TAM251_Chapter1_Stress_prelecture_Johnson

Sunday, June 13, 2021 11:39 PM

TAM251_Chapter1_Stress_prelecture_Johnson

Chapter 1: Stress

Chapter Objectives

- ✓ Understand concepts of normal and shear stress
- ✓ Analyze and design with axial (normal) and shear loads

Review of statics - Equilibrium

1) External Loads

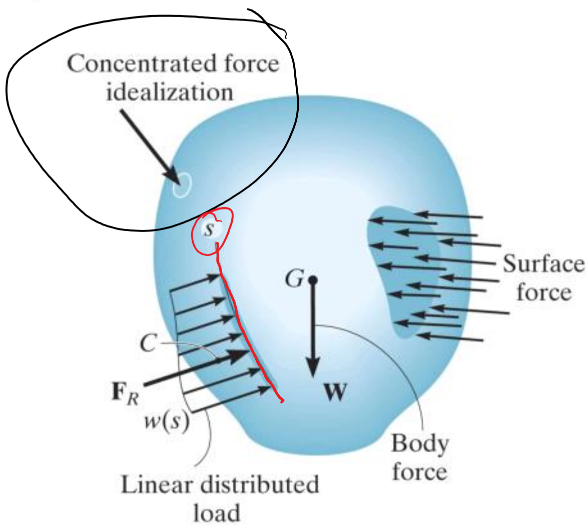
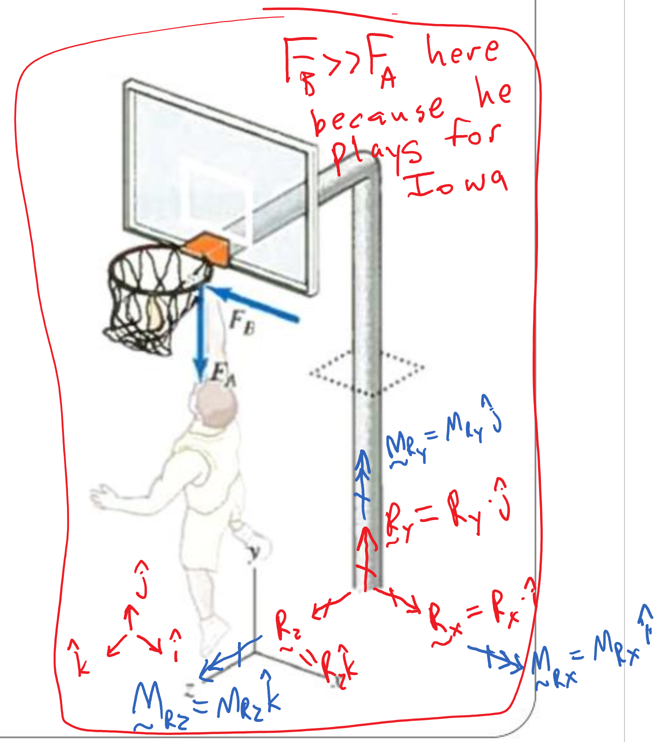
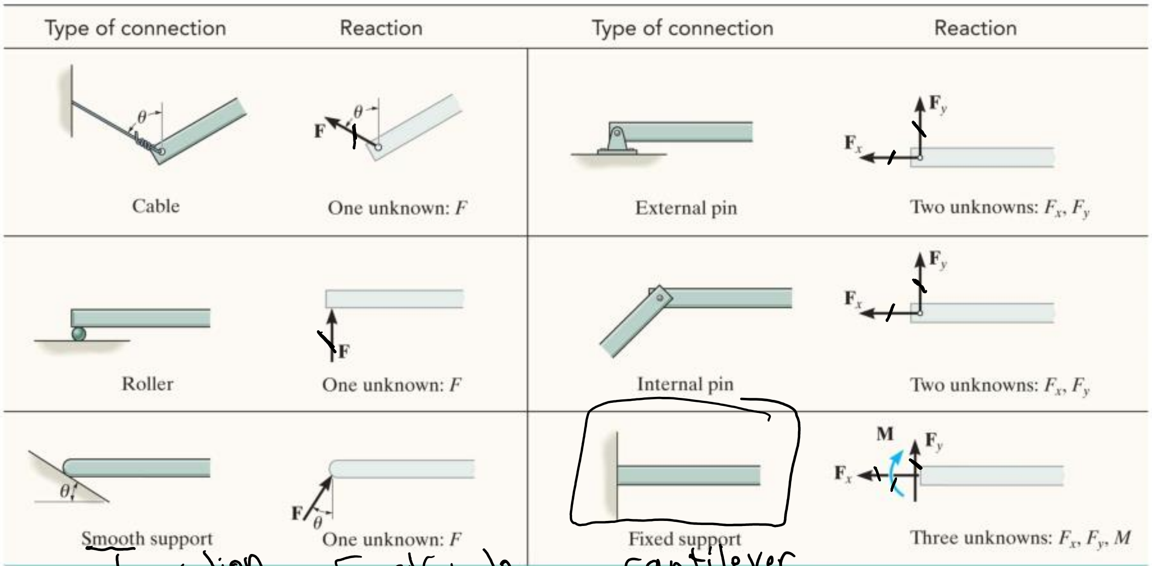


Figure: 01_01



2) Support reactions

Reactions occur where motion is constrained.



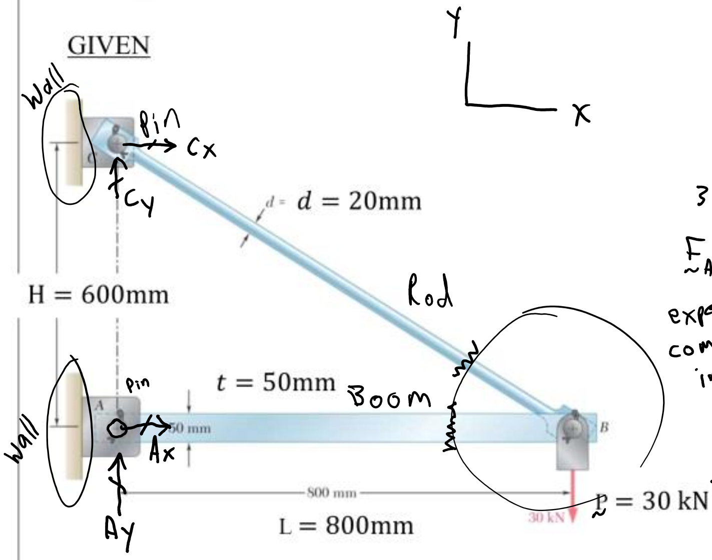
friction is small (neglect)

F acts normal to the surface

cantilever beam cannot rotate at the fixed support ⇒ Reaction moment constrains against rotation at the support.

Example 1

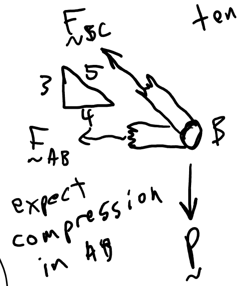
GIVEN



FIND

(a) Internal forces in the boom and rod ✓

(b) Reactions at A & C



expect tension in BC

expect compression in AB

$\Sigma F_y = 0$

$-P + \frac{3}{5} F_{BC} = 0$

$\boxed{F_{BC} = \frac{5}{3} P}$

Note $F_{BC} > 0 \Rightarrow$ Tension

$F_{BC} = 50$ kN

$\Sigma F_x = 0 \Rightarrow -F_{AB} - \frac{4}{5} F_{BC} = 0$

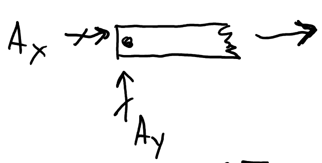
$F_{AB} = -\frac{4}{5} F_{BC} = -\frac{4}{5}\left(\frac{5}{3} P\right)$

$\boxed{F_{AB} = -\frac{4}{3} P}$

$F_{AB} = -40$ kN

$F_{AB} < 0 \Rightarrow$ compression

Find Reactions at A and C



$F_{AB} = -40 \text{kN}\ \hat{i}$

$= -F_{AB} \cdot \hat{i}$

$\Sigma F_y = 0$

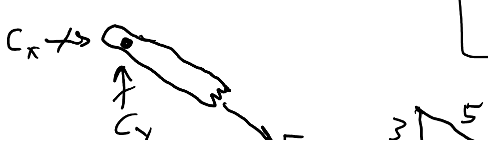
$\boxed{A_y = 0}$

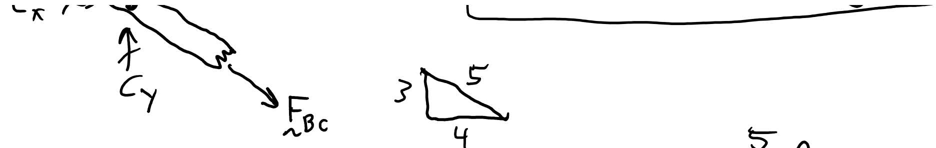
$\Sigma F_x = 0 \Rightarrow A_x + F_{AB} = 0$

$A_x = -F_{AB}$

$A_x = -\left(-\frac{4}{3} P\right) = 40$ kN

$\boxed{\begin{array}{l} A_x = \frac{4}{3} P \text{ to the right} \\ A_x = 40 \text{ kN to the right} \end{array}}$





$$F_{BC} = 50\,kN = \frac{5}{3}P$$

$$\Sigma F_y = 0 \Rightarrow C_y - F_{BC} \cdot \frac{3}{5} = 0$$

$$C_y = \frac{3}{5} F_{BC} = \frac{3}{5}\left(\frac{5}{3}P\right)$$

$$\boxed{C_y = P} \quad \text{acts upward}$$

$$C_y = 30\,kN \text{ upward}$$

$$\Sigma F_x = 0 \Rightarrow C_x + F_{BC} \cdot \frac{4}{5} = 0$$

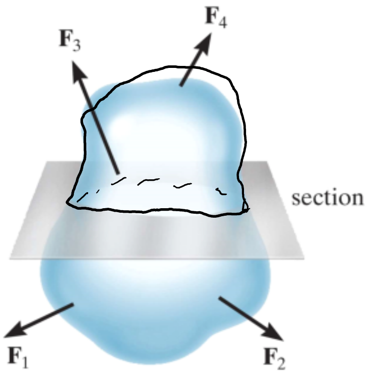
$$C_x = -\frac{4}{5} F_{BC} = -\frac{4}{5}\left(\frac{5}{3}P\right)$$

$$\boxed{C_x = -\frac{4}{3}P}$$

$$C_x = \frac{4}{3}P \text{ acting to the left}$$

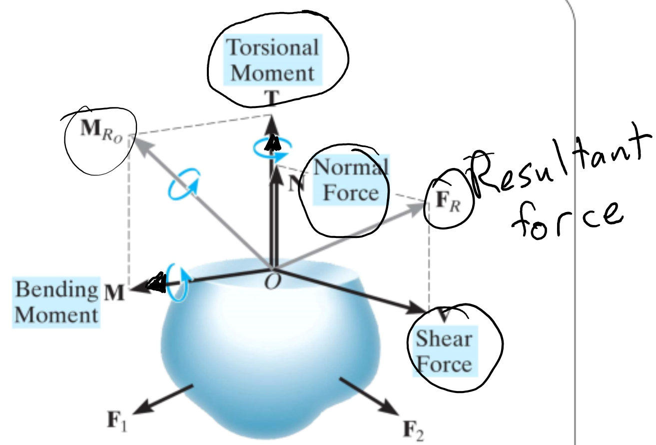
$$C_x = 40\,kN \text{ to the left}$$

Equilibrium and Free-body diagram



(a)

Figure: 01_02a



(d)

Figure: 01_02d

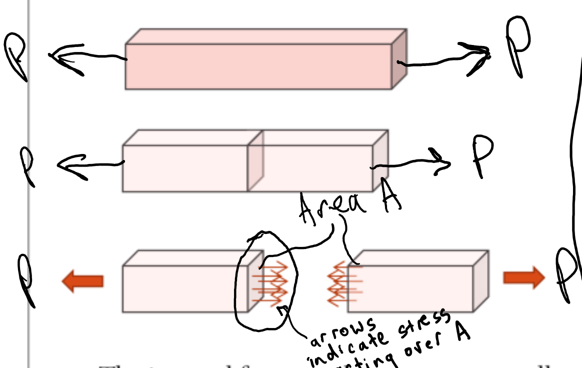
M_{R_O} = Torsional Moment + Bending Moment

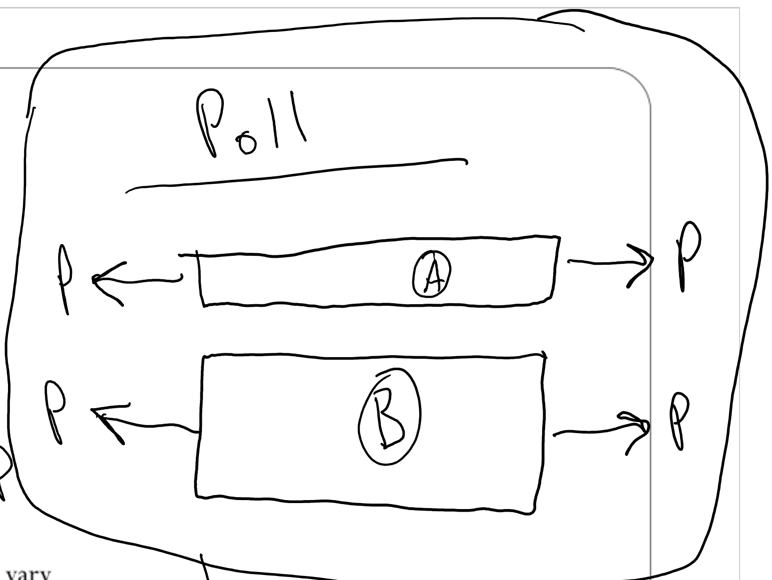
F_R = Normal force + Shear force

Statics course → assume rigid bodies

Now, we assume that bodies are deformed under the actions of forces!

Stress





- The internal forces and moments generally vary from point to point.
- Obtaining this distribution is of primary importance in mechanics of materials.
- The total force in a cross-section, divided by the cross-sectional area, is the **stress**
- We use stress to **normalize forces** with respect to the size of the geometry

normal force

$\sigma = \frac{P}{A}$

normal

In which is the stress greater?

A !

cross-sectional area, is the **stress**

- We use stress to **normalize forces** with respect to the size of the geometry

σ ← normal stress

given A !

Average normal stress – axial loading

St. Venant's Principle
– far from where the component is loaded, the stress state becomes uniform

$\sigma > 0$: tension
$\sigma < 0$: compression

$$\boxed{\sigma_{avg}} = \frac{F_{Rod}}{A_{Rod}} = \frac{50\,kN}{\frac{\pi}{4}(20\,mm)^2}$$

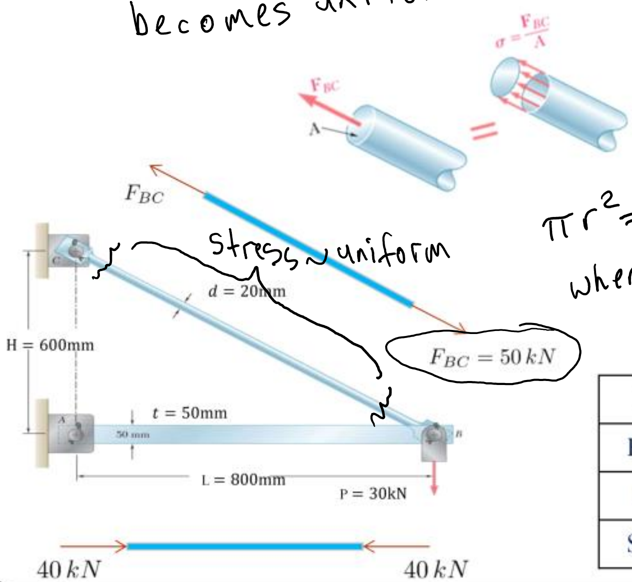
$$= \frac{50{,}000\,N}{\frac{\pi}{4}(0.020\,m)^2}$$

$$= 159 \times 10^6\ N/m^2$$

$$= \boxed{159\ MPa}$$

Positive stress ⇒ tension

$\pi r^2 = \frac{\pi}{4} d^2$ where $d = 2r$



Units	SI sytem	BG system (US)
FORCE	[N]	[lb]
AREA	[m^2]	[in^2]
STRESS	[Pa]=[N/m^2]	[psi]=[lb/in^2]

kPa
MPa
GPa

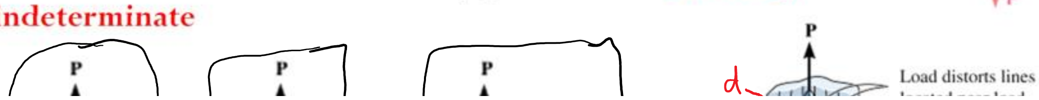
$ksi = \frac{kips}{in^2} = 1000\,psi$

Average normal stress – axial loading

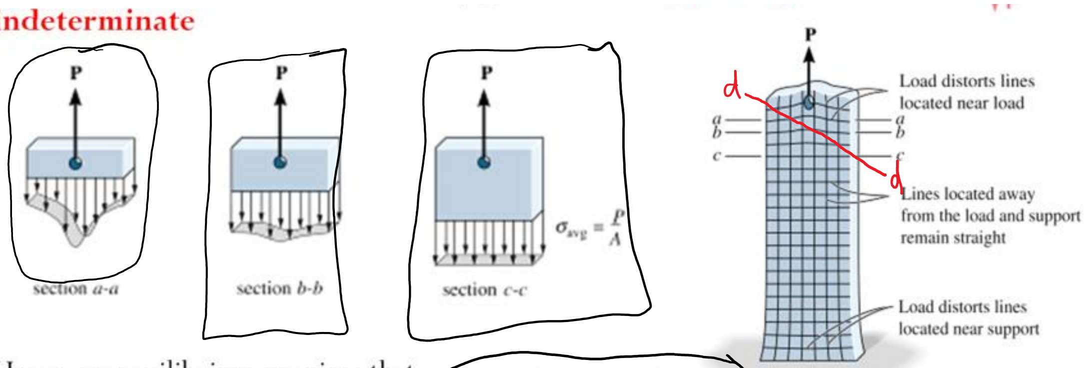
- We should note that $\sigma = \frac{F}{A}$ is the average value of the stress over the cross-sectional area, not the stress at a specific point of the cross section
- Recall that the stress at any given point Q of the cross section is given by

$$\sigma = \lim_{\Delta A \to 0} \frac{\Delta F}{\Delta A}$$

- The actual distribution of stresses in any given section is **statically indeterminate**



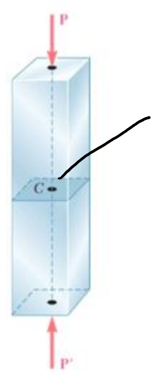
indeterminate



- However, equilibrium requires that $P = \int dF = \int \sigma dA$ ← true at any cross-section

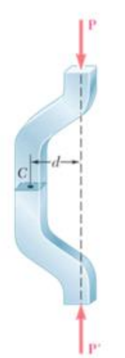
Average normal stress – axial loading

- Here we assume that the **distribution of normal stresses** in an axially loaded member is **uniform**
- Stress is calculated away from the points of application of the concentrated loads
- Uniform distribution of stress is possible only if the line of action of the concentrated load P passes through the centroid of the section considered



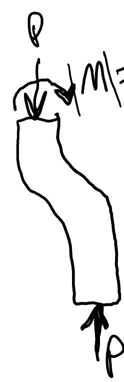
C is at the centroid

Centric axial loading
(stress distribution is uniform)



Internal Bending Moment + Normal Force

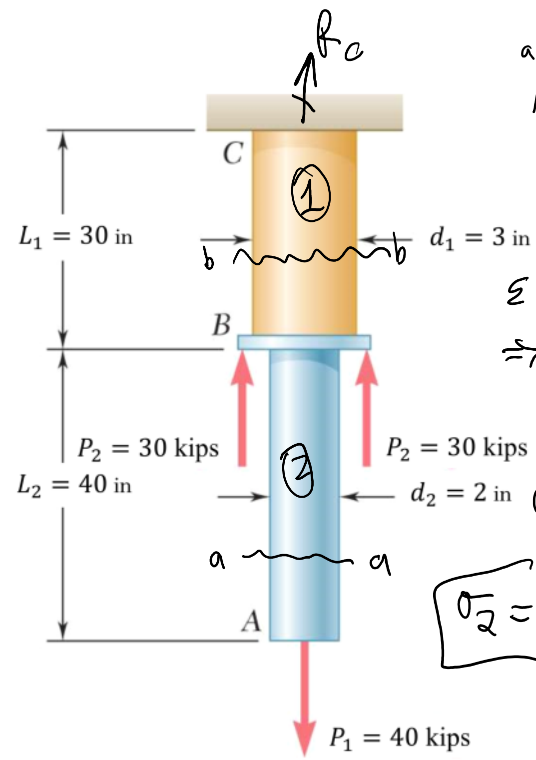
Eccentric axial loading
(stress distribution is not uniform)

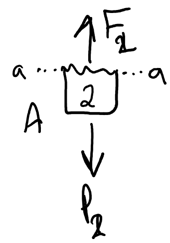


$|M| = |P \cdot d|$

Example 2

Obtain the normal stresses in each rod





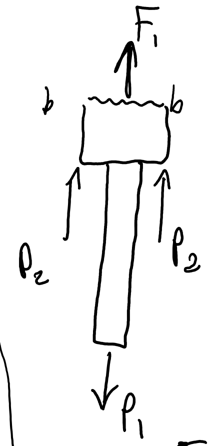
$\Sigma F_y = 0$

$\Rightarrow F_2 - P_1 = 0$

$F_2 = P_1$

$$\sigma_2 = \frac{F_2}{A_2} = \frac{P_1}{\frac{\pi}{4} d_2^2}$$

$$\boxed{\sigma_2 = \frac{40\text{ kips}}{\frac{\pi}{4}(2\text{ in})^2} = 12.74\text{ ksi}}$$



$\Sigma F_y = 0$

$F_1 + 2P_2 - P_1 = 0$

$F_1 = P_1 - 2P_2$

$F_1 = -20$ kips
(compression)

$$\sigma_1 = \frac{F_1}{\frac{\pi}{4} d_1^2}$$

$$\sigma_1 = \frac{-20\text{ kips}}{\frac{\pi}{4}(3\text{ in})^2}$$

$$\boxed{\sigma_1 = -2.83\text{ ksi}}$$

Average Shear stress

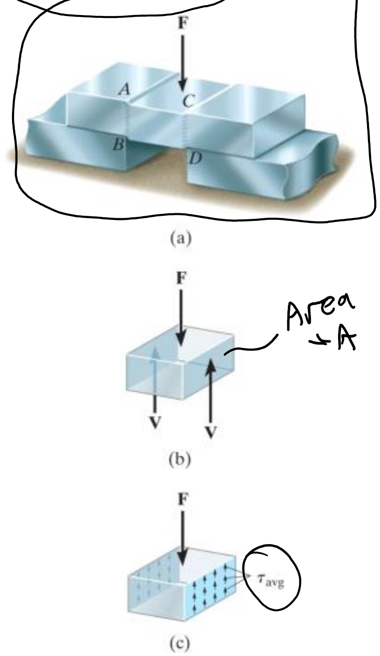


Figure: 01_19

- Obtained when transverse forces are applied to a member
- The distribution of shear stresses <u>cannot</u> be assumed uniform
- Common in bolts, pins and rivets used to connect various structural members

$$\Sigma F_z = -F + 2V = 0$$

$$V = \frac{1}{2}F$$

$$\tau_{avg} = \frac{V}{A} = \frac{F}{2A}$$

τ_{ave}: average shear stress
V: shear force
A: cross section area

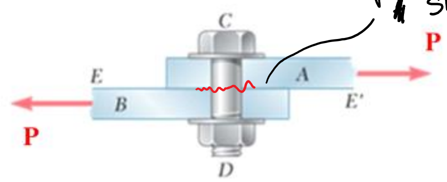
In reality (for beams in bending)

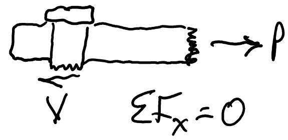
parabolic distribution

$$\tau_{max} = \frac{3}{2}\tau_{avg}$$

Single Shear

pin carries shear at one location



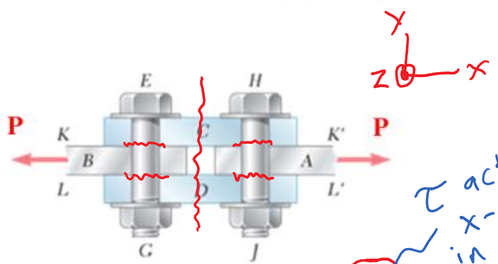


$$\Sigma F_x = 0$$

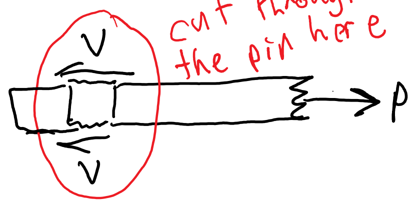
$$P - V = 0 \Rightarrow V = P$$

$$\boxed{\tau_{avg} = \frac{V}{A} = \frac{P}{A}}$$

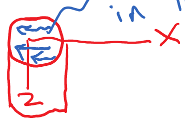
Double Shear

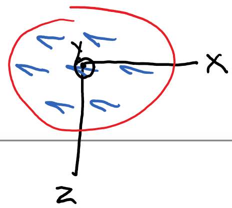


cut through the pin here



τ acts in the x-direction in the x-z plane





$$\Sigma F_x = 0$$

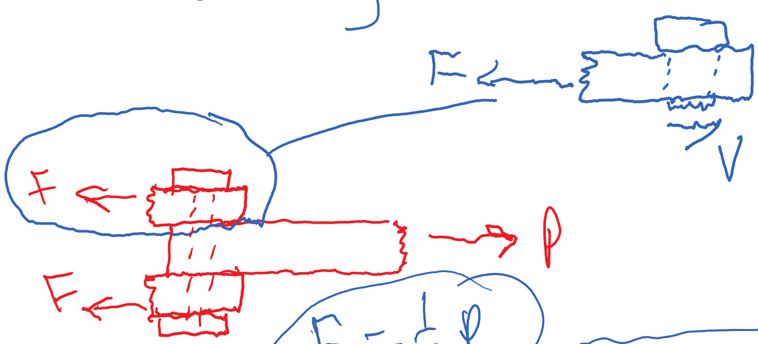
$$P - 2V = 0$$

$$V = \tfrac{1}{2}P$$

$$\boxed{\tau_{avg} = \frac{V}{A} = \frac{P}{2A}}$$

pin less likely to fail than in single shear

Alternatively, we could do a FBD with the following cut:



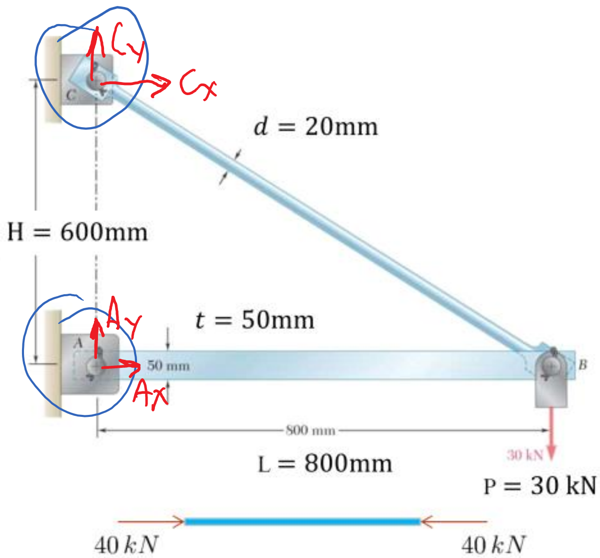
$$F = V$$

$$F = \tfrac{1}{2}P$$

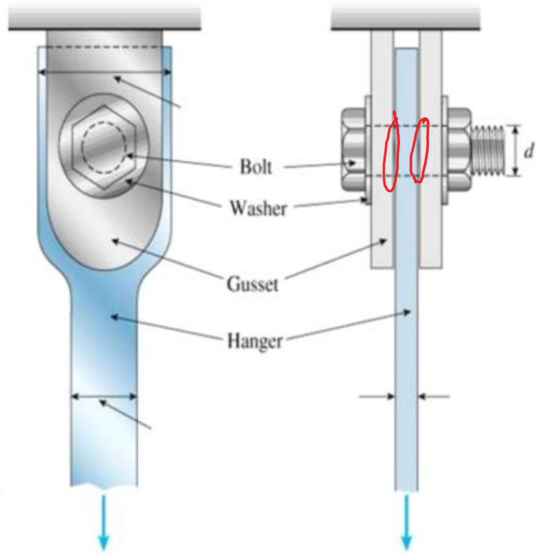
$$V = \tfrac{1}{2}P$$

Example 3

Given:



Pins at A and C



Is this
(A) single shear
(B) double shear

Find: Shear stresses in pins A & C

From example

$A_x = 40\text{ kN}$ $A_y = 0$

$C_x = -40\text{kN}$ $C_y = 30\text{kN}$

$|F_A| = \sqrt{A_x^2 + A_y^2} = 40\text{kN}$

$|F_C| = \sqrt{C_x^2 + C_y^2} = 50\text{kN}$

$V_{pin,A} = \frac{F_A}{2} = 20\text{kN}$ but in two locations along the pin b.c. double shear

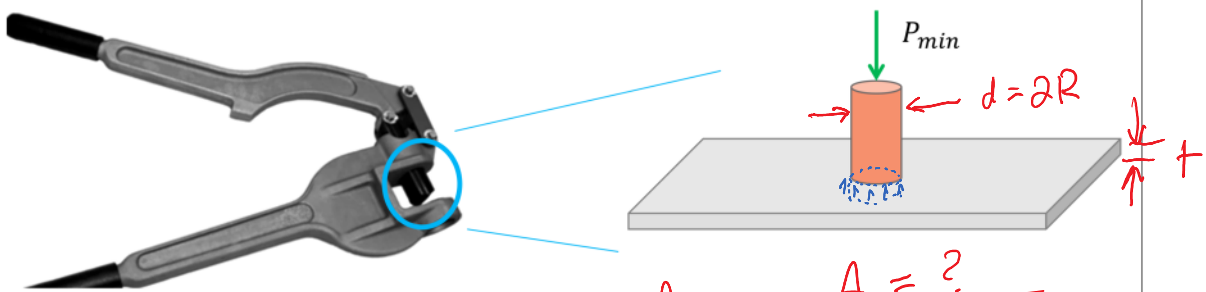
$\tau_{pin,A} = \frac{20\text{kN}}{\frac{\pi}{4} d_{pin}^2}$

$V_{pin,C} = \frac{F_c}{2} = 25\text{kN}$, double shear

$\tau_{pin,C} = \frac{25\text{kN}}{\frac{\pi}{4} d_{pin}^2}$

Example 4 http://www.youtube.com/watch?v=9sMXItQjHkE

A cylindrical punch of radius R is used to perforate a hole in a metal plate of thickness t. If τ_{max} is the maximum shear stress that the metal will sustain before breaking, what is the minimum force P_{min} that must be applied on the punch in order to perforate the paper?



Puncture occurs when $\tau = \tau_{max} = \dfrac{P_{min}}{A}$

property of the steel stud

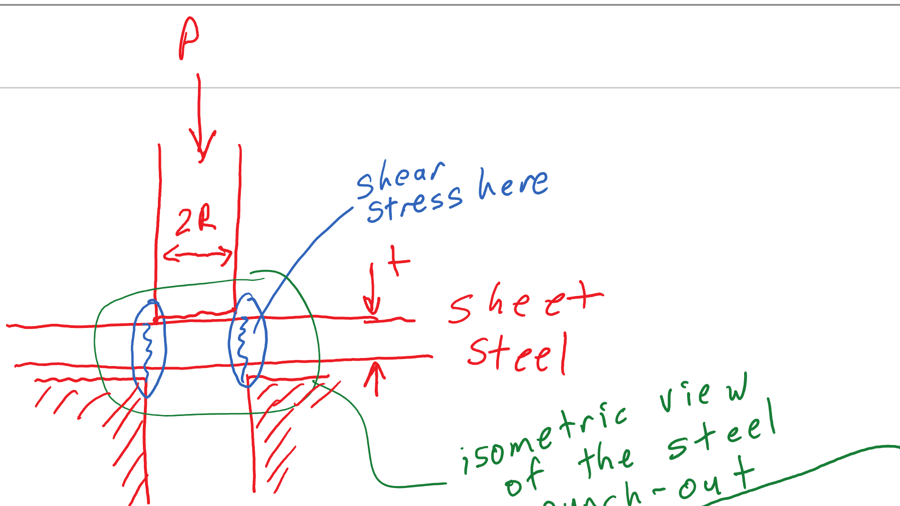
$A = ?$

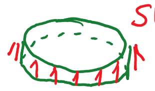
A) πR^2

B) $2\pi R^2$

C) $2\pi R t$

D) $\pi R^2 t$





Shear stresses act around the punch-out on this cylindrical surface

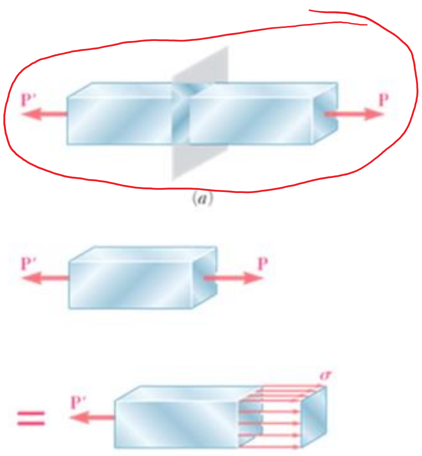
surface Area of a cylindrical surface $= \underbrace{(2\pi R)}_{\text{circumference}} \cdot t$ "Height of the cylinder" of cylinder

Stress on an oblique plane under axial loading

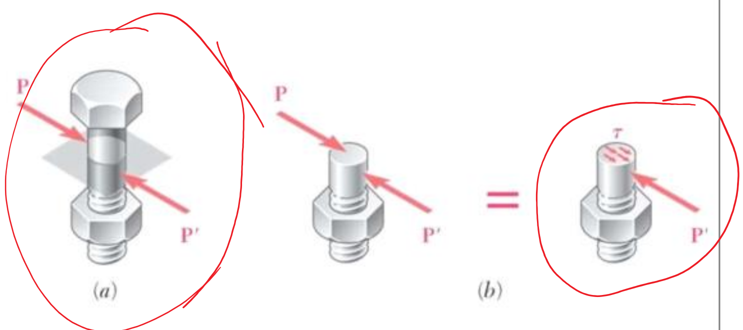
So far…

- Axial forces: NORMAL STRESS
- Transverse forces: SHEAR STRESS

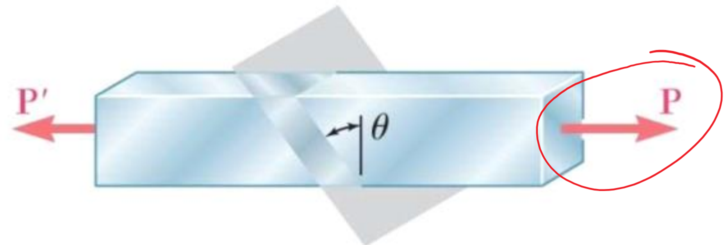
➡ This relation is observed only on planes perpendicular to the axis of the member or connection

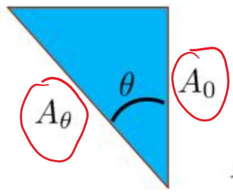


(a) (b)

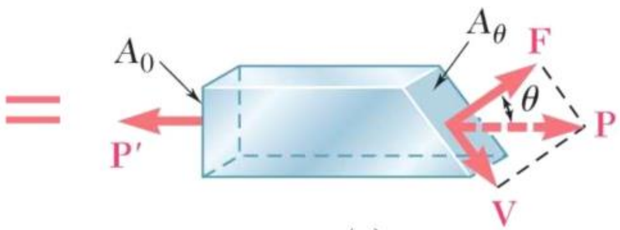


(a) (b)





$A_\theta = \frac{A_0}{\cos\theta}$

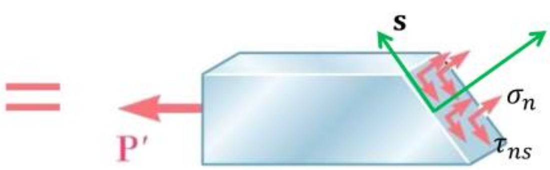


(c)

Resolving P into components F and V

F is normal to the oblique plane

V is parallel to the oblique plane



(d)

$F = P \cdot \cos\theta$

$\sigma_n = \frac{F}{A_\theta} = \frac{P \cdot \cos\theta}{A_0 / \cos\theta}$

$\sigma_n = \frac{P}{A_0} \cdot \cos^2\theta$

$V = P \cdot \sin\theta$

$\tau_{ns} = \frac{V}{A_\theta} = \frac{P \sin\theta}{A_0/\cos\theta}$

$\tau_{ns} = \frac{P}{A_0} \sin\theta \cdot \cos\theta$

$$\boxed{\sigma_n = \sigma_0 \cdot \cos^2\theta \qquad \tau_{ns} = \sigma_0 \cdot \underbrace{\sin\theta \cdot \cos\theta}}$$

$\sin\theta \cdot \cos\theta$ is maximized at $\theta = 45°$

Design of structures

- **<u>Design Requirement</u>**: A structural design is intended to support and/or transmit loads while maintaining safety and utility: ***don't break***
- <u>Strength</u> of a structure reflects its ability to resist failure.
- **<u>Ultimate Load</u>** (P_u): force when specimen fails (breaks).
- **<u>Ultimate normal stress</u>** (σ_u):

$$\sigma_u = \frac{P_u}{A}$$

- A structure is safe if its strength exceeds the required strength
- Factor of Safety:
  Ratio of structural strength to maximum (allowed) applied load (P_{all})

all = allowed

F.S. > 1 always

typically F.S. ≥ 2

FS = factor of safety

$$\boxed{P_{all} = \frac{P_u}{FS}}$$

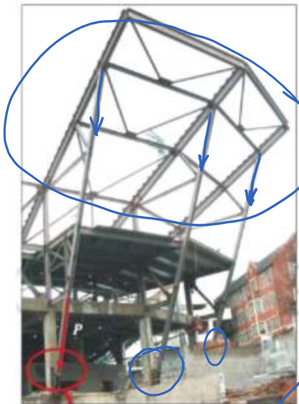
Allowable stress design

$$\sigma_{all} = \frac{\sigma_u}{FS}$$

$$\tau_{all} = \frac{\tau_u}{F.S.}$$

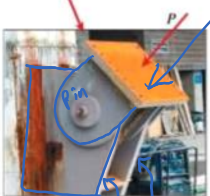
Example 5 The upper deck of a football stadium is supported by braces each of which transfer a load P to the base of the column, as illustrated in the figure below. A cap plate at the bottom of the brace evenly distributes the load P to four flange plates through a pin of diameter d_p=2 in to two gusset plates . The ultimate shear stress in all pins is τ_u = 30 ksi, the ultimate normal stress in each brace is σ_u = 80 ksi and the cross-sectional area of each brace is A_b =80 in^2. Determine the allowable P if a factor of safety FS = 3.0 is required.

ultimate shear stress

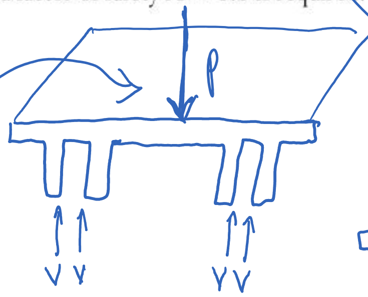


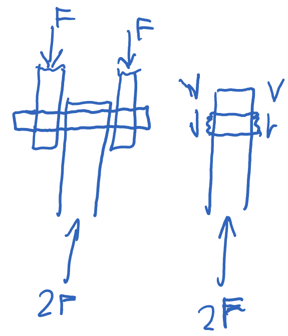
seats will eventually go here

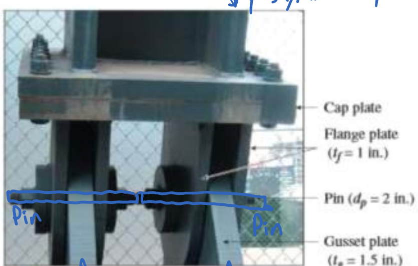
(a) Stadium brace



gussets







gussets

$\Sigma F_y = 0$

By symmetry

$V = \frac{P}{4}$

For pins

$V_{p,all} = \tau_{all} \cdot A_{pin}$

$\frac{P_{all}}{4} = \left(\frac{\tau_u}{F.S.}\right)\frac{\pi}{4} d_{pin}^2$

$\boxed{P_{all}} = \frac{(30\text{ksi})}{3} \cdot \frac{\pi}{\cancel{4}}(2\text{in})^2 \times \cancel{4}$

$= \boxed{125.6 \text{ kips}}$ for the pins

For brace (experience normal stress)

$F_{b,all} = \sigma_{all} \cdot A_b$

$\boxed{P_{all}} = \left(\frac{\sigma_u}{F.S.}\right) A_b$

$= \frac{(80\text{ksi})(80\text{in}^2)}{3}$

$= \boxed{2133 \text{ kips}}$

pick the smaller one

$P_{all} = 125.6$ kips is the largest load that can be allowed.

$= 2133$ kips

$$\boxed{P_{all} = 125.6 \text{ kips}}$$

smaller is the largest load that can be allowed.