TAM 251/AL2: Final Exam. Spring 2012

NAME:

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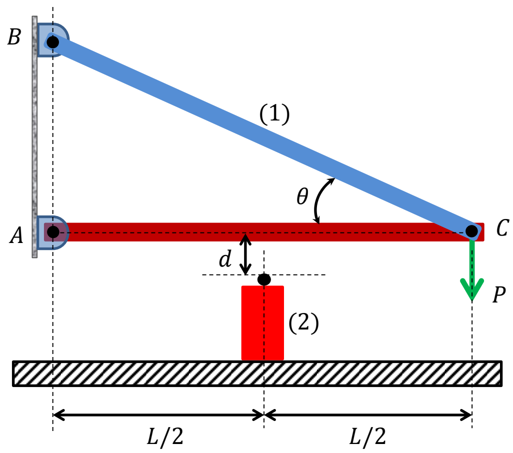
This is a closed book, closed note exam. You have 3 hours to complete it. Answer the questions in the space provided. THE REVERSE SIDE OF THE PAGES WILL NOT BE GRADED; use them for scratch calculations if you wish. PLEASE BOX YOUR ANSWERS AND WRITE YOUR NAME IN EVERY PAGE!

PROBLEM 1) Consider the structure shown in the figure below. The horizontal bar AC of length L is rigid. Bar (1) has stiffness k, cross section area $2A$ and makes an angle θ with the rigid bar. When no load is applied, a vertical gap of length d exists between the rod (2) and the rigid bar. Rod (2) has cross sectional area A, length H, Elasticity modulus E, Poisson's ration ν. The point of contact of rod (2) is located at $L/2$ from the pin A.



(a) A vertical force P is applied at C as indicated. If rod (2) is to remain <u>stress free</u>, obtain an expression for the normal stress in bar (1), denoted as σ_1. [4 points]

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(b) If rod (2) is to remain stress free, obtain an expression for the average shear stress at pin A. Assume that pin A is in double shear and has radius R. [4 points]

(c) What is the maximum load P_{max} that can be applied at C so that rod (2) remains stress free? [4 points]

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(d) If we now apply a larger load P_L such that $P_L > P_{max}$, obtain an expression for the normal stress at bar (1) and the normal stress at rod (2). [8 points]

(e) Find the normal strain and the lateral strain on rod (2) when load P_L is applied. [4 points]

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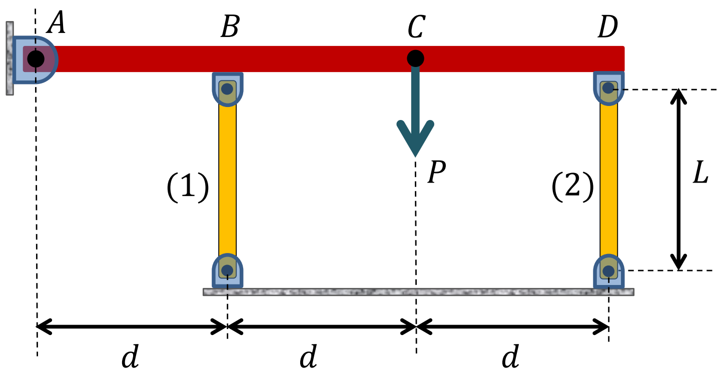
PROBLEM 2) A rigid horizontal beam AD is supported by two pin-ended columns as shown below. The columns have the same dimensions and are made of the same material (i.e., same I, E and L). Determine an expression for the load P_{cr} that will cause elastic buckling of one of the columns, and indicate which column will buckle first. [16 points]



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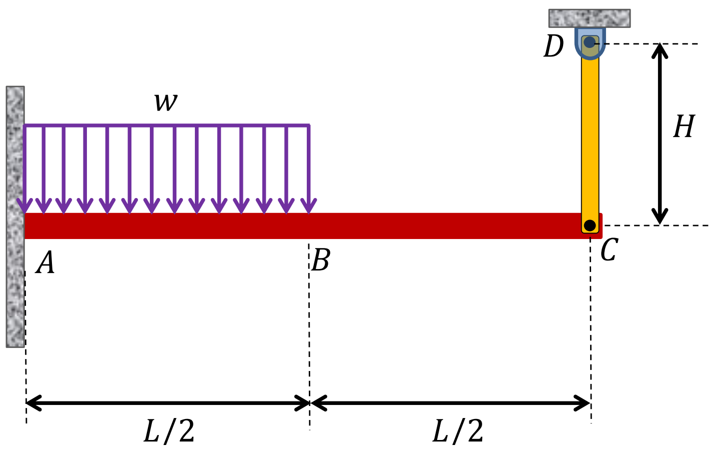
PROBLEM 3) The cantilever beam AC has additional support from cable CD. The beam has elasticity modulus E, cross-section with moment of inertia I and supports a uniform distributed load of intensity w over half of its lenght. Before the load is applied, the cable is taut, but stress free. The cable has cross-section area A and elasticity modulus E. Find an expression for the cable tension T as a function of the given variables. Use the table provided at the end of the test. [16 points]



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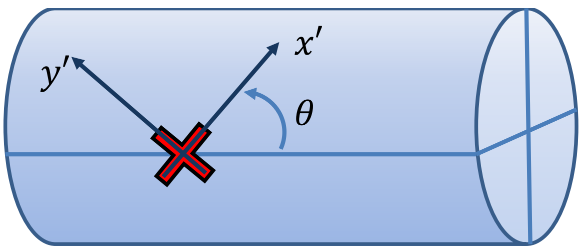
PROBLEM 4) A cylindrical pressure vessel has inner diameter $d = 500$ mm, thickeness $t = 3$ mm, elasticity modulus $E = 210$ GPa and Poisson's ratio $\nu = 0.3$. A strain gage placed on the outside surface of the vessel at an unknown angle θ gives the following measures:

$$\epsilon_{x'} = 220 \times 10^{-6} \qquad \epsilon_{y'} = 480 \times 10^{-6}$$



(a) Obtain the normal stresses $\sigma_{x'}$ and $\sigma_{y'}$. [8 points]

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(b) Using the Morh's circle, obtain the principal stresses in the outside surface of the pressure vessel. [6 points]

(c) Determine the pressure of the vessel. [4 points]

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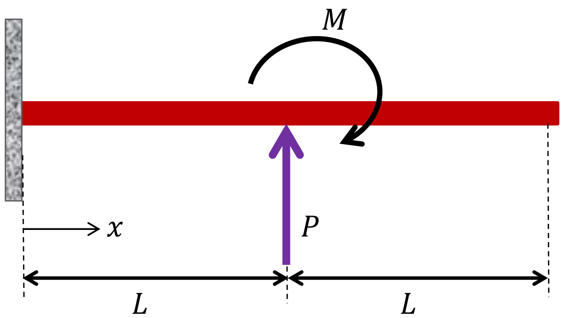
PROBLEM 5) Consider the beam of length $2L$ shown in the figure below. The beam has elasticity modulus E and moment of inertia I. A force P and a moment M are applied at midpoint (i.e., $x = L$)



(a) Determine the reactions (show them in the figure or in a free-body diagram).[4 points]

(b) Obtain an expression for the bending moment $M_1(x)$ valid for $0 \leq x < L$ and for the bending moment $M_2(x)$ valid for $L < x \leq 2L$. [6 points]

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(c) By solving the ODE $y'' = M/(EI)$, obtain the deflection $y_1(x)$ for $0 \leq x < L$ and $y_2(x)$ for $L < x \leq 2L$. Indicate clearly the boundary conditions you impose. Verify that $y_2(2L) = ...$ [8 points]

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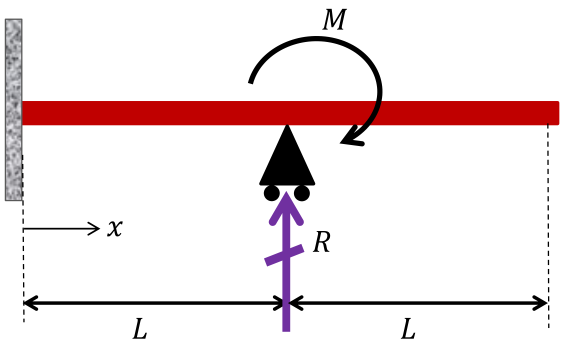
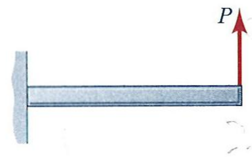
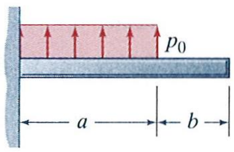
(d) Obtain an expression for the reaction R in the structure shown below. This structure is identical to that of Part (a) above, except that is has a simple support at $x = L$. Indicate any previous results that you use. [4 points]



(e) Without solving any ODE, obtain an expression for the deflection $y_2(x)$ for $L < x \leq 2L$ for the beam of Part (d) above. [4 points]

3		$v = \frac{Px^2}{6EI}(3L - x)$ $\quad v' = \frac{Px}{2EI}(2L - x)$ $\delta_B = \frac{PL^3}{3EI}$ $\quad \theta_B = \frac{PL^2}{2EI}$
6		$v = \frac{p_0 x^2}{24EI}(6a^2 - 4ax + x^2) \quad 0 \leq x \leq a$ $v' = \frac{p_0 x}{6EI}(3a^2 - 3ax + x^2) \quad 0 \leq x \leq a$ $v = \frac{p_0 a^3}{24EI}(4x - a) \quad v' = \frac{p_0 a^3}{6EI} \quad a \leq x \leq L$ $\delta_B = \frac{p_0 a^3}{24EI}(4L - a) \quad \theta_B = \frac{p_0 a^3}{6EI}$

EQUATIONS

For circular cross-section: $I_p = \frac{\pi r^4}{2}$ and $I_z = I_y = \frac{\pi r^4}{4}$

Normal stress: $\sigma_x = \frac{F}{A} + \frac{M_y\, z}{I_y} - \frac{M_z\, y}{I_z}$

Shear stress (due to torsion): $\tau = \frac{T\, r}{I_p}$

Shear stress (due to shear force): $\tau = \frac{V\, Q}{I\, t}$

Mohr's circle center:

$$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2}$$

Mohr's circle radius:

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + (\tau_{xy})^2}$$

Relations among distributed load, shear and bending moment:

$$\frac{dV}{dx} = -w$$

$$\frac{dM}{dx} = V$$