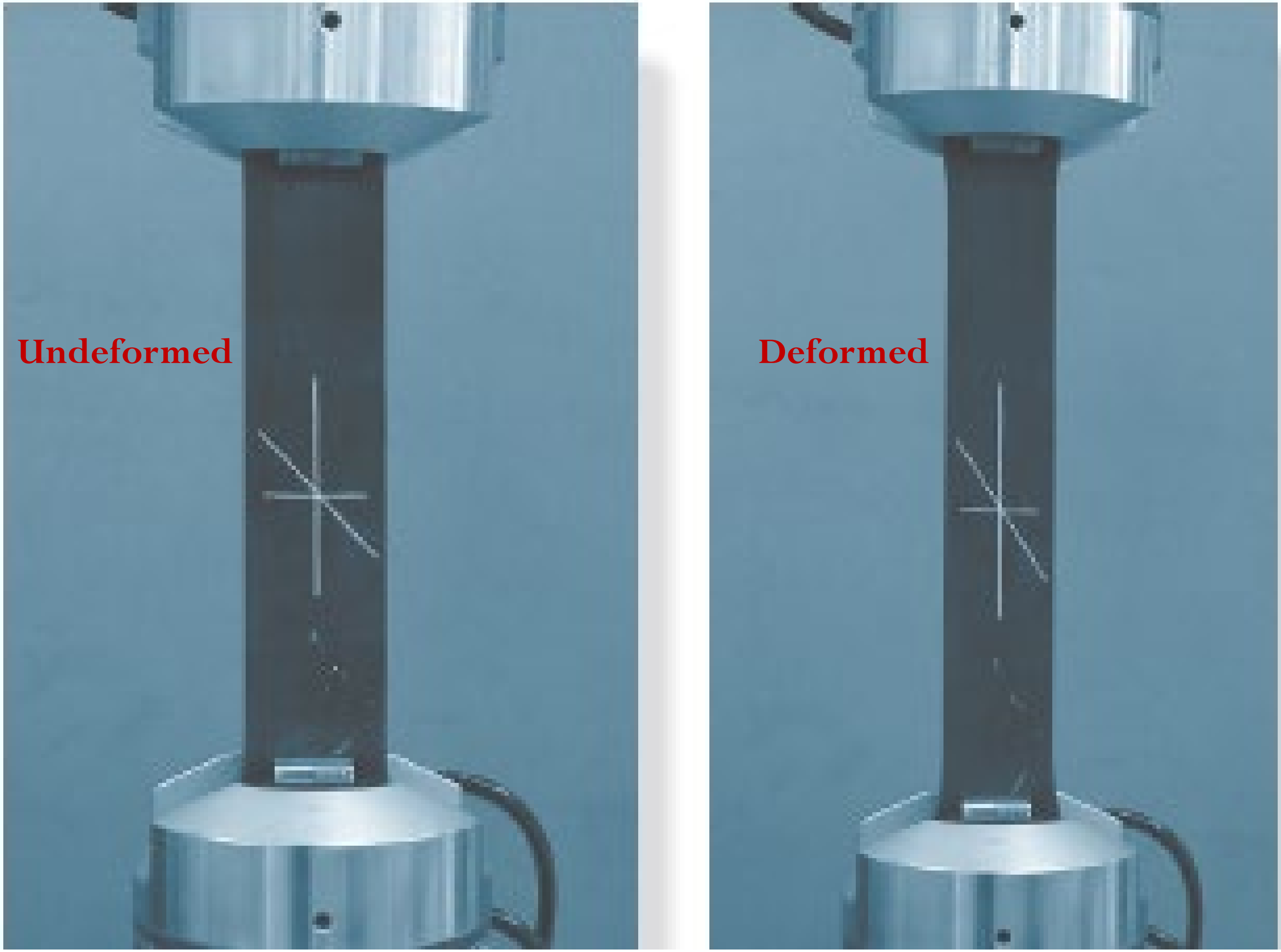
Chapter 2: Strain

Chapter Objectives

- ✓ Understand the concepts of normal and shear strain
- ✓ Apply the concept to determine the strain for various types of problems

DEFORMATION: change in length or shape of a body when forces are applied (or change in temperature)

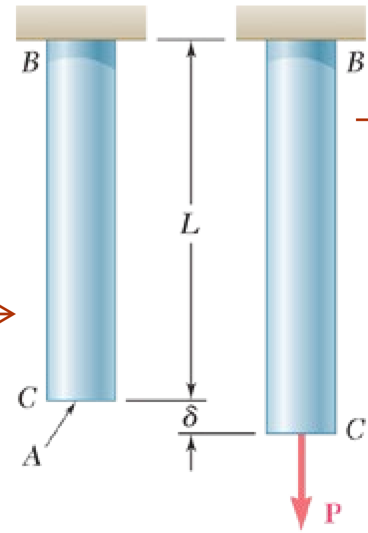


Rubber membrane subject to tension

Extensional strain

Change in length of a member divided by its original length (i.e., deformation per unit length)

$$\epsilon = \frac{\delta}{L} = \frac{L_{final} - L_{initial}}{L_{initial}}$$



Deformed configuration

Uniform strain along member AB

Undeformed configuration

Strain is dimensionless!

Recall point-wise definition of stress: $\sigma = \lim_{\Delta A \to 0} \frac{\Delta F}{\Delta A}$

Similarly, we have a point-wise definition of strain: $\epsilon = \lim_{\Delta x \to 0} \frac{\Delta \delta}{\Delta x} = \frac{d\delta}{dx}$

True vs Engineering Strain

We just defined "engineering strain", $\epsilon = \frac{\delta}{L_i}$

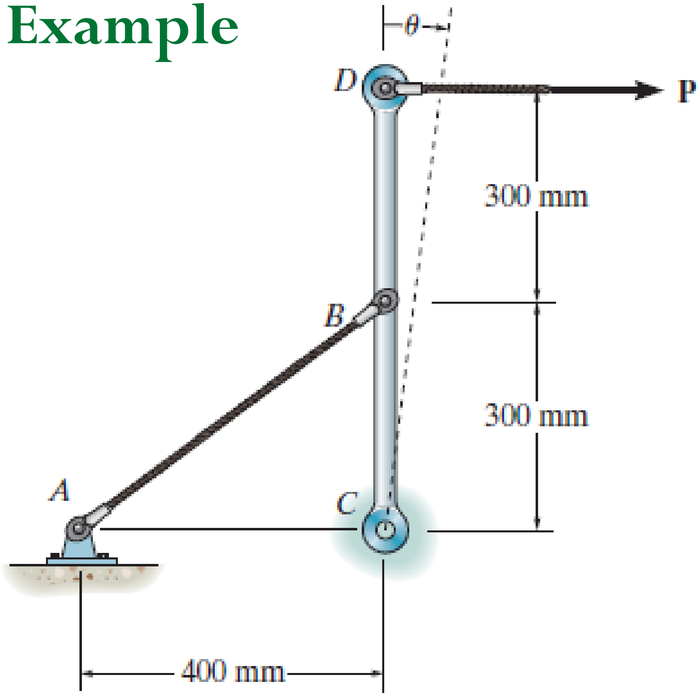
"True strain" accounts for change in length of the bar as strain increases

True vs Engineering Strain

For $L_i = 10$

$\boldsymbol{\delta}$	$\epsilon_{eng} = \dfrac{\delta}{L_i}$	$\epsilon_{true} = ln\left(\dfrac{L_f}{L_i}\right)$	**Error**
0.01	0.001	0.00099	0.05%
0.05	0.005	0.00498	0.25%
0.1	0.01	0.00995	0.5%
1	0.1	0.0953	4.9%
5	0.5	0.4054	23.3%

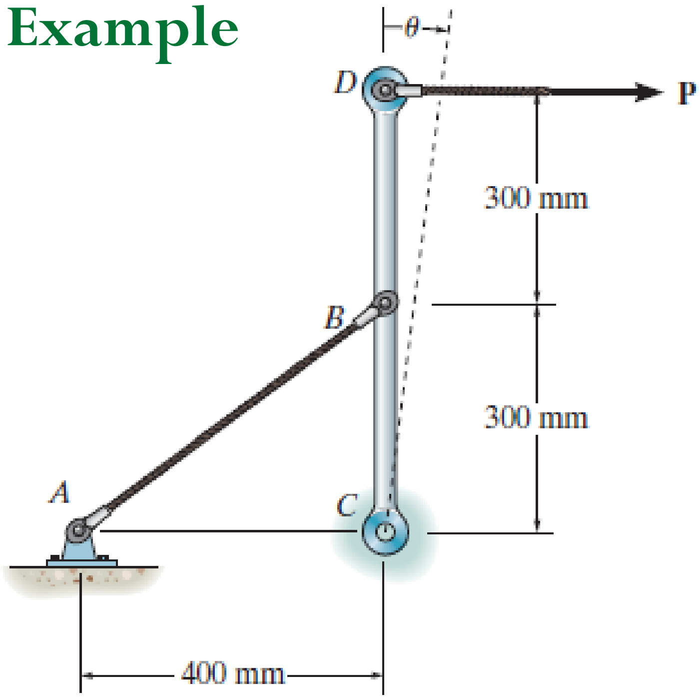
Example



Part of a control linkage of an airplane consists of a rigid member CDB and a flexible cable AB. If a force is applied at the end D of the member and causes a normal strain in the cable of 0.0035 mm/mm, determine the displacement of point D. Originally the cable is unstretched.

Method 1: Trigonometry

Example



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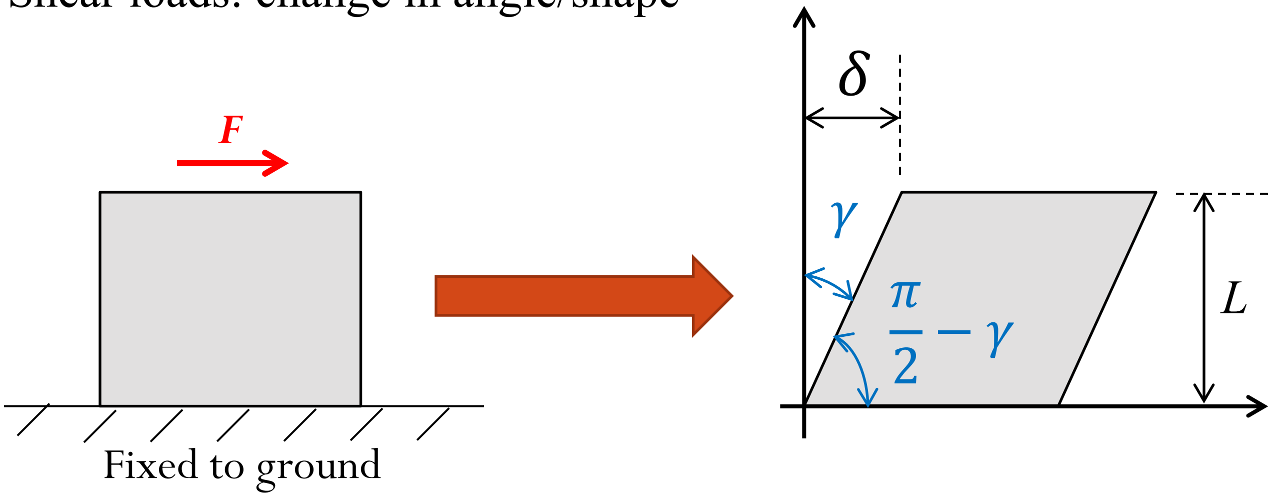
Method 2: Assume rotations are <u>small</u>

Shear Strain

Axial loads: change in length

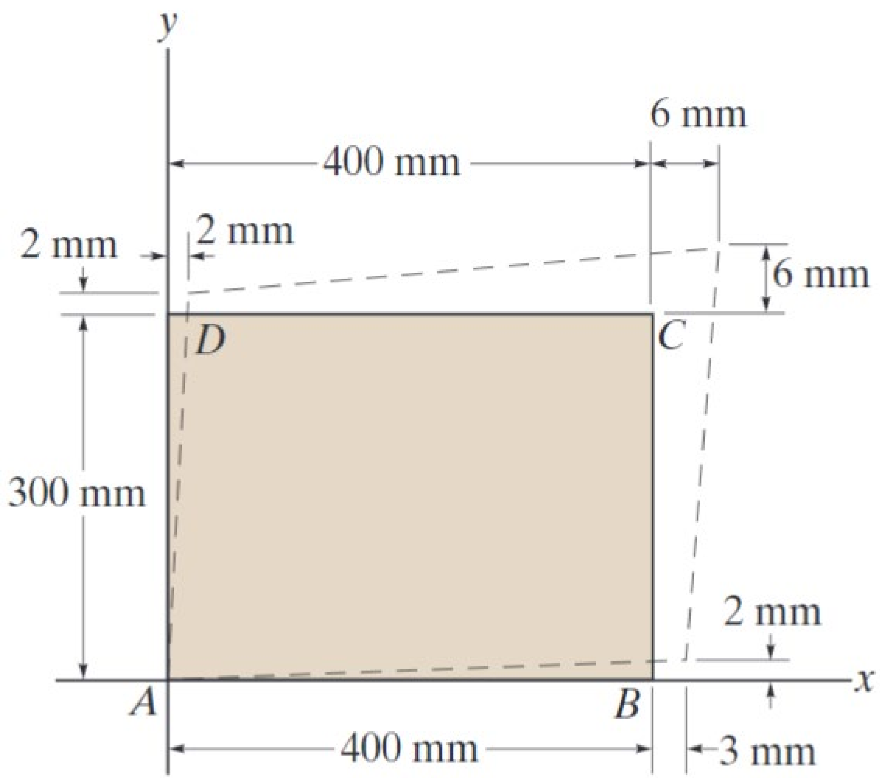
Shear loads: change in angle/shape



Shear strain $=$ Change in angle that was originally at 90 degrees $(\frac{\pi}{2})$

$= \gamma$ (for now, we consider shear strain **magnitudes** only)

Example



The rectangular plate is deformed into the shape shown by the dashed lines.

Determine

a) the average normal strain along diagonal BD

b) the average shear strain at corner B

Measurement of Strain

- **Direct measurement:**
  - Initial and final lengths of some section of the specimen are measured, perhaps by some handheld device such as a ruler
  - Axial strain computed directly by following formula:

$$\epsilon = \frac{\delta}{L} = \frac{L_{final} - L_{initial}}{L_{initial}}$$

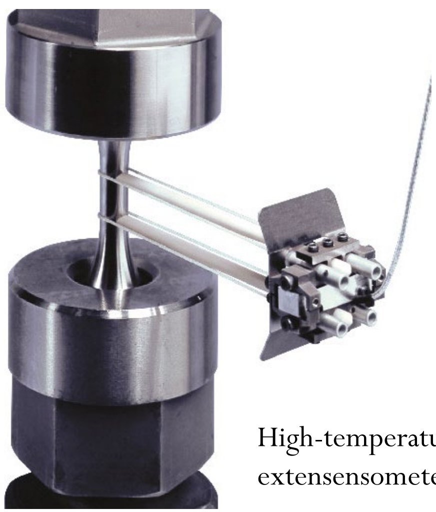
  - Accurate measurements of strain in this way may require a fairly large initial length

Measurement of Strain

- **Contact Extensometer:**
  - A clip-on device that can measure very small deformations
  - Two clips attach to a specimen before testing
  - The clips are attached to a transducer body

$$\epsilon = \frac{\delta}{L} = \frac{L_{final} - L_{initial}}{L_{initial}}$$

  - The transducer outputs a voltage
  - Changes in voltage output are converted to strain

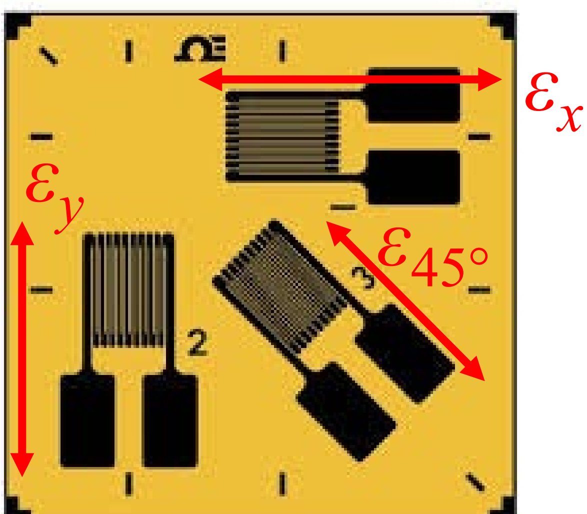


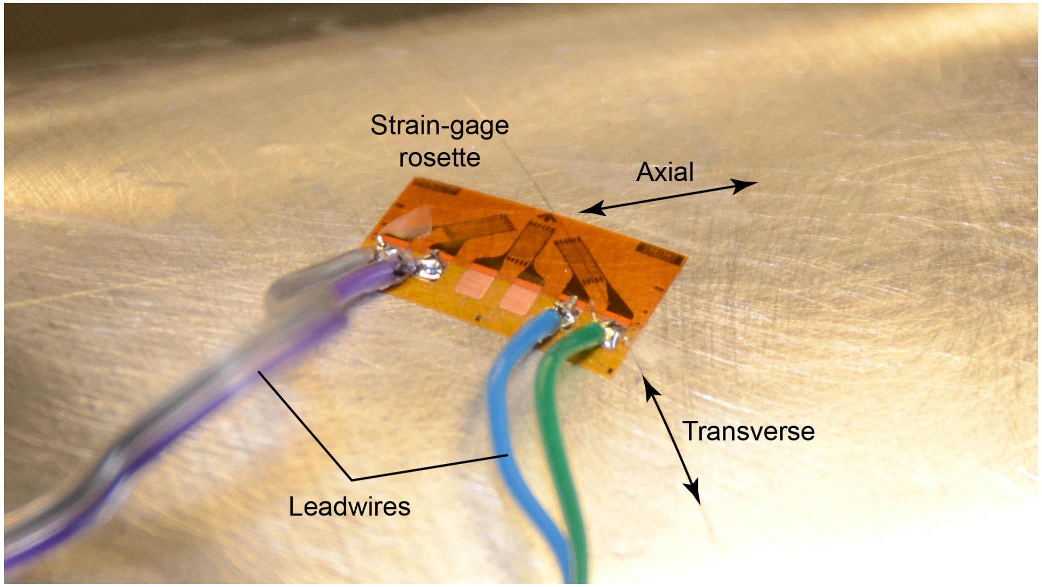
High-temperature contact extensensometer. From instron.com

A tensile test in the Materials Testing Instructional Laboratory, Talbot Lab, UIUC

Measurement of Strain

- Strain gages
  - Small electrical resistors whose resistance changes with strain
  - Change in resistance can be converted to strain measurement
  - Often sold as "rosettes," which can measure normal strain in two or more directions
  - Can be bonded to test specimen

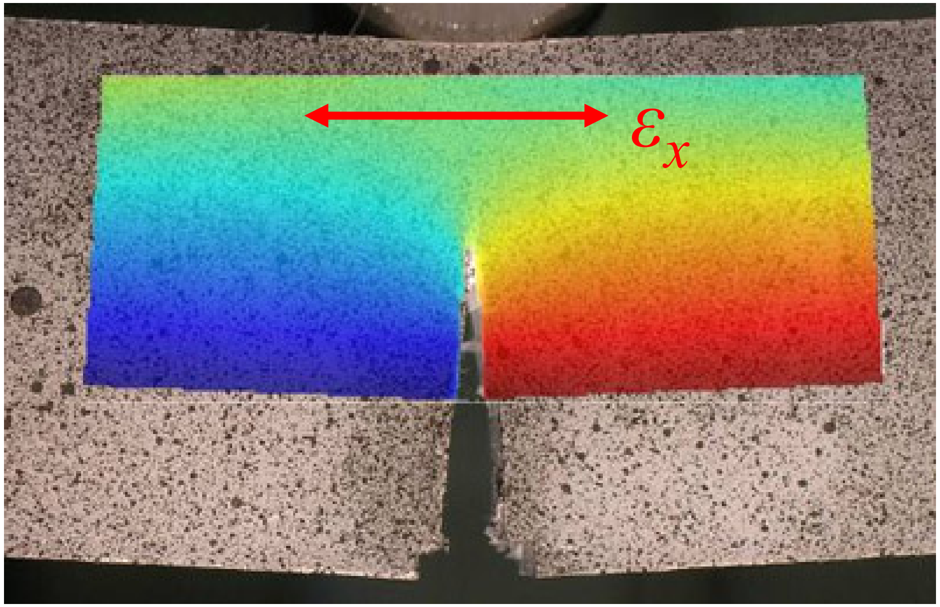




Measurement of Strain

- **Digital Image Correlation (DIC)**
  - Image placed on surface of test specimen
  - Image may consist of speckles or some regular pattern
  - Deformation of image tracked by digital camera
  - Image analysis used to determine multiple strain components

DIC system analyzing a notch fracture test, from trilion.com



Strain field in a notch fracture test, as measured using DIC.
From barthelat-lab.mcgill.ca