TAM251_Chapter2_Strain_prelecture_Johnson

Thursday, June 17, 2021 10:41 AM

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Chapter 2: Strain

Chapter Objectives

- ✓ Understand the concepts of normal and shear strain
- ✓ Apply the concept to determine the strain for various types of problems

Strain is a measure of geometric deformation.

Critical

become proficient in symbolic algebra

key: understand dimensional analysis

$$[\sigma] = \frac{\text{force}}{\text{area}} = \text{stress}$$

$$[\tau] = \frac{\text{force}}{\text{area}}$$

$$[A] = \text{area}$$

$A = \frac{\pi}{4} d^2$ $\quad [d] = \text{length}$

$[\pi/4] = 1$ $\quad [d^2] = \text{area} = \text{length}^2$

$\sigma = E \cdot \varepsilon$

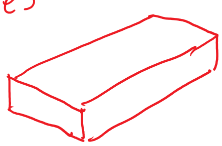
Young's Modulus

$[E] = \text{stress}$ $\quad [\varepsilon] = 1 = \text{length}^0 = 1$

Dimensions must match on both sides of any equation. $\Rightarrow [\sigma] = [E \cdot \varepsilon] = [E] \cdot [\varepsilon]$

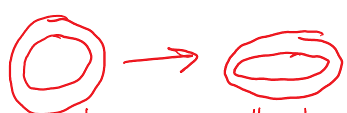
Question

If a rectangular bar of some metal is heated uniformly, does its shape change?



$\Rightarrow$ presence of normal strain, but no shear strain

If a ring is held upright and stepped on, ~~it~~ will its shape change?

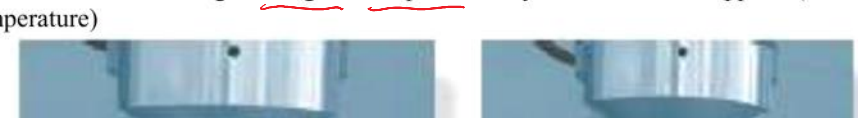


$\Rightarrow$ presence of shear strain (and normal strain)

due to normal strain

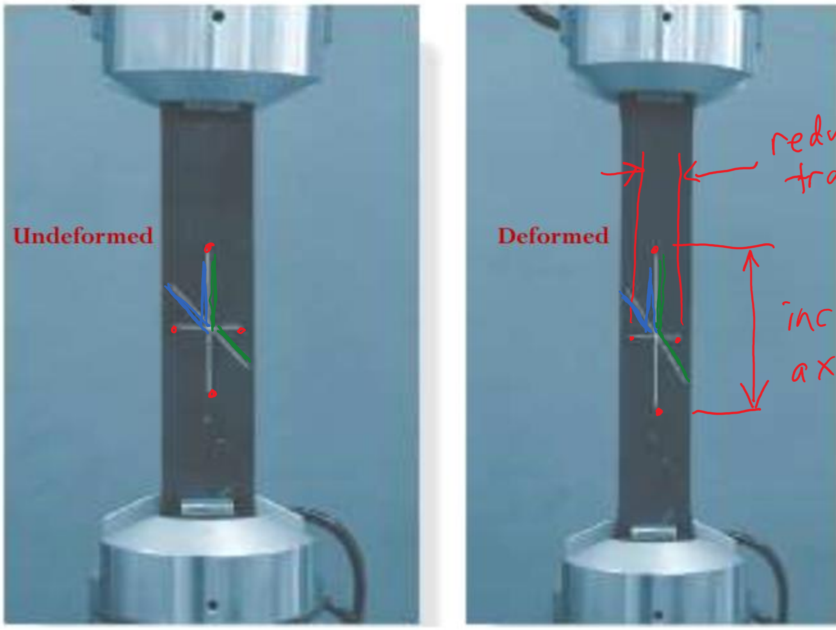
due to shear strain

DEFORMATION: change in length or shape of a body when forces are applied (or change in temperature)



due to

DEFORMATION: change in length or shape of a body when forces are applied (or change in temperature)



reduction in transverse length

increase in axial length

Rubber membrane subject to tension

Rectangle of one aspect ratio to a rectangle of a different aspect ration

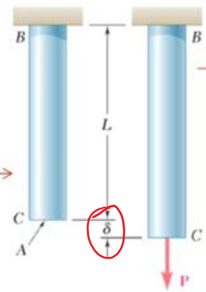
Extensional strain (normal strain)

Change in length of a member divided by its original length (i.e., deformation per unit length)

change in length

$$\epsilon = \frac{\delta}{L} = \frac{L_{final} - L_{initial}}{L_{initial}}$$

$[\varepsilon] = 1$



Deformed configuration

Uniform strain along member AB

Undeformed configuration

Strain is dimensionless!

Recall point-wise definition of stress: $\sigma = \lim_{\Delta A \to 0} \frac{\Delta F}{\Delta A}$

infinitesimal increase in length at x

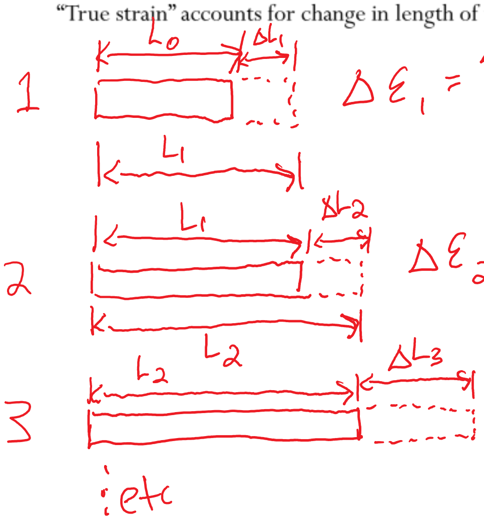
Similarly, we have a point-wise definition of strain: $\epsilon = \lim_{\Delta x \to 0} \frac{\Delta \delta}{\Delta x} = \frac{d\delta}{dx}$

MBS1

True vs Engineering Strain

We just defined "engineering strain", $\epsilon = \frac{\delta}{L_i}$ ← change in length, ← initial length

"True strain" accounts for change in length of the bar as strain increases



$$\Delta \varepsilon_1 = \frac{\Delta L_1}{L_0}$$

$$\Delta \varepsilon_2 = \frac{\Delta L_2}{L_1}$$

$$\Delta \varepsilon_3 = \frac{\Delta L_3}{L_2}$$

$$\boxed{\varepsilon_{true}} = \int_0^{\varepsilon_{final}} d\varepsilon \qquad [d\varepsilon] = 1$$

$$= \int_{L_0}^{L_f} \frac{dL}{L} \qquad \left[\frac{dL}{L}\right] = 1$$

$$= \ln\left(\frac{L_f}{L_0}\right)$$

$$= \ln\left(\frac{L_0 + \delta}{L_0}\right)$$

$$= \ln\left(1 + \frac{\delta}{L_0}\right)$$

$$= \boxed{\ln(1 + \varepsilon_{engineering})}$$

For most $\underbrace{\text{practical}}_{\text{(or "structural")}}$ engineering purposes, the true strain is very, very, very small.

For those cases $\ln(1 + \varepsilon_{eng.}) \approx \varepsilon_{eng.}$

$$\varepsilon_{true} \approx \varepsilon_{eng.}$$

Taylor expansion

$$\ln(1 + \varepsilon_{eng}) = \varepsilon_{eng} - \frac{1}{2}\varepsilon_{eng}^2 + \frac{1}{3}\varepsilon_{eng}^3 - \frac{1}{4}\varepsilon_{eng}^4 + \dots$$

when $\varepsilon_{eng} \underbrace{\ll}_{\text{much smaller than}} 1$, $\varepsilon_{eng}^2, \varepsilon_{eng}^3$, etc. $= 0$

(get rid of higher orders of ε_{eng})

$$\ln(1 + \varepsilon_{eng}) = \varepsilon_{eng} \text{ for small } \varepsilon_{eng}$$

True vs Engineering Strain

ε_{eng}

True vs Engineering Strain

For $L_i = 10''$

δ	$\epsilon_{eng} = \dfrac{\delta}{L_i}$	$\epsilon_{true} = ln\left(\dfrac{L_f}{L_i}\right)$	Error
$0.01''$	0.001	0.00099	0.05%
$0.05''$	0.005	0.00498	0.25%
$0.1''$	0.01	0.00995	0.5%
$1''$	0.1	0.0953	4.9%
$5''$	0.5	0.4054	23.3%

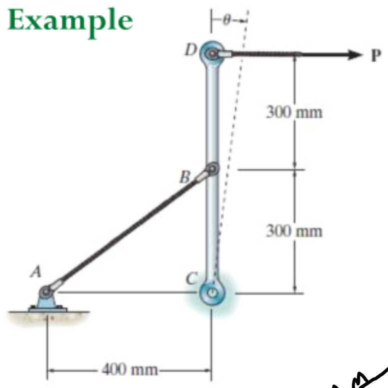
clearly acceptable for TAM251 analysis

borderline

X

usualy in engineering 5% error and less is acceptable

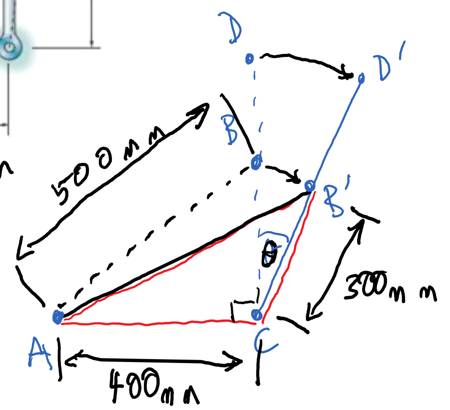
Example



Part of a control linkage of an airplane consists of a rigid member CDB and a flexible cable AB. If a force is applied at the end D of the member and causes a normal strain in the cable of 0.0035 mm/mm, determine the displacement of point D. Originally the cable is unstretched.

Method 1: Trigonometry

$L_{AB} = 500\,mm$



$$\varepsilon_{AB} = 0.0035 = \frac{L_{AB'} - L_{AB}}{L_{AB}}$$

$$L_{AB'} = L_{AB}(1 + 0.0035)$$

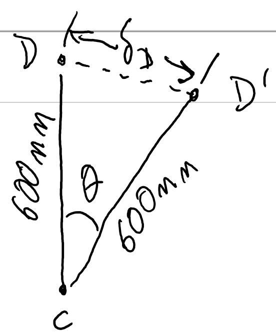
$$= 501.75\,mm$$

law of cosines to red triangle

$$(501.75\,mm)^2 = (300\,mm)^2 + (400\,mm)^2 - 2(300\,mm)(400\,mm)\cos(90° + \theta)$$

$$\therefore \theta = \cos^{-1}(\ldots) - 90° = 0.419°$$

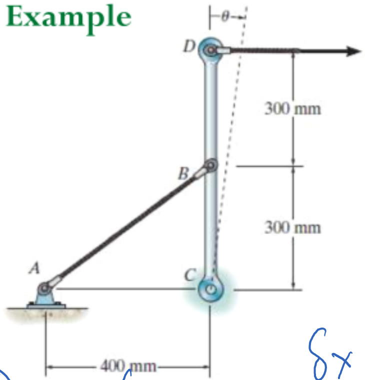
$$= 0.0073^{rad}$$



$$\delta_D^2 = (600\,mm)^2 + (600\,mm)^2 - 2(600\,mm)^2 \cdot \cos(0.0073^{rad})$$

$$\boxed{\delta_D = 4.38\,mm}$$

Example



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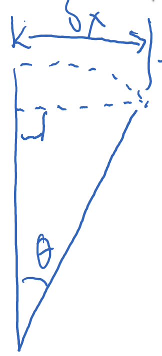
Method 2: Assume rotations are small

1



$\delta_x = L \cdot \sin\theta$

$\delta_y = L - L\cos\theta$

For small θ

$\sin\theta \approx \theta$

$\cos\theta \approx 1$

$\tan\theta \approx \theta$

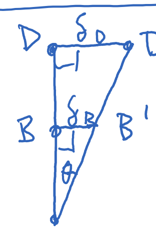
$\delta_x \approx L\cdot\theta \quad \delta_y \approx 0$

2

$$\frac{\delta_{AB}}{L_{AB}} = 0.0035$$

$\delta_{AB} \approx 1.75\,mm$

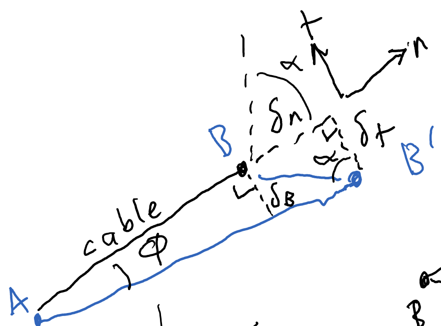
3

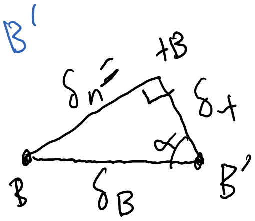


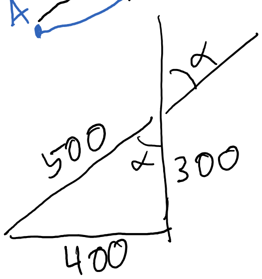
Triangle CDD' is similar with CBB'

$$\frac{\delta_D}{600\,mm} = \frac{\delta_B}{300\,mm}$$

$\Rightarrow \delta_D = 2\cdot\delta_B$







δ_n extension of cable $= \delta_{AB}$

δ_t due to rotation of the cable through small angle ϕ

$\delta_{AB} = \delta_B \cdot \sin\alpha$

$$\delta_B = \frac{\delta_{AB}}{\sin\alpha}$$

$$\delta_D = 2\cdot\delta_B = \frac{2\cdot\delta_{AB}}{(4/5)} = \frac{2\cdot(1.75\,mm)}{0.8}$$

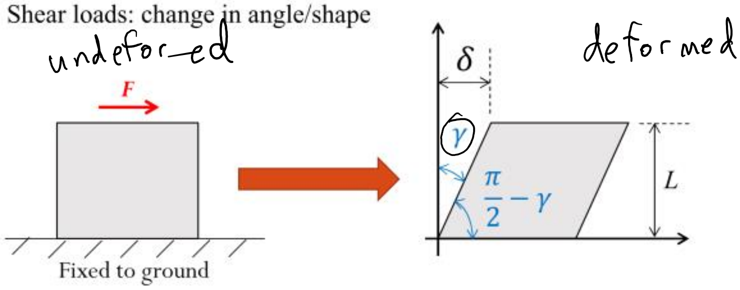
$$\boxed{\delta_D = 4.375\,mm}$$

Shear Strain

Shear Strain

Axial loads: change in length

Shear loads: change in angle/shape

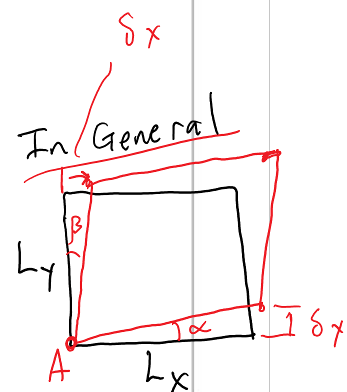


Shear strain = Change in angle that was originally at 90 degrees ($\frac{\pi}{2}$)

= γ (for now, we consider shear strain **magnitudes** only)

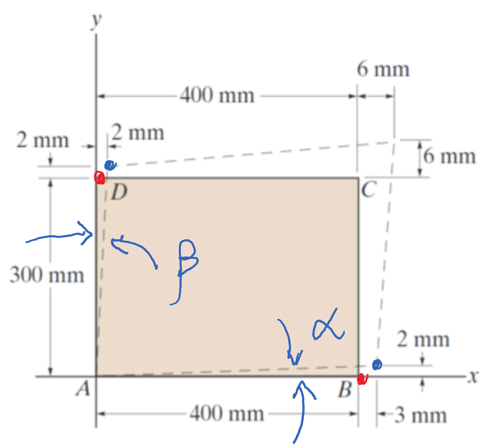
$$\tan\gamma = \boxed{\frac{\delta}{L} \approx \gamma} \quad \text{for small } \gamma$$

In General



$$\gamma_A = \alpha + \beta = \frac{\delta_y}{L_x} + \frac{\delta_x}{L_y}$$

Example



The rectangular plate is deformed into the shape shown by the dashed lines.

Determine

a) the average normal strain along diagonal BD

b) the average shear strain at corner B

a) $\varepsilon_{BD} = \dfrac{L_{B'D'} - L_{BD}}{L_{BD}}$

$L_{BD} = 500\,mm$

$B' = (403, 2)\,mm$

$D' = (2, 302)\,mm$

$L_{B'D'} = \sqrt{(403-2)^2 + (2-302)^2}\,mm$

$= 500.8\,mm$

$$\boxed{\varepsilon_{BD} = \frac{500.8 - 500}{500} = 1.6\times10^{-3}}$$

γ β α:

$$\gamma_B = \alpha + \beta$$

α:

$$\alpha = \tan^{-1}\left(\frac{2}{403}\right) = 0.0050$$

$$\beta = \tan^{-1}\left(\frac{2}{302}\right) = 0.0066$$

$$\Rightarrow \boxed{\gamma_B = 0.0050 + 0.0066 = 0.0116 \text{ rad}}$$

Measurement of Strain

- **Direct measurement:**
  - Initial and final lengths of some section of the specimen are measured, perhaps by some handheld device such as a ruler
  - Axial strain computed directly by following formula:

$$\epsilon = \frac{\delta}{L} = \frac{L_{final} - L_{initial}}{L_{initial}}$$

  - Accurate measurements of strain in this way may require a fairly large initial length

Measurement of Strain

CEE 300 ME 330
TAM 324

- **Contact Extensometer:**
  - A clip-on device that can measure very small deformations
  - Two clips attach to a specimen before testing
  - The clips are attached to a transducer body

$$\epsilon = \frac{\delta}{L} = \frac{L_{final} - L_{initial}}{L_{initial}}$$

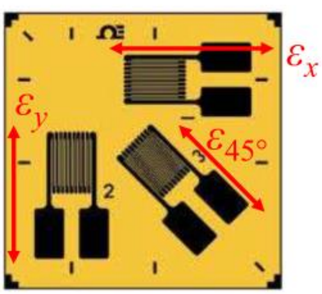
  - The transducer outputs a voltage
  - Changes in voltage output are converted to strain

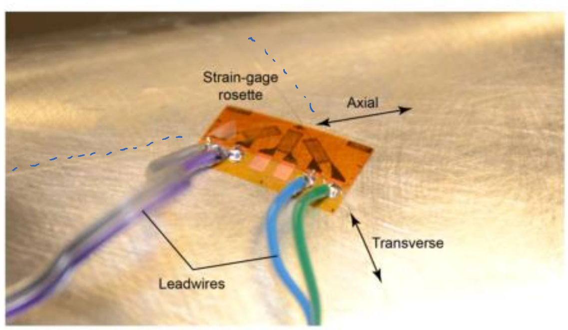
A tensile test in the Materials Testing Instructional Laboratory, Talbot Lab, UIUC

High-temperature contact extensensometer. From instron.com

Measurement of Strain

- Strain gages
  - Small electrical resistors whose resistance changes with strain
  - Change in resistance can be converted to strain measurement
  - Often sold as "rosettes," which can measure normal strain in two or more directions
  - Can be bonded to test specimen

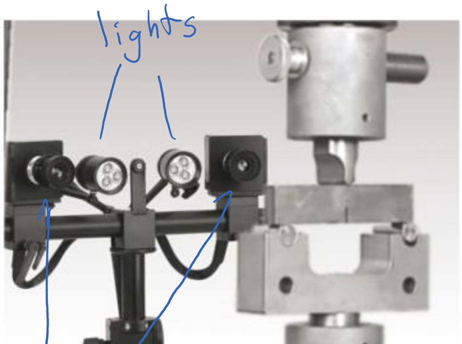




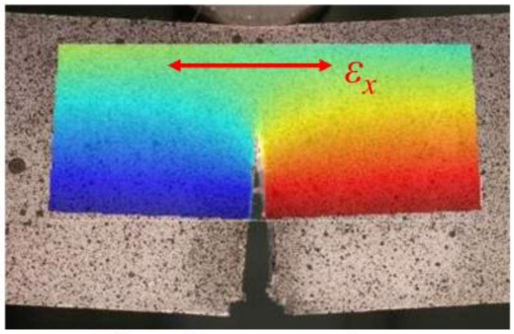
Measurement of Strain

- **Digital Image Correlation (DIC)**
  - Image placed on surface of test specimen
  - Image may consist of speckles or some regular pattern
  - Deformation of image tracked by digital camera
  - Image analysis used to determine multiple strain components

lights



DIC system analyzing a notch fracture test, from trilion.com



Strain field in a notch fracture test, as measured using DIC.
From barthelat-lab.mcgill.ca

2 cameras
at an angle