Chapter 1: Stress

Chapter Objectives

- ✓ Understand concepts of normal and shear stress
- ✓ Analyze and design with axial (normal) and shear loads

Review of statics - Equilibrium

1) External Loads

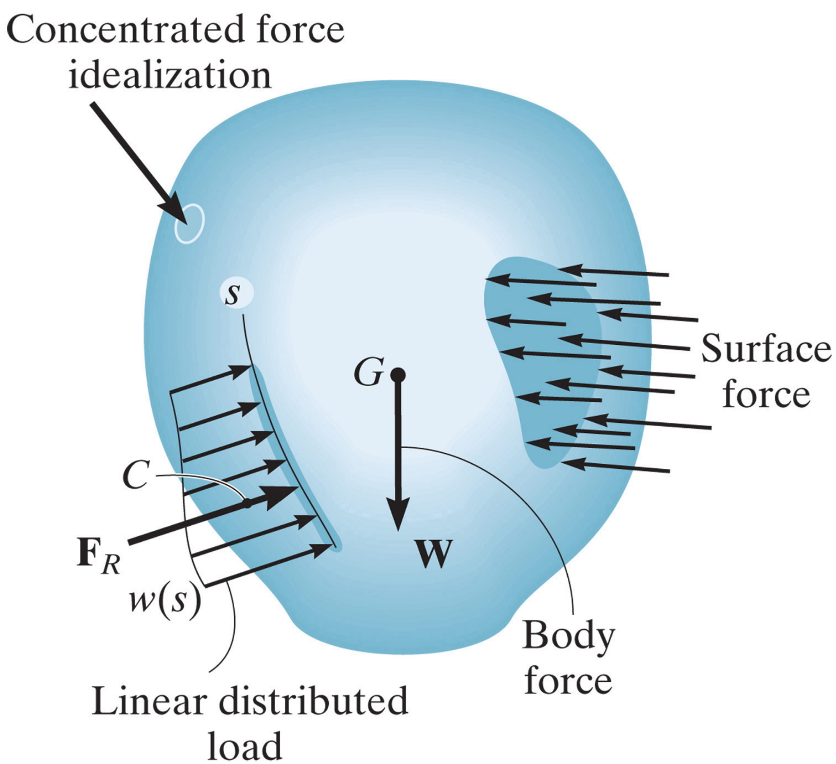
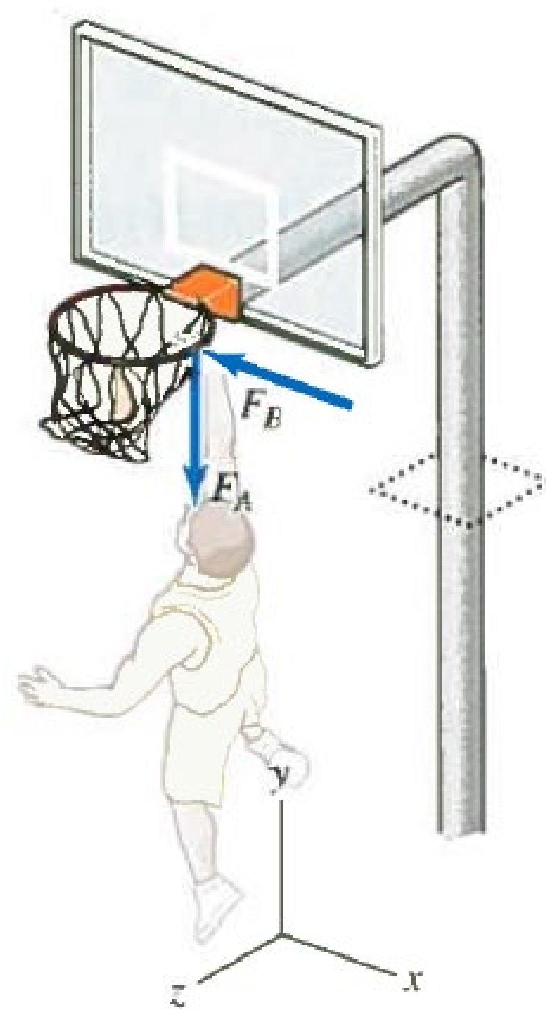
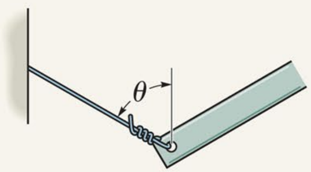
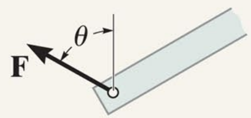
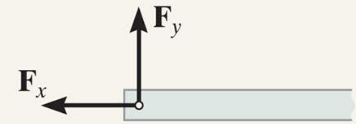
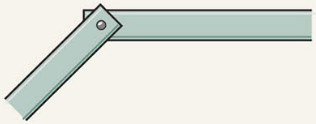
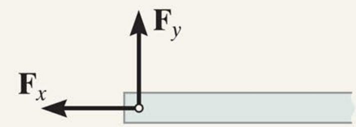
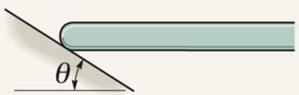
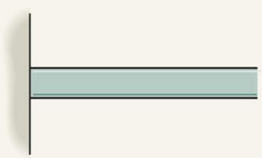
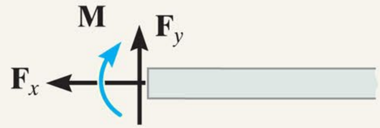


Figure: 01_01

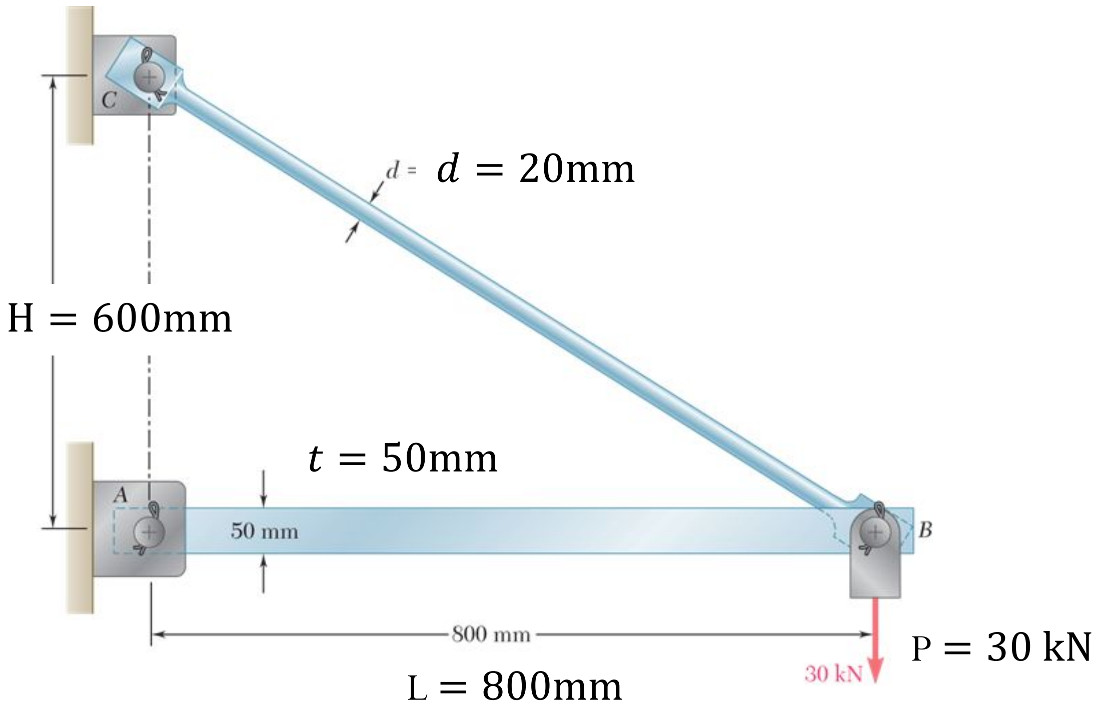


2) Support reactions

Type of connection	Reaction	Type of connection	Reaction
 Cable	 One unknown: F	 External pin	 Two unknowns: F_x, F_y
 Roller	 One unknown: F	 Internal pin	 Two unknowns: F_x, F_y
 Smooth support	 One unknown: F	 Fixed support	 Three unknowns: F_x, F_y, M

Example 1

GIVEN



FIND

(a) Internal forces in the boom and rod

(b) Reactions at A & C

Equilibrium and Free-body diagram

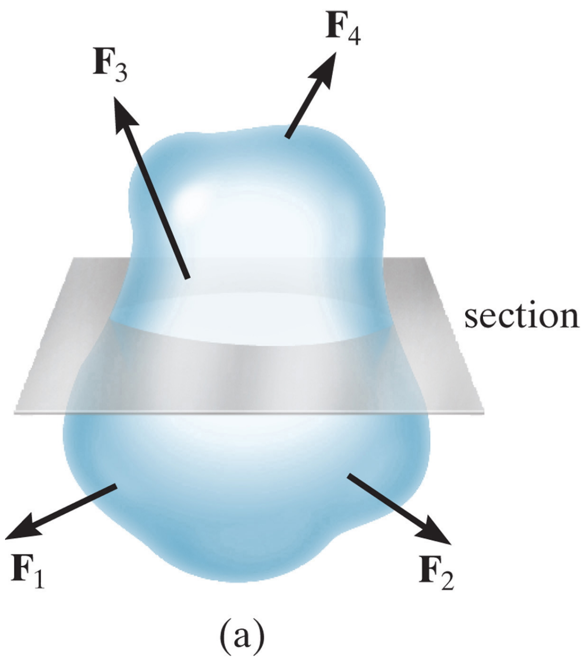


Figure: 01_02a

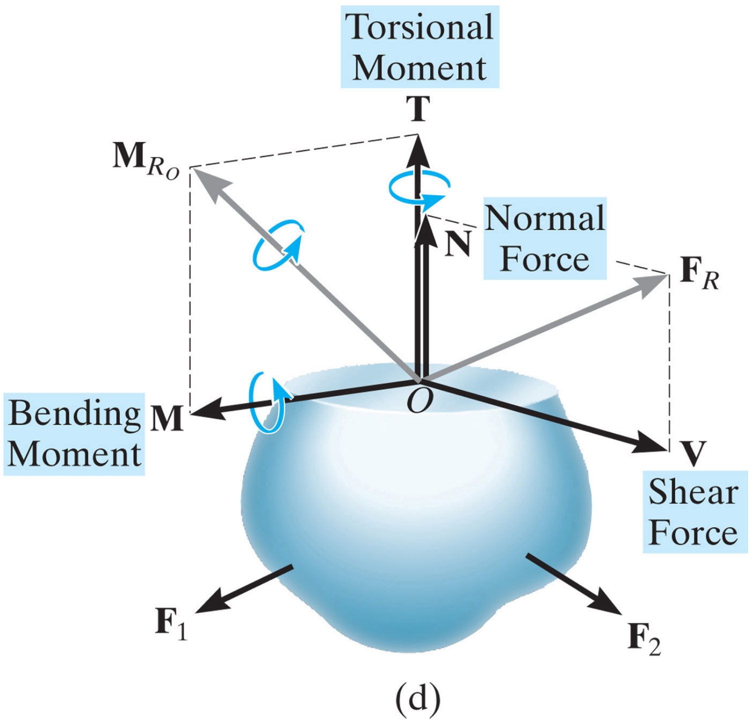
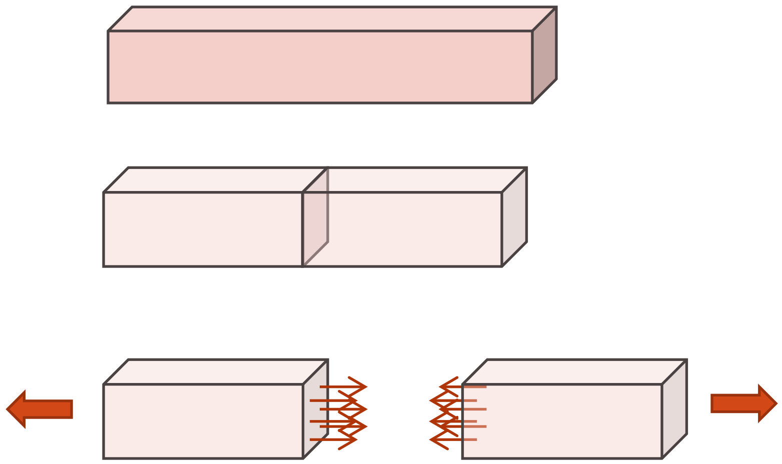


Figure: 01_02d

Statics course → assume rigid bodies

Now, we assume that bodies are deformed under the actions of forces!

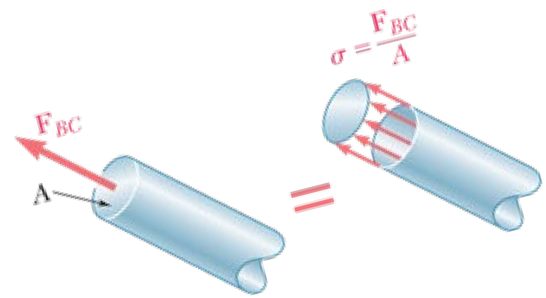
Stress

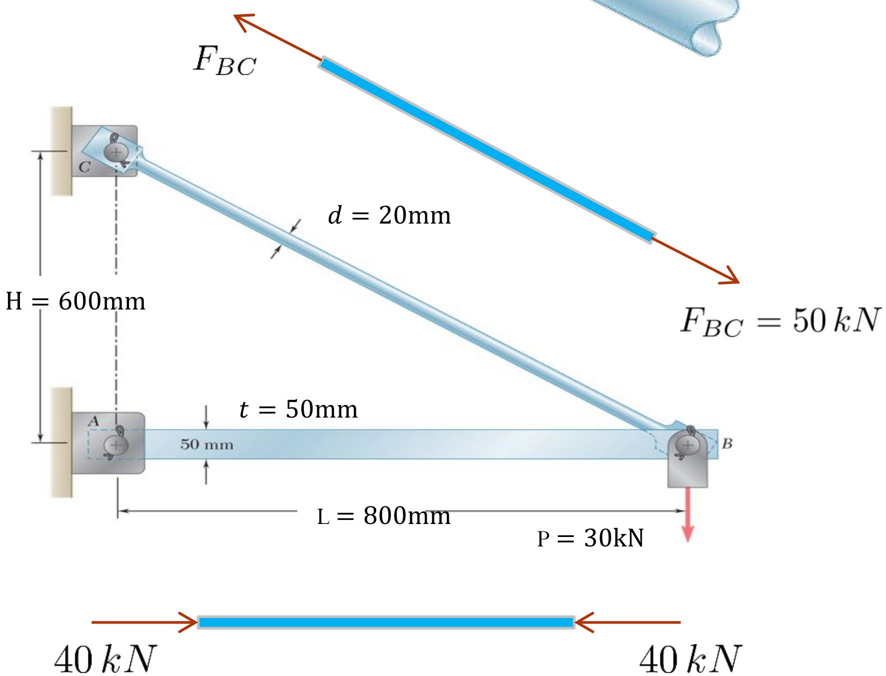


- The internal forces and moments generally vary from point to point.
- Obtaining this distribution is of primary importance in mechanics of materials.
- The total force in a cross-section, divided by the cross-sectional area, is the **stress**
- We use stress to **normalize forces** with respect to the size of the geometry

Average normal stress – axial loading

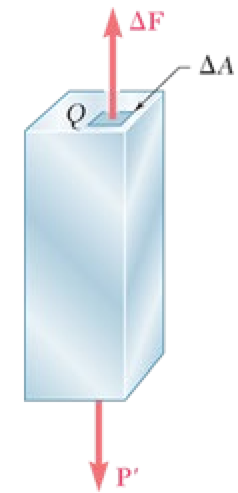
$\sigma > 0$: tension
$\sigma < 0$: compression





Units	SI sytem	BG system (US)
FORCE	[N]	[lb]
AREA	[m^2]	[in^2]
STRESS	[Pa]=[N/m^2]	[psi]=[lb/in^2]

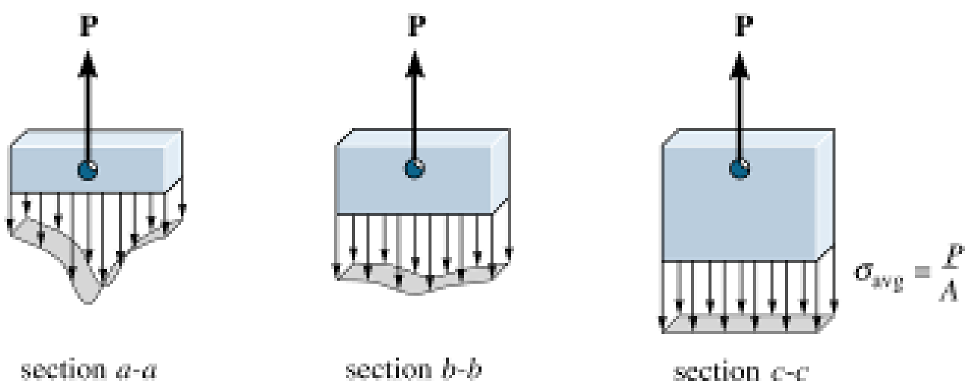
Average normal stress – axial loading

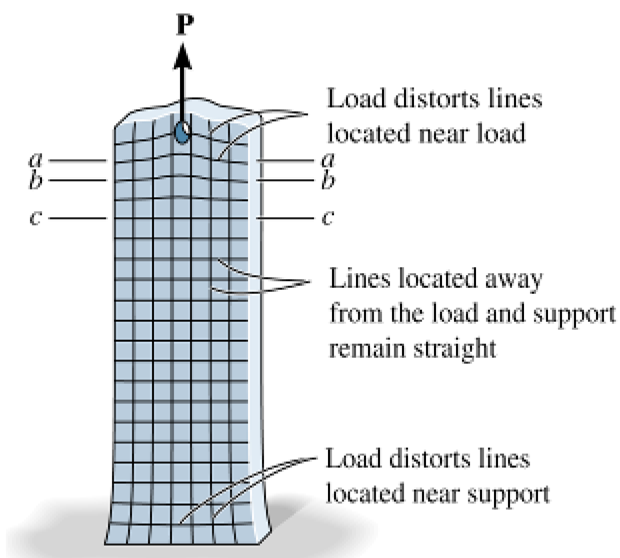


- We should note that $\sigma = \frac{F}{A}$ is the average value of the stress over the cross-sectional area, not the stress at a specific point of the cross section
- Recall that the stress at any given point Q of the cross section is given by

$$\sigma = \lim_{\Delta A \to 0} \frac{\Delta F}{\Delta A}$$

- The actual distribution of stresses in any given section is **statically indeterminate**



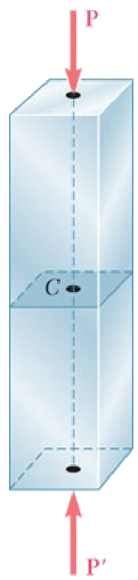


- However, equilibrium requires that

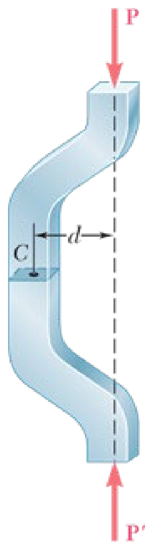
$$P = \int dF = \int \sigma dA$$

Average normal stress – axial loading

- Here we assume that the **distribution of normal stresses** in an axially loaded member is **uniform**
- Stress is calculated away from the points of application of the concentrated loads
- Uniform distribution of stress is possible only if the line of action of the concentrated load *P* passes through the centroid of the section considered



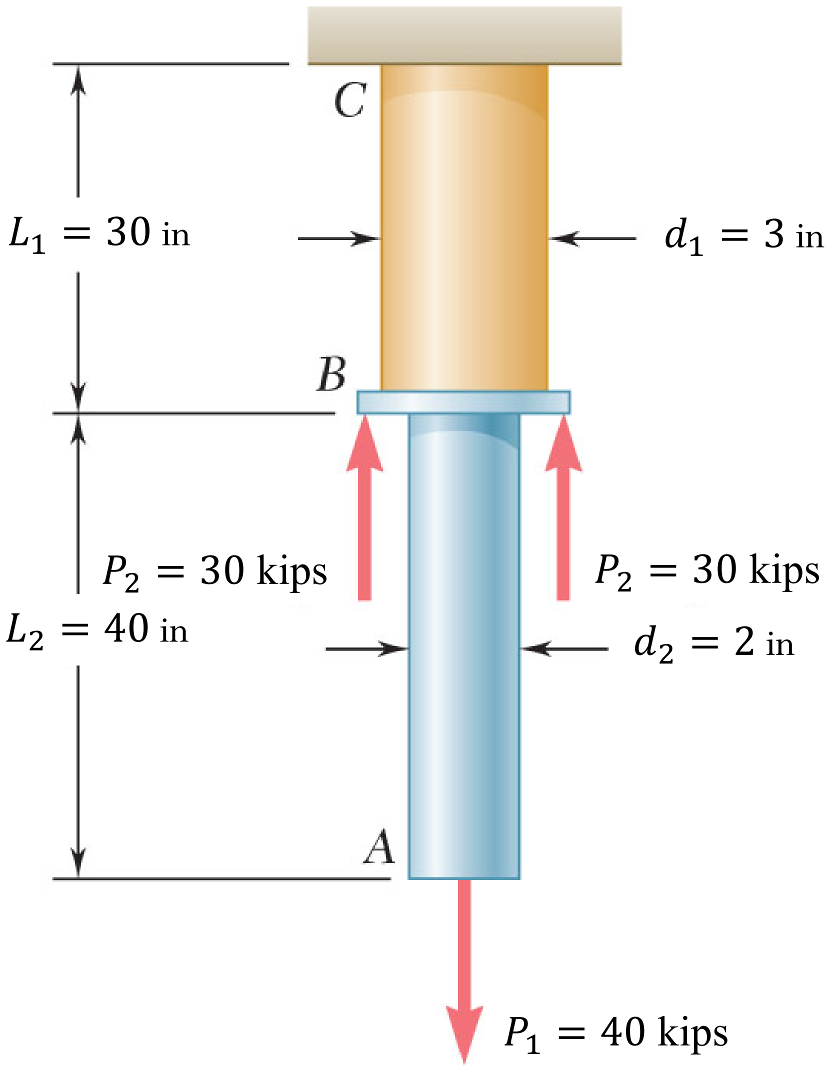
Centric axial loading
(stress distribution is uniform)



Eccentric axial loading
(stress distribution is not uniform)

Example 2

Obtain the normal stresses in each rod



Average Shear stress

- Obtained when transverse forces are applied to a member
- The distribution of shear stresses <u>cannot</u> be assumed uniform
- Common in bolts, pins and rivets used to connect various structural members

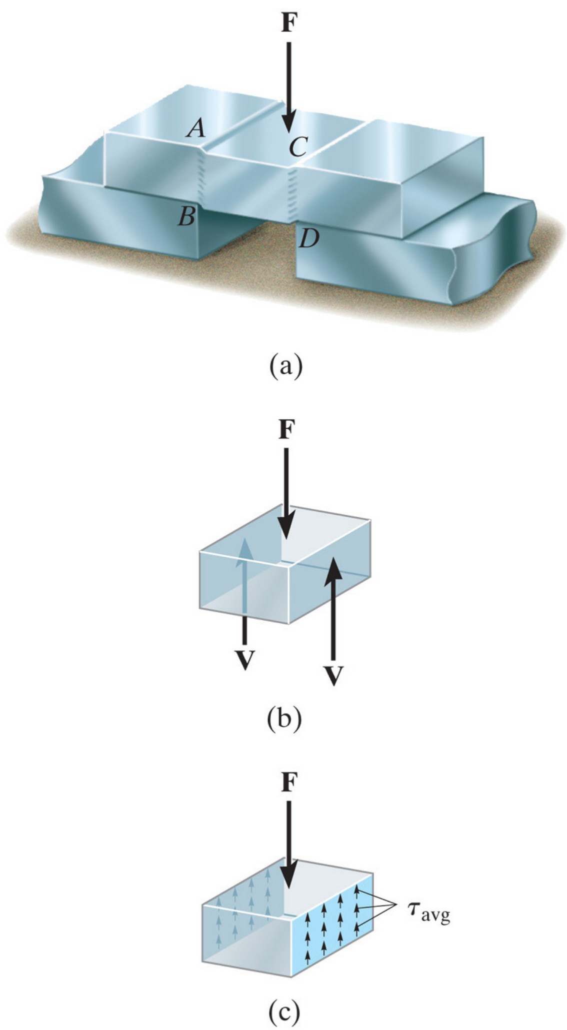
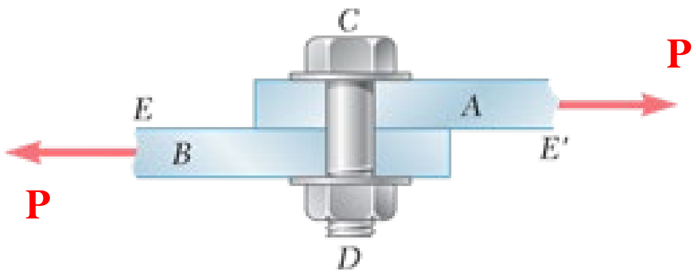


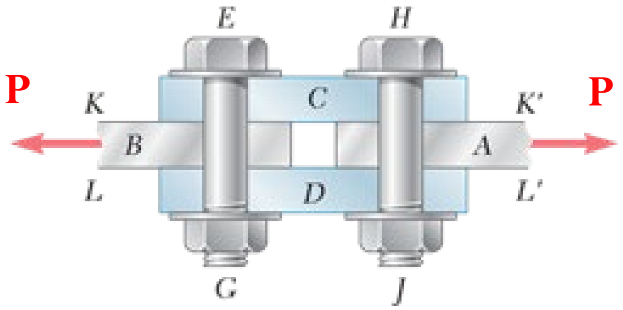
Figure: 01_19

τ_{ave}: average shear stress
V: shear force
A: cross section area

Single Shear

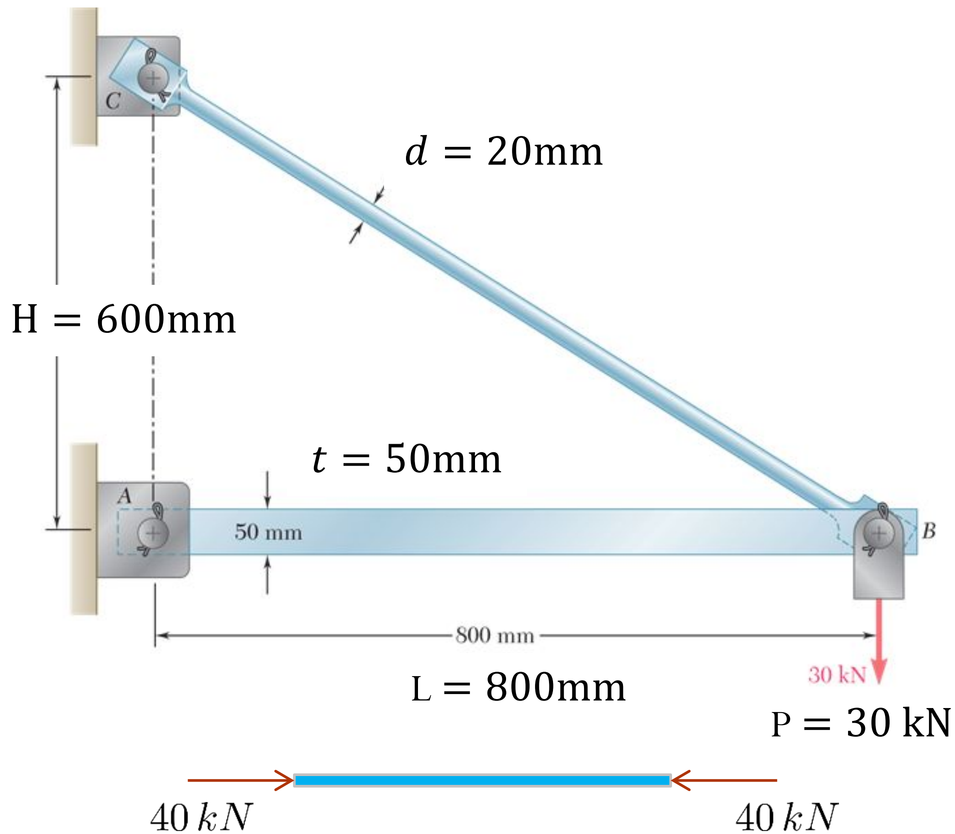


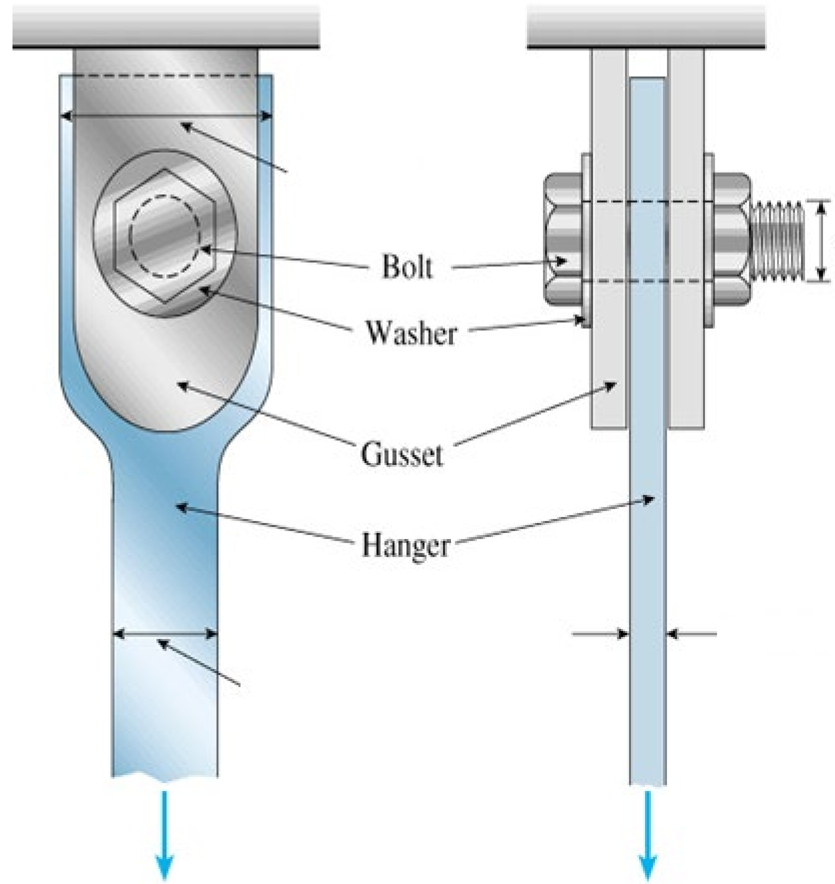
Double Shear



Example 3

Given:



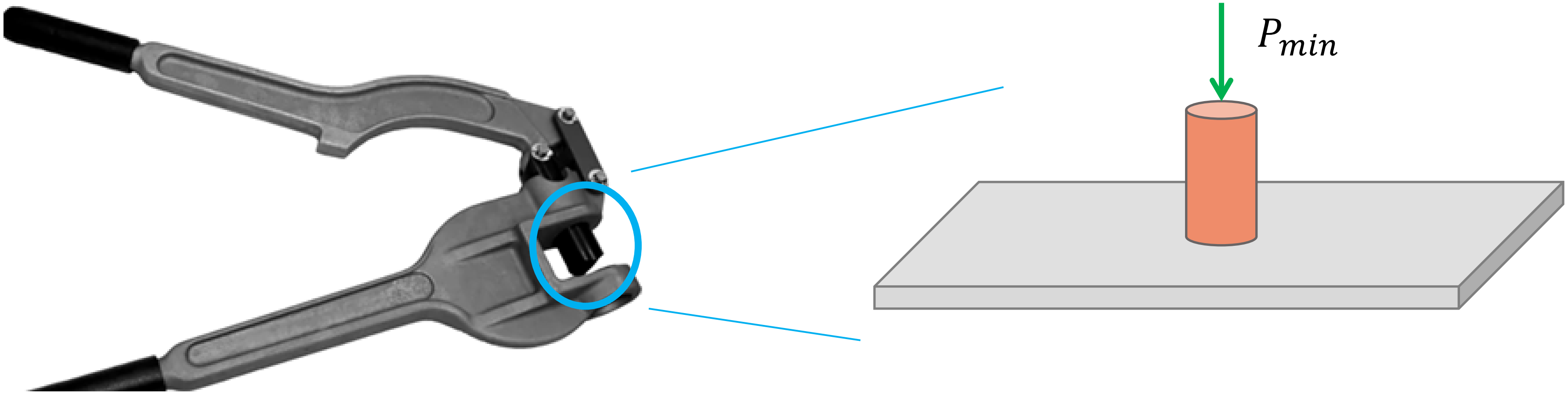


Find: Shear stresses in pins A & C

Example 4

http://www.youtube.com/watch?v=9sMXItQjHkE

A cylindrical punch of radius R is used to perforate a hole in a metal plate of thickness t. If τ_{max} is the maximum shear stress that the metal will sustain before breaking, what is the minimum force P_{min} that must be applied on the punch in order to perforate the paper?

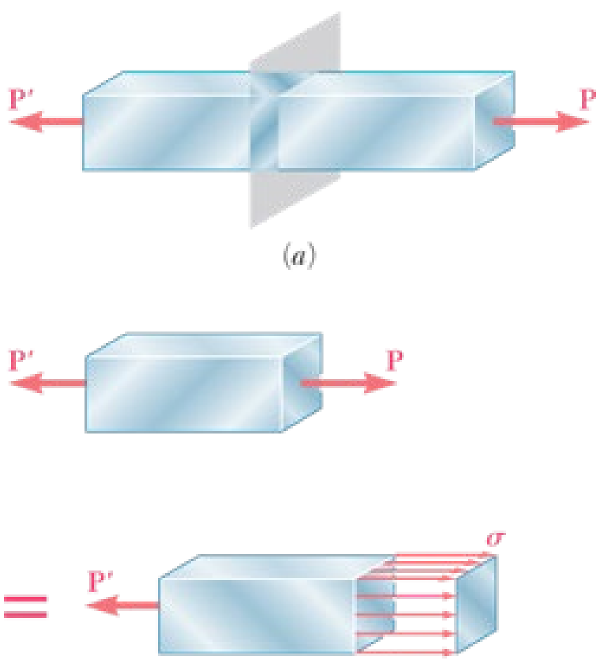


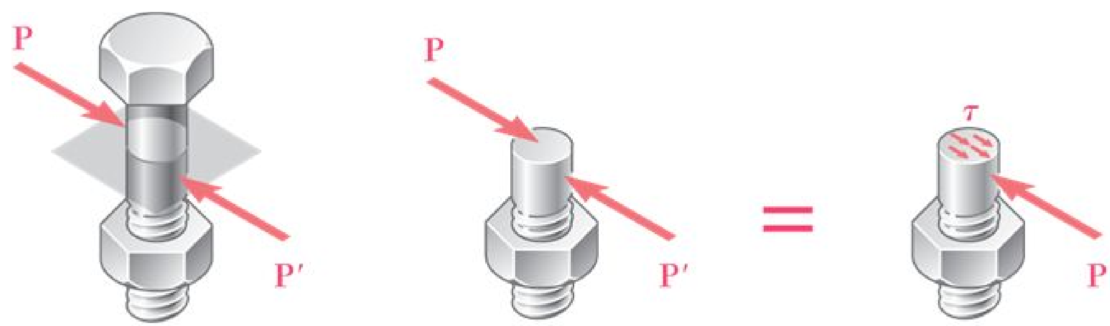
Stress on an oblique plane under axial loading

So far…

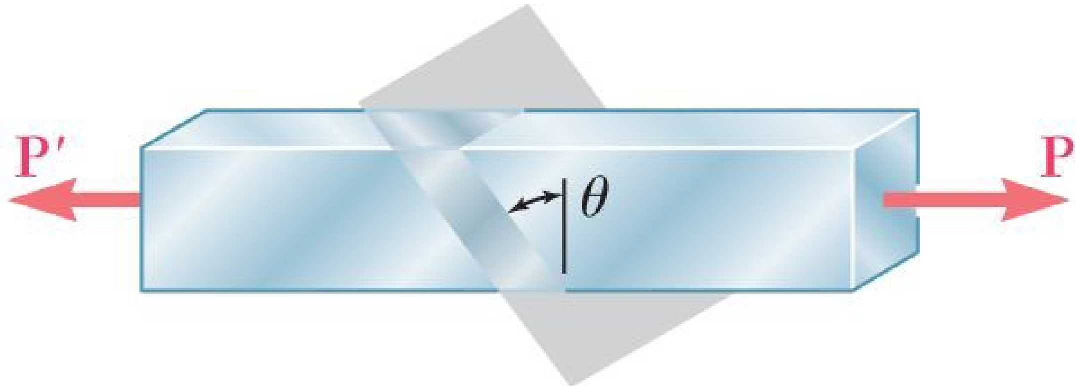
- Axial forces: NORMAL STRESS
- Transverse forces: SHEAR STRESS

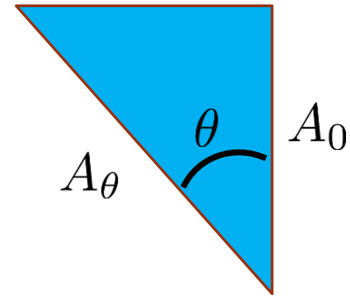
→ This relation is observed only on planes perpendicular to the axis of the member or connection





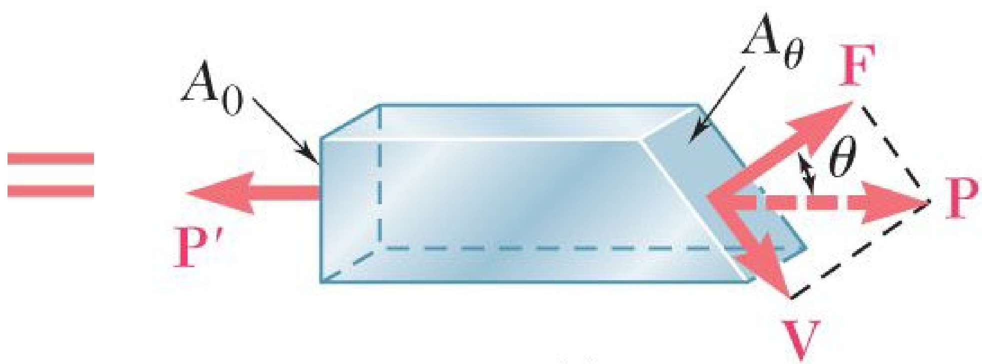
(a) (b)



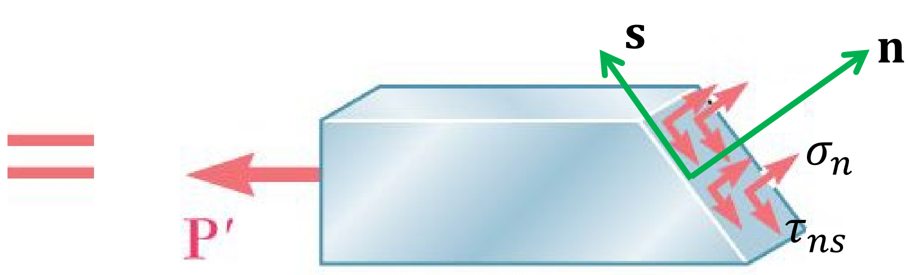


$$A_\theta = \frac{A_0}{\cos\theta}$$

Resolving P into components F and V



(c)



(d)

Design of structures

- <u>**Design Requirement**</u>: A structural design is intended to support and/or transmit loads while maintaining safety and utility: ***don't break***
- <u>Strength</u> of a structure reflects its ability to resist failure.
- <u>**Ultimate Load**</u> (P_u): force when specimen fails (breaks).
- <u>**Ultimate normal stress**</u> (σ_u):

$$\sigma_u = \frac{P_u}{A}$$

- A structure is safe if its strength exceeds the required strength
- Factor of Safety:
  Ratio of structural strength to maximum (allowed) applied load (P_{all})

FS = factor of safety

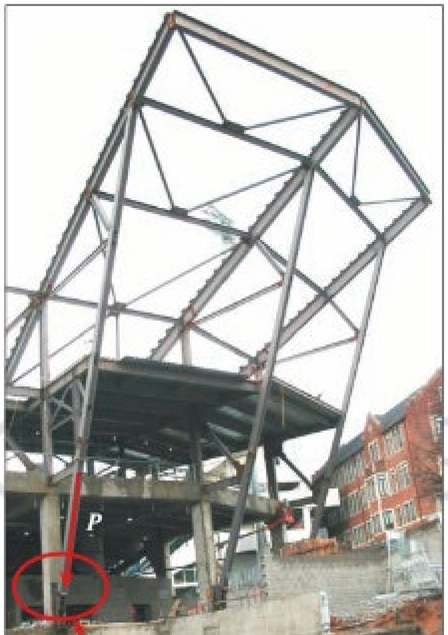
Allowable stress design

$$P_{all} = \frac{P_u}{FS}$$

$$\sigma_{all} = \frac{\sigma_u}{FS}$$

Example 5

The upper deck of a football stadium is supported by braces each of which transfer a load P to the base of the column, as illustrated in the figure below. A cap plate at the bottom of the brace evenly distributes the load P to four flange plates through a pin of diameter d_p=2 in to two gusset plates . The ultimate shear stress in all pins is $\tau_u = 30$ ksi, the ultimate normal stress in each brace is $\sigma_u = 80$ ksi and the cross-sectional area of each brace is A_b =80 in^2. Determine the allowable P if a factor of safety FS = 3.0 is required.



(a) Stadium brace