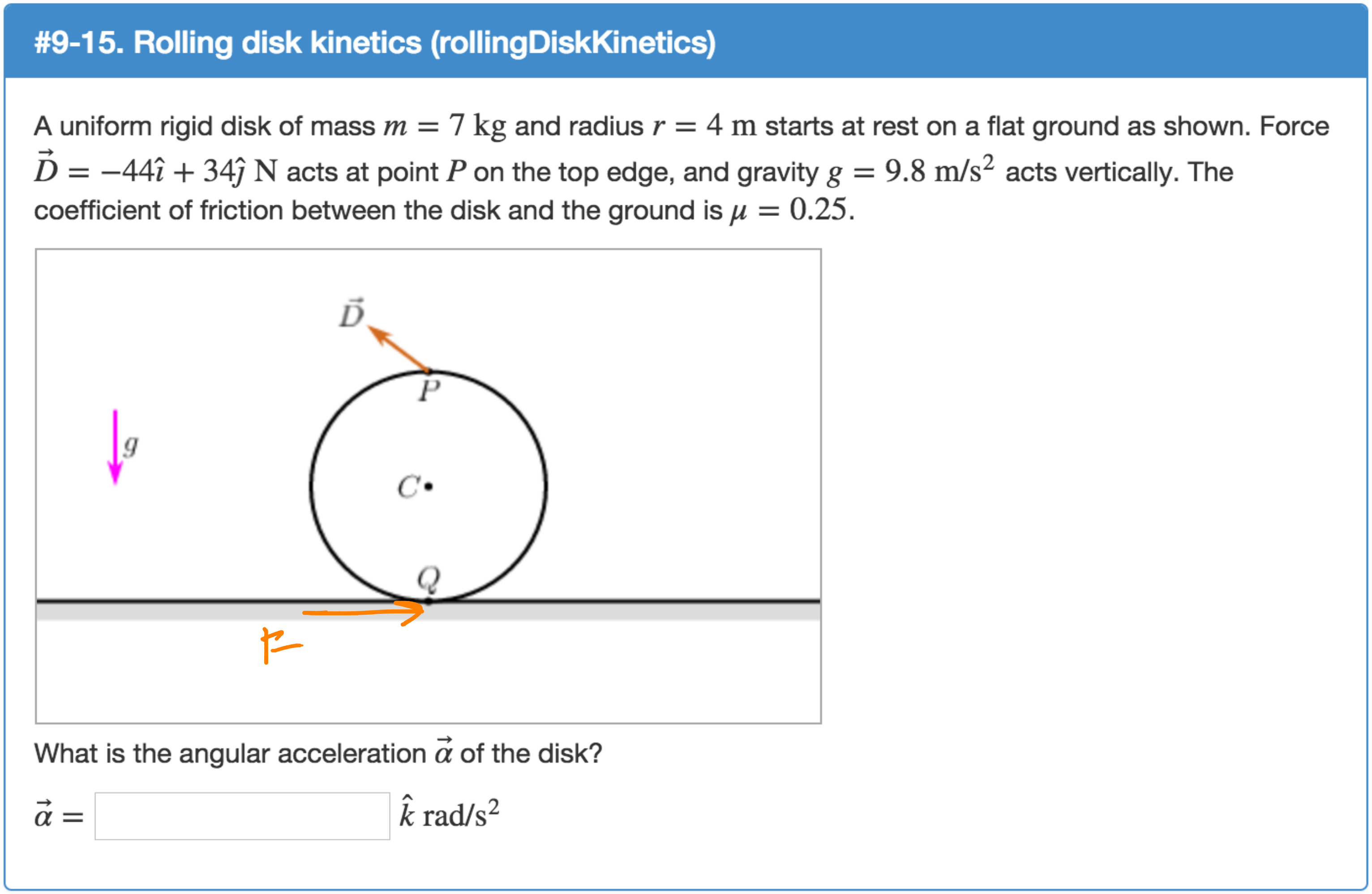
[✓ B. No]

$\vec{F} = -14.7\,\hat{\imath}\ N$

$\vec{N} = 34.6\,\hat{\jmath}\ N$

$\vec{a}_c = -8.38\,\hat{\imath}\ m/s^2$

$\alpha_z = 2.10\ rad/s^2$



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What is the angular acceleration $\vec{\alpha}$ of the disk?

$\vec{\alpha} =$ ☐ $\hat{k}\ \mathrm{rad/s^2}$

Slip left

Which var do we NOT solve for?

- A. a_{cx}
- B. a_{Qx}
- C. α_z
- D. F
- E. N

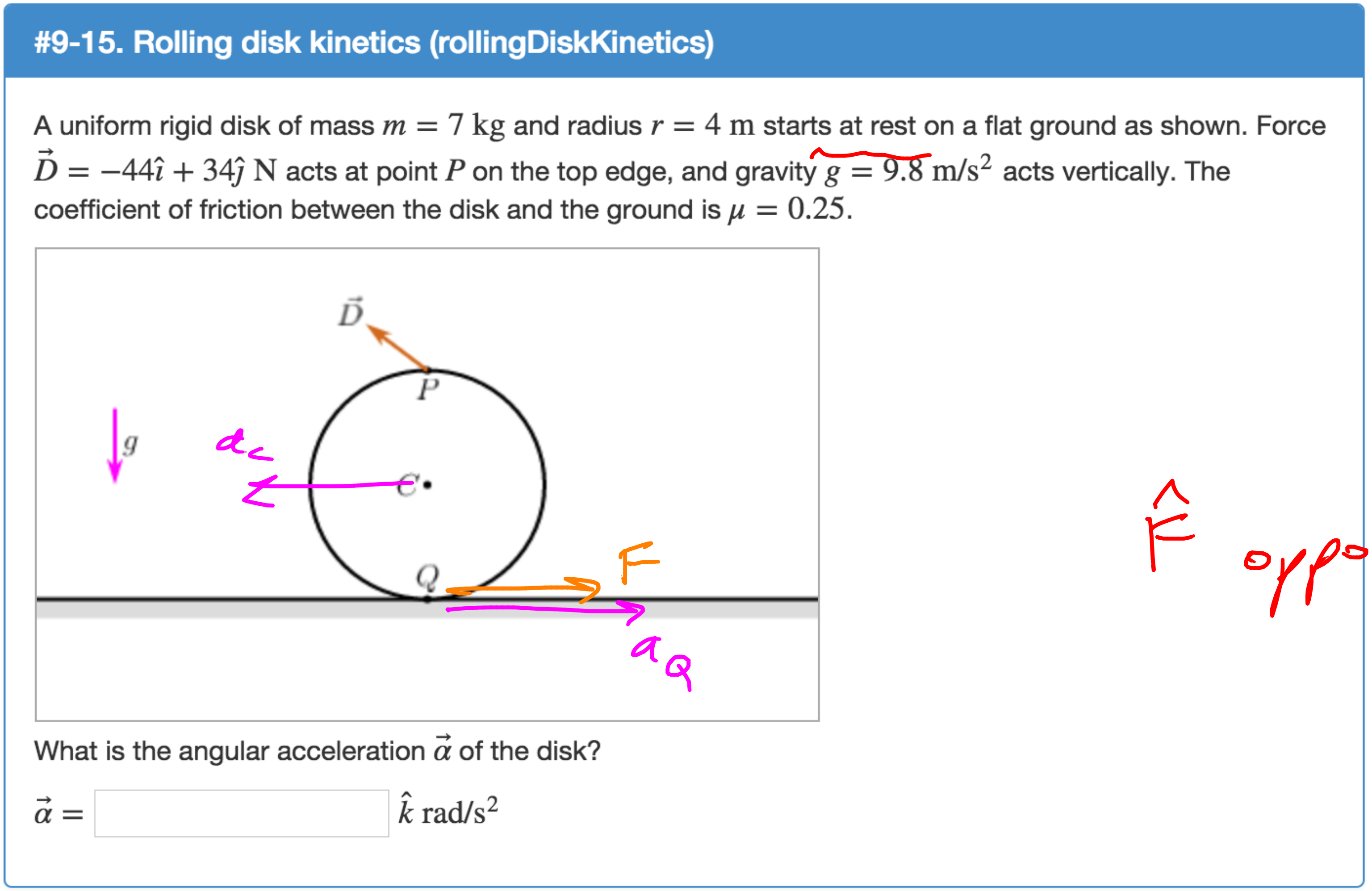
Solve for all

Which eqn do we NOT use?

- A. $\sum \vec{F} = m\vec{a}_c$ ✓
- [✓ B.] $\sum M_{Qz} = I_{Qz}\alpha_z$
- C. $\sum M_{cz} = I_{cz}\alpha_z$ ✓
- [✓ D.] $\vec{a}_Q = 0$
- E. $F = \mu N$ ✓

What do we check?

- A. $|F| \leq \mu |N|$
- [✓ B.] $\hat{F}$ opposes $\hat{v}$ or $\hat{a}$
- C. $a_{cx} = 0$
- D. $a_{Qx} = 0$
- E. $|F| = \mu |N|$



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A uniform rigid disk of mass $m = 7\ \mathrm{kg}$ and radius $r = 4\ \mathrm{m}$ starts at rest on a flat ground as shown. Force $\vec{D} = -44\hat{\imath} + 34\hat{\jmath}\ \mathrm{N}$ acts at point P on the top edge, and gravity $g = 9.8\ \mathrm{m/s^2}$ acts vertically. The coefficient of friction between the disk and the ground is $\mu = 0.25$.

What is the angular acceleration $\vec{\alpha}$ of the disk?

$\vec{\alpha} =$ ______ $\hat{k}\ \mathrm{rad/s^2}$

$\hat{F}$ opposes $\hat{a}$

$\vec{F} = 8.65\,\hat{\imath}\ N$

$\vec{N} = 34.6\,\hat{\jmath}\ N$

$\vec{a}_c = -5.05\,\hat{\imath}\ m/s^2$

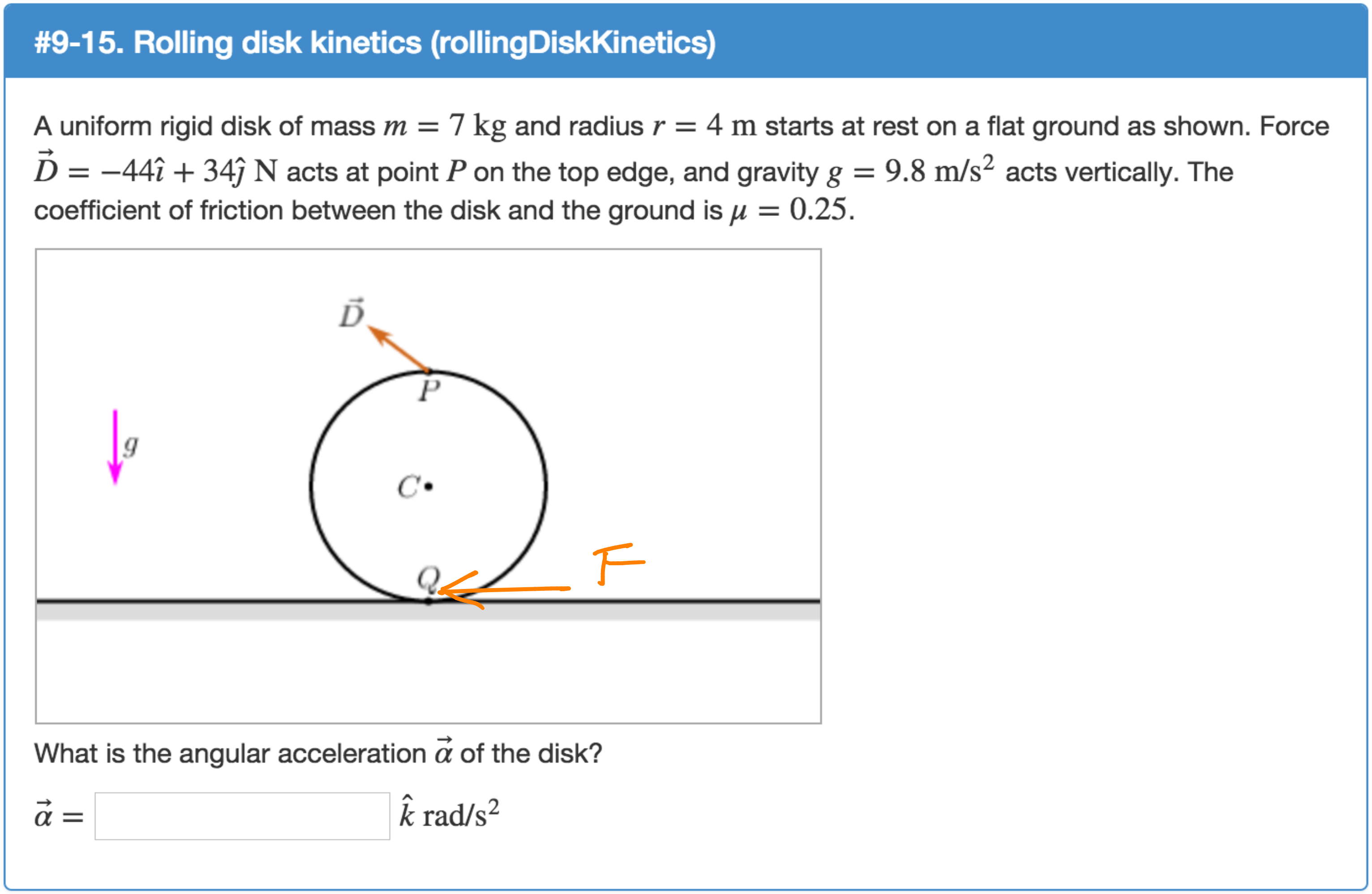
$\alpha_z = 3.76\ rad/s^2$

$\vec{a}_Q = 9.44\,\hat{\imath}\ m/s^2$

Is this valid?

A. Yes

[✓ B. No]



#9-15. Rolling disk kinetics (rollingDiskKinetics)

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What is the angular acceleration $\vec{\alpha}$ of the disk?

$\vec{\alpha} =$ ________ $\hat{k}\ \mathrm{rad/s^2}$

slip right

Which var do we NOT solve for?

A. a_{cx}

B. a_{Qx}

C. α_z

D. F

E. N

Solve for all

Which eqn do we NOT use?

A. $\sum \vec{F} = m\vec{a}_c$

[✓ B.] $\sum M_{Qz} = I_{Qz}\alpha_z$

C. $\sum M_{cz} = I_{cz}\alpha_z$

[✓ D.] $\vec{a}_Q = 0$

E. $F = \mu N$

What do we check?

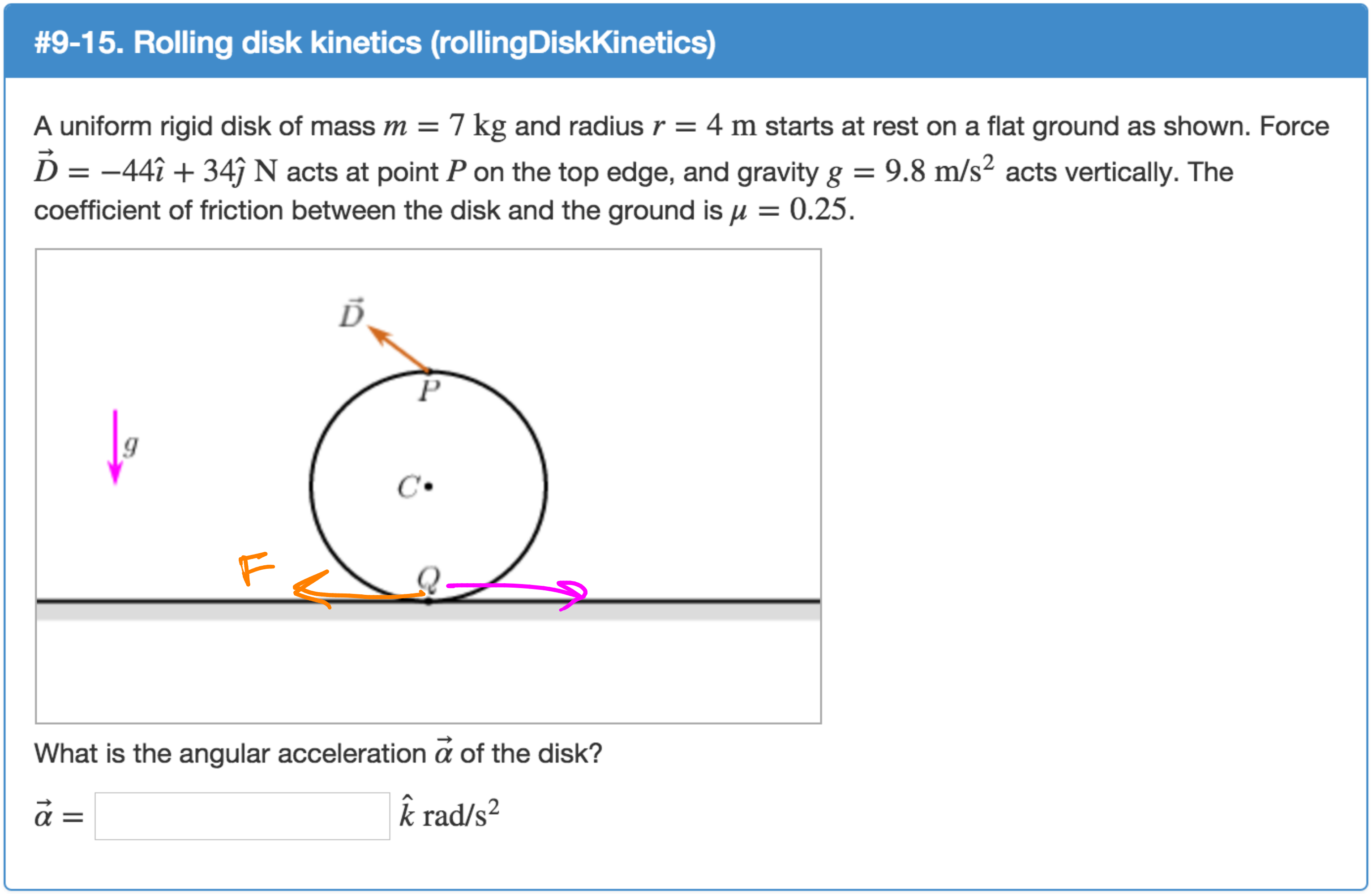
A. $|F| \leq \mu |N|$

[✓ B.] $\hat{F}$ opposes $\hat{v}$ or $\hat{a}$

C. $a_{cx} = 0$

D. $a_{Qx} = 0$

E. $|F| = \mu |N|$



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What is the angular acceleration $\vec{\alpha}$ of the disk?

$\vec{\alpha} =$ ☐ $\hat{k}$ rad/s^2

$\vec{F} = -8.65\,\hat{\imath}\ N$

$\vec{N} = 34.6\,\hat{\jmath}\ N$

$\vec{a}_c = -7.52\,\hat{\imath}\ m/s^2$

$\alpha_z = 2.52\ rad/s^2$

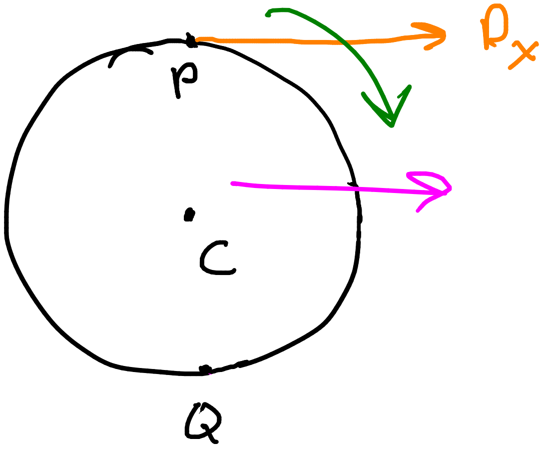
$\vec{a}_Q = 2.58\,\hat{\imath}\ m/s^2$

Valid?

[✓ A. Yes]

B. No

Ex



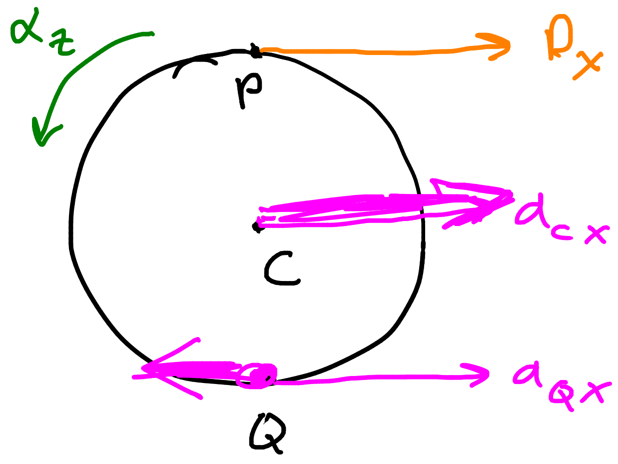
Disk starts at rest.

No gravity.

Force $D_x > 0$ is applied at P.

Q accelerates
- A. left
- B. zero
- C. right

Ex



Disk starts at rest.

No gravity.

Force $D_x > 0$ is applied at P.

Q accelerates
A. left
B. zero
C. right

$I_c = \frac{1}{2} m r^2$ $M_c = D_x r$ $\alpha = \frac{M}{I}$

$a_{cx} =$
A. $\frac{2 D_x}{m}$
[✓ B.] $\frac{D_x}{m}$
C. 0
D. $-\frac{D_x}{m}$
E. $-\frac{2 D_x}{m}$

$\frac{F}{m} = a$

$\alpha_z =$
A. $\frac{2 D_x}{m r}$
B. $\frac{D_x}{m r}$
C. 0
D. $-\frac{D_x}{m r}$
[✓ E.] $-\frac{2 D_x}{m r}$

$a_{Qx} =$
A. $\frac{2 D_x}{m}$
B. $\frac{D_x}{m}$
C. 0
[✓ D.] $-\frac{D_x}{m}$
E. $-\frac{2 D_x}{m}$