Chapter 10: Theories of Failure

Chapter Objectives

✓Theories of failure for ductile and brittle materials
✓Predict failure of material

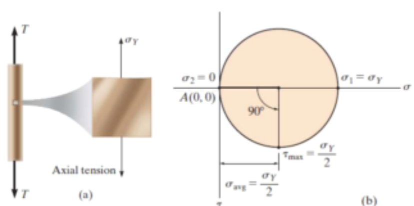
Theories of failures for ductile materials

If a material is **ductile**, failure is specified by the initiation of yielding

Maximum-shear-stress criterion (Tresca criterion)

- Typically, **ductile** materials fail by shear stress (slipping that occurs between the crystals of microstructure)
- Yielding begins when the absolute maximum shear stress reaches the shear stress that causes the same material to yield when subject only to axial tension

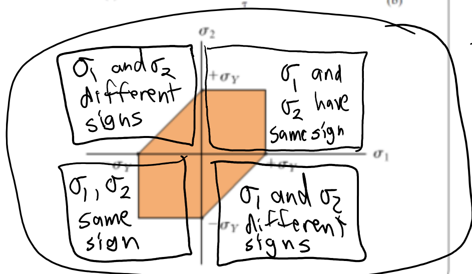
$$\boxed{\tau_{max}^{abs} = \frac{\sigma_Y}{2}}$$



Failure occurs when:

$$\left.\begin{array}{l}|\sigma_1| = \sigma_Y \\ |\sigma_2| = \sigma_Y\end{array}\right\} \sigma_1, \sigma_2 \text{ have same signs}$$

$$|\sigma_1 - \sigma_2| = \sigma_Y\} \quad \sigma_1, \sigma_2 \text{ have opposite signs}$$

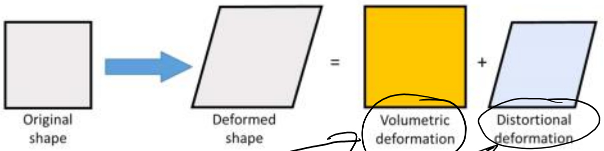


any load where (σ_1, σ_2) is outside the shaded area ⇒ yielding failure!

Theories of failures for ductile materials (cont.)

Maximum-distortion-energy criterion (Von Mises criterion)

All elastic deformations can be broken down into a combination of volumetric and distortional deformations:

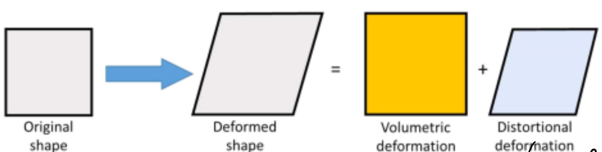


The total strain energy W (analogous to stored potential energy in a spring) in the material is the sum of the volumetric energy W_v and distortional energy W_d associated with each fundamental deformation mode: $W = W_v + W_d$

Experimental evidence suggests that ductile materials do not fail due to states of stress which impose a volume change only (e.g. pure pressure loading – metals in deep ocean waters, rocks deep beneath earth's crust, etc, due not fail even though they experience high pressure): therefore, **the distortion strain energy is hypothesized to govern failure**

Theories of failures for ductile materials (cont.)

Maximum-distortion-energy criterion (Von Mises criterion)

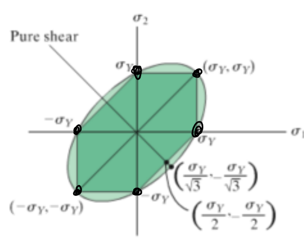


- For plane stress: $W_d = \frac{1+\nu}{3E}(\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2)$ } 2D loading (in-plane)
- For uniaxial tensile test, at the moment of yield: $W_{d,yield} = \frac{1+\nu}{3E}\sigma_Y^2$ } 1D material testing experiments

Equating these expressions gives the condition at yield:

$$\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 = \sigma_Y^2$$

plane stress



Theories of failures for brittle materials

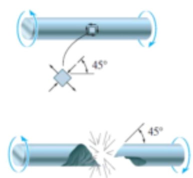
If a material is brittle, failure is specified by fracture

Maximum-normal-stress criterion

- The fracture of a **brittle** material is caused only by the maximum tensile stress in the material, and not the compressive stress.
- Maximum principle stress σ_1 in the material reaches a limiting value that is equal to the ultimate normal stress the material can sustain when it is subjected to simple tension.

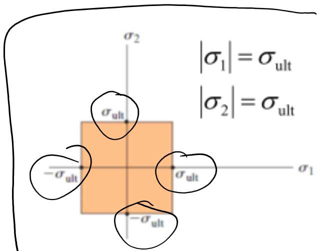
Failure of a brittle material in tension

(a)



Failure of a brittle material in torsion

(b)

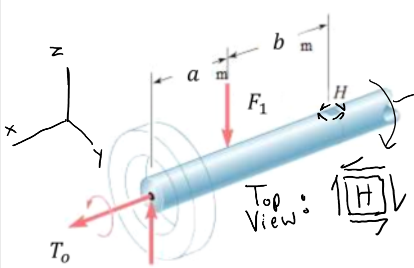


$$|\sigma_1| = \sigma_{ult}$$
$$|\sigma_2| = \sigma_{ult}$$

- Due to material imperfections, tensile fracture of a brittle material is difficult to predict, and so theories of failure for brittle materials should be used with caution

Example: The axle of an automobile is subjected to the forces and couple shown. Knowing the diameter of the solid axle is D = 32 mm and the yield strength of the material is $\sigma_Y = 500$ MPa, determine

(a) The principal stresses at point H located on the top of the axle
(b) The maximum shear stress at the same point
(c) Whether failure occurs at point H according to the von Mises criterion
(d) Whether failure occurs at point H according to the Tresca criterion



$T_x = T_o$

$F_1 = 3\ KN$ $\quad a = 0.15\ m$

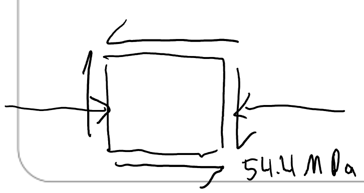
$T_o = 350\ N.m$ $\quad b = 0.2\ m$

Top View:

$$T_x:\ \tau_{xy} = \frac{-T_o\left(D/2\right)}{J} = \frac{-T_o \cdot \frac{D}{2}}{\frac{\pi}{32}D^4} = \frac{-16T_o}{\pi D^3} = -54.4\ MPa$$

$$M_y:\ \sigma_x = \frac{-(F_1 \cdot a)\left(\frac{D}{2}\right)}{I} = \frac{-F_1 \cdot a \cdot D/2}{\frac{\pi}{64}D^4} = \frac{-32F_1 a}{\pi D^3} = -139.9\ MPa$$

$$\sigma_y = 0$$
$$\sigma_z = 0$$



$$\sigma_{avg} = 70\ MPa$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 88.6\ MPa$$

$$\sigma_1 = 18.6\ MPa$$

(a) $\begin{cases} \sigma_1 = 18.6\ MPa \\ \sigma_2 = \sigma_{avg} - R = -158.6\ MPa \end{cases}$

(b) $|\tau_{max}| = R = 88.6\ MPa$

(c) $\sigma_1^2 + \sigma_2^2 - \sigma_1 \times \sigma_2 = \sigma_Y^2$

$$\sqrt{(18.6)^2 + (-158.6)^2 - (18.6)(-158.6)}\ MPa = 168.7\ MPa < \sigma_Y = 500\ MPa$$

$\Rightarrow$ Material does not fail, according to the V-M criterion

(d) σ_1 and σ_2 have opposite sign

$$\Rightarrow |\sigma_1 - \sigma_2| = |18.6 - (-158.6)|\ MPa = 177.2\ MPa$$

$$177.2\ MPa < 500\ MPa$$

$\Rightarrow$ Mat'l does not fail!