

Chapter 10: Theories of Failure

Chapter Objectives

- ✓ Theories of failure for ductile and brittle materials
- ✓ Predict failure of material

Theories of failures for ductile materials

If a material is **ductile**, failure is specified by the initiation of yielding

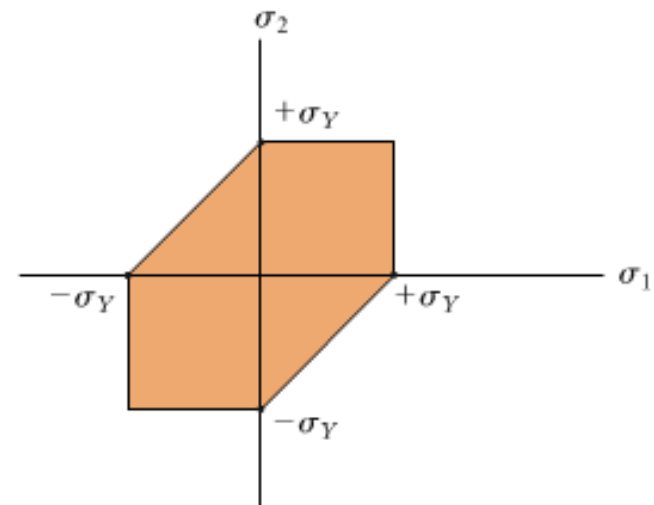
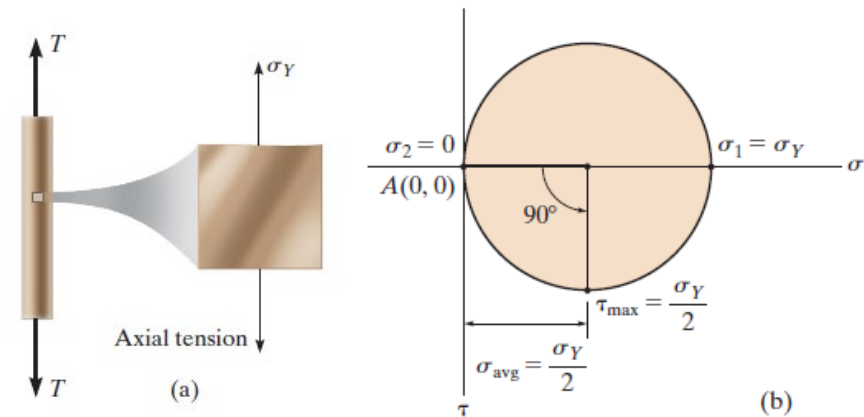
Maximum-shear-stress criterion (Tresca criterion)

- Typically, **ductile** materials fail by shear stress (slipping that occurs between the crystals of microstructure)
- Yielding begins when the absolute maximum shear stress reaches the shear stress that causes the same material to yield when subject only to axial tension

$$\tau_{max}^{abs} = \frac{\sigma_Y}{2}$$

Failure occurs when:

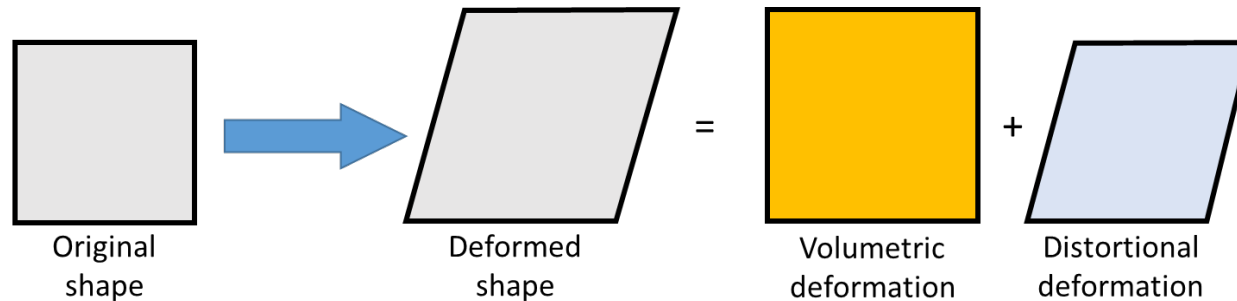
$$\left. \begin{array}{l} |\sigma_1| = \sigma_Y \\ |\sigma_2| = \sigma_Y \end{array} \right\} \sigma_1, \sigma_2 \text{ have same signs}$$
$$|\sigma_1 - \sigma_2| = \sigma_Y \quad \sigma_1, \sigma_2 \text{ have opposite signs}$$



Theories of failures for ductile materials (cont.)

Maximum-distortion-energy criterion (Von Mises criterion)

All elastic deformations can be broken down into a combination of volumetric and distortional deformations:

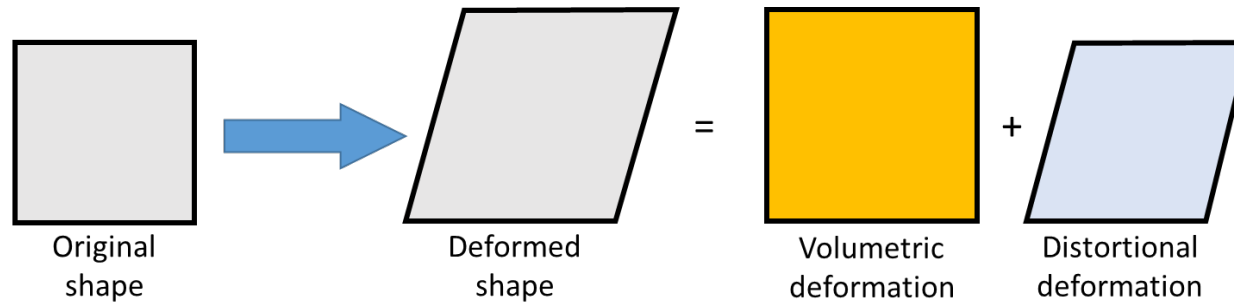


The total strain energy W (analogous to stored potential energy in a spring) in the material is the sum of the volumetric energy W_v and distortional energy W_d associated with each fundamental deformation mode: $W = W_v + W_d$

Experimental evidence suggests that ductile materials do not fail due to states of stress which impose a volume change only (e.g. pure pressure loading – metals in deep ocean waters, rocks deep beneath earth's crust, etc, due not fail even though they experience high pressure): therefore, **the distortion strain energy is hypothesized to govern failure**

Theories of failures for ductile materials (cont.)

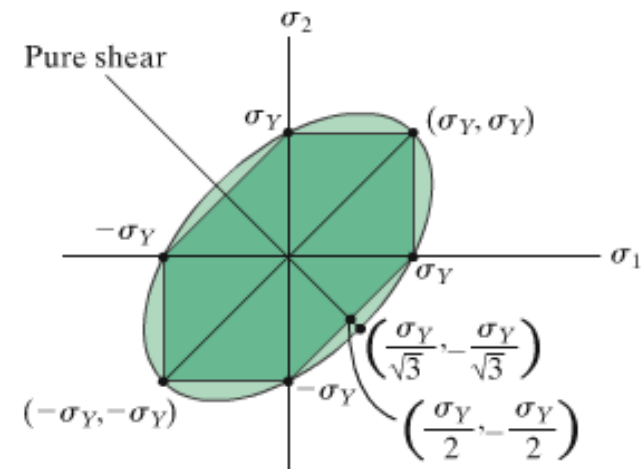
Maximum-distortion-energy criterion (Von Mises criterion)



- For plane stress: $W_d = \frac{1+\nu}{3E} (\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2)$
- For uniaxial tensile test, at the moment of yield: $W_{d,yield} = \frac{1+\nu}{3E} \sigma_Y^2$

Equating these expressions gives the condition at yield:

$$\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 = \sigma_Y^2$$

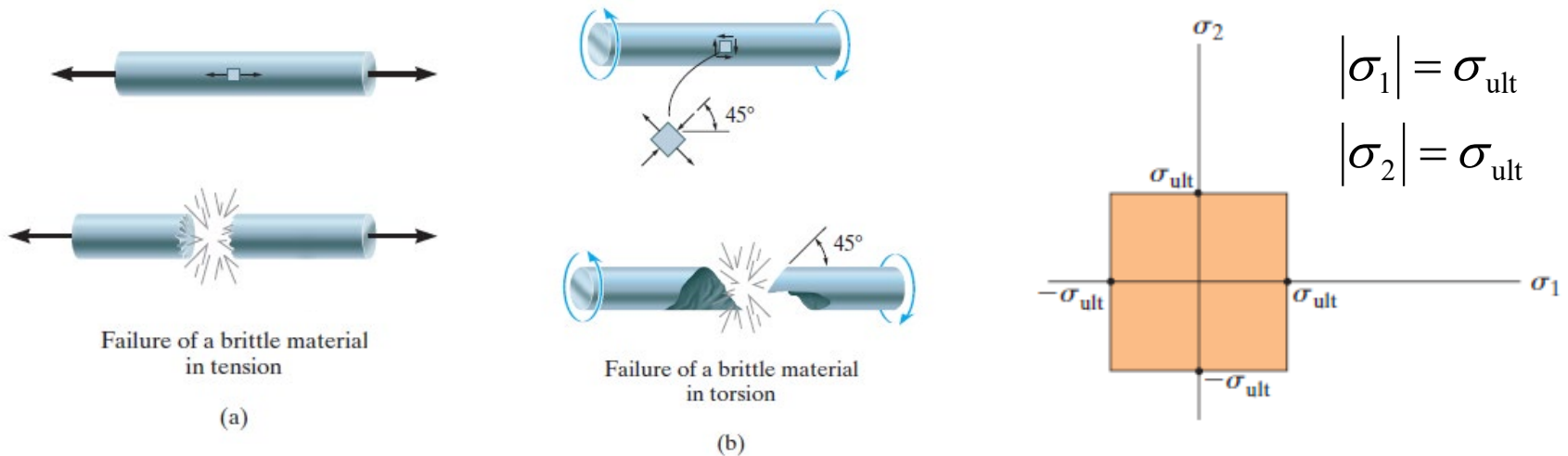


Theories of failures for brittle materials

If a material is brittle, failure is specified by fracture

Maximum-normal-stress criterion

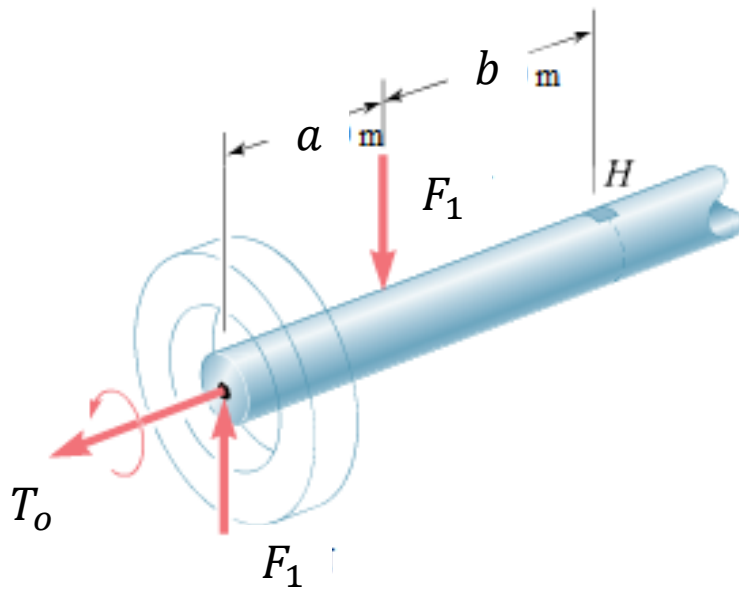
- The fracture of a **brittle** material is caused only by the maximum tensile stress in the material, and not the compressive stress.
- Maximum principle stress σ_1 in the material reaches a limiting value that is equal to the ultimate normal stress the material can sustain when it is subjected to simple tension.



- Due to material imperfections, tensile fracture of a brittle material is difficult to predict, and so theories of failure for brittle materials should be used with caution

Example: The axle of an automobile is subjected to the forces and couple shown. Knowing the diameter of the solid axle is $D = 32 \text{ mm}$ and the yield strength of the material is $\sigma_Y = 500 \text{ MPa}$, determine

- The principal stresses at point H located on the top of the axle
- The maximum shear stress at the same point
- Whether failure occurs at point H according to the von Mises criterion
- Whether failure occurs at point H according to the Tresca criterion



$$F_1 = 3 \text{ KN}$$

$$a = 0.15 \text{ m}$$

$$T_o = 350 \text{ N.m}$$

$$b = 0.2 \text{ m}$$