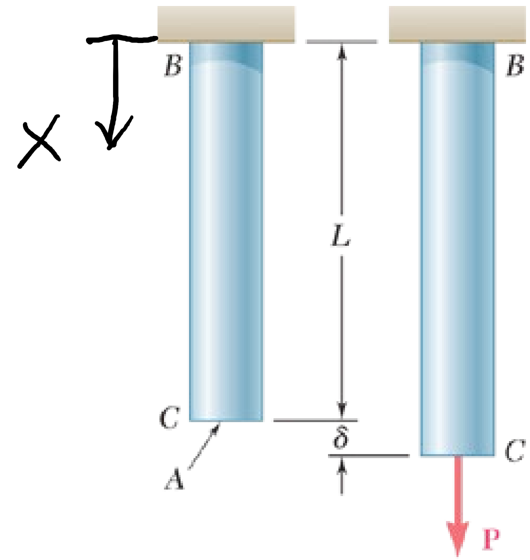
# Chapter 4: Axial Load

## Chapter Objectives

- ✓ Determine the elastic deformation of axially loaded members
- ✓ Apply the principle of superposition for total effect of different loading cases
- ✓ Deal with compatibility conditions
- ✓ Deal with thermal stresses
- ✓ Misfit problems

# Axial deformation




So far…

$$\underbrace{\sigma = \frac{F}{A} \qquad \epsilon = \frac{\delta}{L} \qquad \sigma = E\,\epsilon}$$

$$\frac{F}{A} = E\,\frac{\delta}{L} \Rightarrow \boxed{\delta = \frac{F \cdot L}{E \cdot A}}$$

$$F = k \cdot \Delta x \Rightarrow F = k \cdot \delta = \left(\frac{EA}{L}\right) \cdot \delta$$

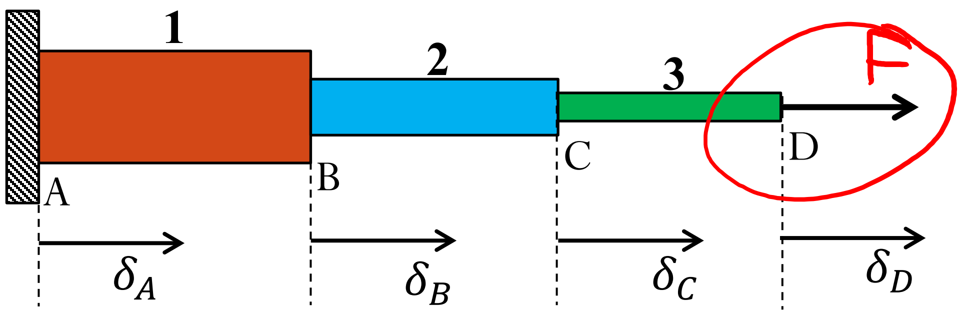
spring const.

In general, $\varepsilon = \frac{du}{dx}$ ← local material displacement

$$\frac{F}{A} = E \cdot \frac{du}{dx} \Rightarrow du = \frac{F(x)}{E(x)\,A(x)} \cdot dx$$

$$\boxed{u(x_2) - u(x_1) = \int_{x_1}^{x_2} \frac{F(x)}{E(x)\,A(x)}\,dx}$$

Superposition principle: 1) The loading must be linearly related to the stress or displacement that is to be determined; 2) Small deformation assumption




$$\delta = \sum_i \delta_i = \sum_i \frac{F_i\, L_i}{E_i\, A_i}$$

$\delta_A = 0$ (A is fixed at wall)

$\delta_{B/A} = \delta_B - \delta_A$ (Relative displacement of two points)

$\delta_1 = \delta_B - \delta_A = \dfrac{F_1 L_1}{E_1 A_1}$

$\delta_{C/B} = \delta_C - \delta_B = \delta_2 = \dfrac{F_2 L_2}{E_2 A_2}$

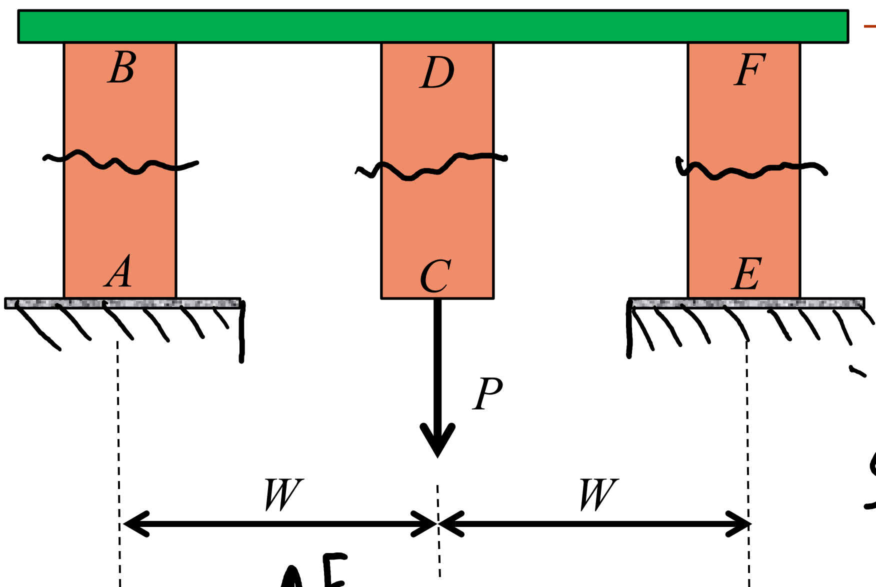
$\delta_{D/C} = \delta_D - \delta_C = \delta_3 = \dfrac{F_3 L_3}{E_3 A_3}$

$F_3 \leftarrow \quad \rightarrow F$

positive, so all rods increase in length $\left\{ \begin{array}{l} F_3 = F \\ F_2 = F \\ F_1 = F \end{array} \right.$

# Example 1

Rigid bar $\Rightarrow$ does not bend



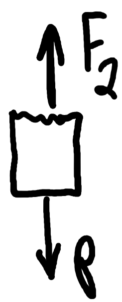

Obtain the displacement of points *C* and *D* when the load P is applied

Ground is fixed

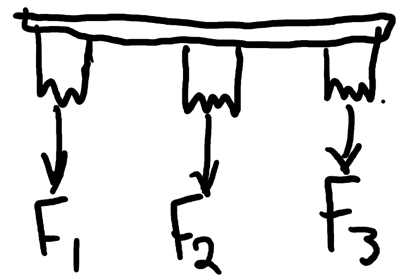
Assume each bar has length $\underbrace{L, E, A}_{\text{all same}}$

Symmetric.

FBD:




$\Rightarrow F_2 = P$ (tension)



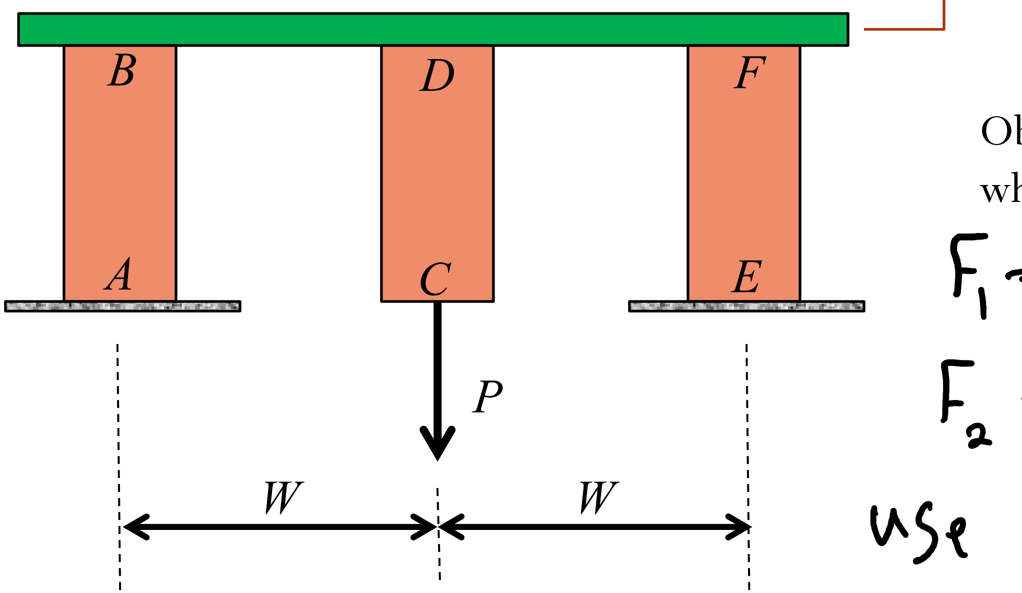

$F_1 = F_3$ (symmetry)

$$\Sigma F_y = 0 \Rightarrow -F_1 - F_2 - F_3 = 0$$

$$F_1 = F_3 = -\frac{1}{2}F_2 = -\frac{P}{2}$$

(compression)

# Example 1




Obtain the displacement of points *C* and *D* when the load P is applied

$F_1 = F_3 = -\frac{1}{2}P$  want $\delta_C$ and $\delta_D$

$F_2 = P$

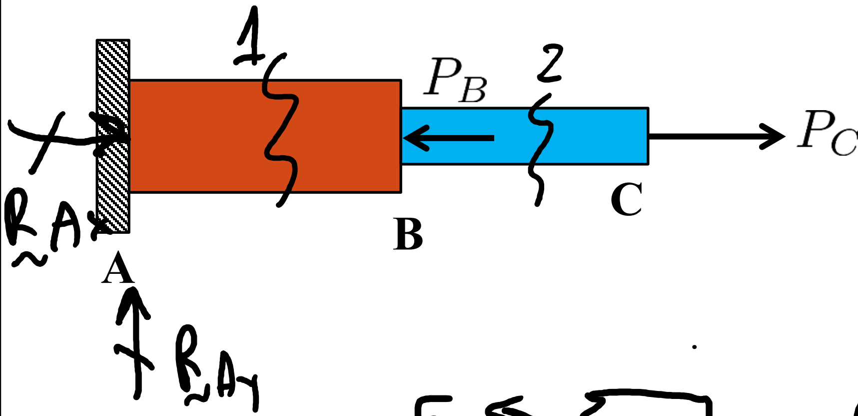
use $\delta_{D/C} = \delta_D - \delta_C = \delta_2 = \frac{F_2 L}{EA} = \frac{P \cdot L}{EA}$

$\delta_D = \delta_B = \delta_F$ because $\delta_A = \delta_E = 0$ and green rod is rigid

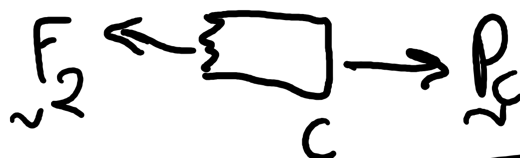
$\delta_{B/A} = \delta_B - \cancel{\delta_A}^{\,0} = \frac{F_1 L}{EA} = -\frac{PL}{2EA} \Rightarrow \delta_B = \boxed{\delta_D = \frac{-PL}{2EA}}$

$\boxed{\delta_C = \delta_D - \delta_{D/C} = \delta_D - \frac{F_2 L}{EA} = \frac{-PL}{2EA} - \frac{2PL}{2EA} = \frac{-3PL}{2EA}}$

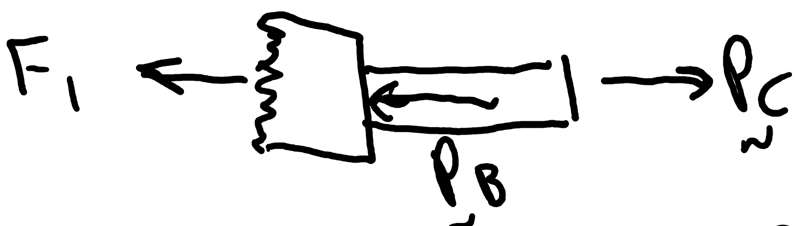
# Statically **Determinate** Problems




can solve for all internal forces and reactions by using equilibrium alone.




$$\sum F_x = 0 \Rightarrow \boxed{F_2 = P_C}$$

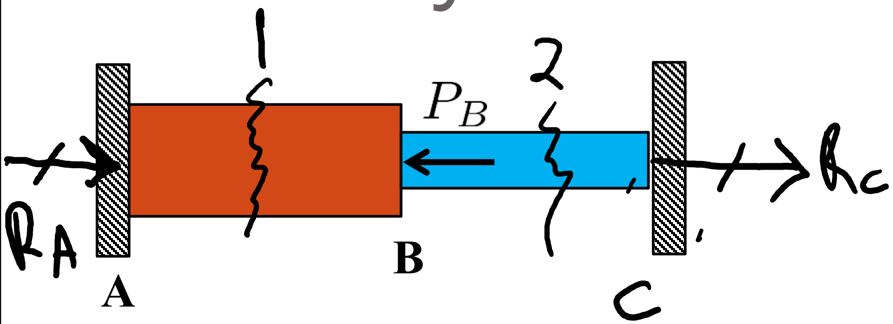



$$\sum F_x = 0 \Rightarrow -F_1 - P_B + P_C = 0$$

$$\boxed{F_1 = P_C - P_B}$$

$$\boxed{R_A = P_B - P_C}$$

# Statically **Indeterminate** Problems




Motion is constrained at both A & C $\Rightarrow$ Two Reactions

$\Sigma F_x = 0$ is the only equil. eqn. we can use

$$R_A - P_B + R_C = 0$$

Apply compatibility condition. $\Rightarrow$ Geometric constraint.

If A & C are fixed, then $\delta_1 + \delta_2 = 0$

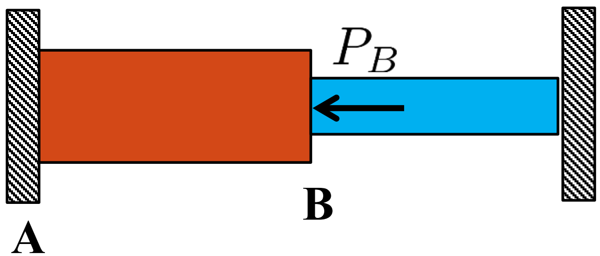
$$\boxed{\frac{F_1 L_1}{E_1 A_1} + \frac{F_2 L_2}{E_2 A_2} = 0}$$

$|F_1| = |R_A|$ $\quad$ $|F_2| = |R_C|$


$R_c = F_2$


$F_1 = -R_A$

# Statically **Indeterminate** Problems



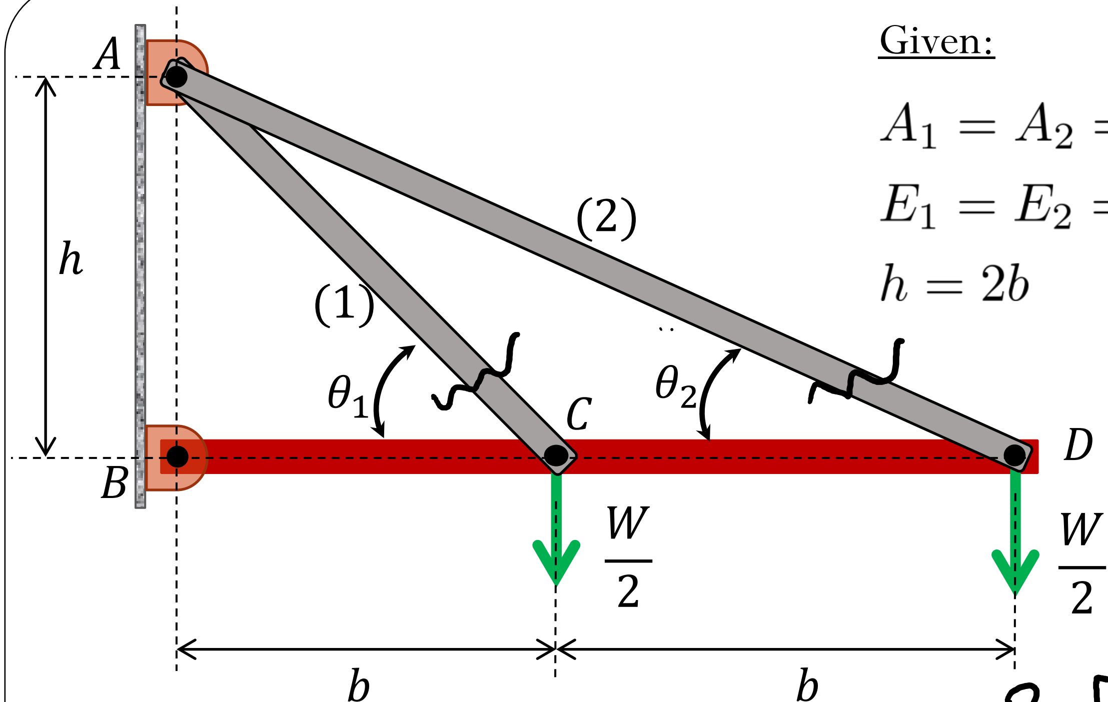

$$\delta_1 + \delta_2 = 0$$

$$F_2 = F_1 + P_B$$

$$\frac{F_1 L_1}{E_1 A_1} + \frac{(F_1 + P_B) \cdot L_2}{E_2 A_2} = 0 \rightarrow \frac{F_1 L_1}{E_1 A_1} + \frac{F_1 L_2}{E_2 A_2} = \frac{-P_B L_2}{E_2 A_2}$$

$$\rightarrow F_1 \left( \frac{L_1}{E_1 A_1} + \frac{L_2}{E_2 A_2} \right) = \frac{-P_B L_2}{E_2 A_2} \rightarrow \boxed{F_1 = \frac{-P_B L_2}{E_2 A_2 \left( \frac{L_1}{E_1 A_1} + \frac{L_2}{E_2 A_2} \right)}}$$

... can also solve for $F_2$




Given:

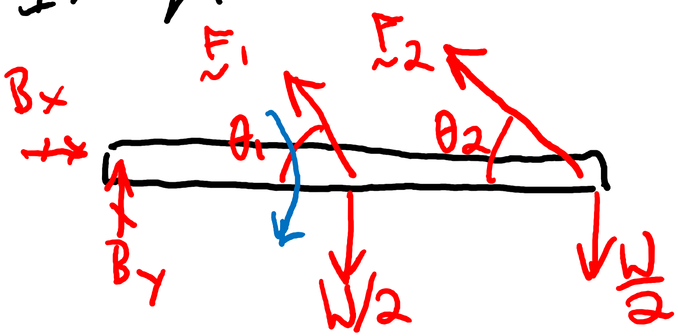
$$A_1 = A_2 = A$$
$$E_1 = E_2 = E$$
$$h = 2b$$

Find:

$$\sigma_1 = ?$$
$$\sigma_2 = ?$$
$$\delta_D = ?$$

Assume that BCD is rigid.

1. Equilibrium




$$\Sigma M_B = 0$$

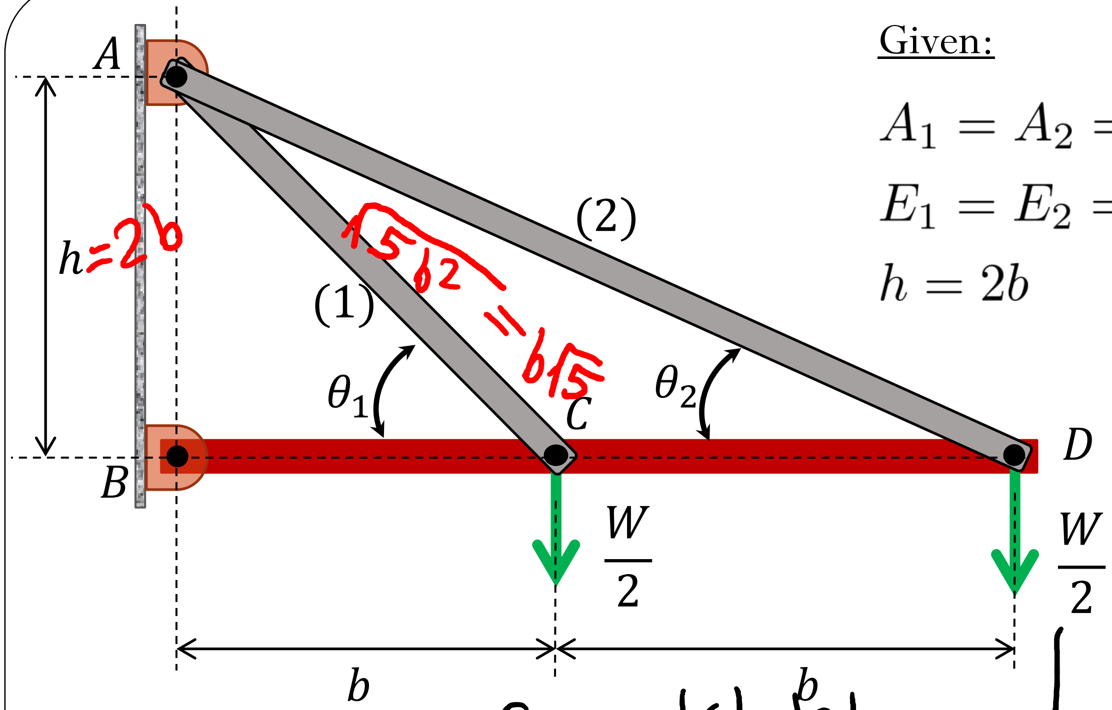
$$-\frac{W}{2}\cdot \cancel{b} - W\cancel{b} + F_1 \cdot \sin\theta_1 \cdot \cancel{b} + F_2 \sin\theta_2 \cdot 2\cancel{b} = 0$$

$$(1) \quad F_1 \cdot \sin\theta_1 + 2 \cdot F_2 \cdot \sin\theta_2 = \frac{3}{2}W$$

2. Force-Elongation

$$(2) \quad e_1 = \frac{F_1 \cdot L_1}{EA} \qquad [e_i] = \text{length}$$

$$(3) \quad e_2 = \frac{F_2 L_2}{EA}$$




**Given:**

$$A_1 = A_2 = A$$
$$E_1 = E_2 = E$$
$$h = 2b$$

**Find:**

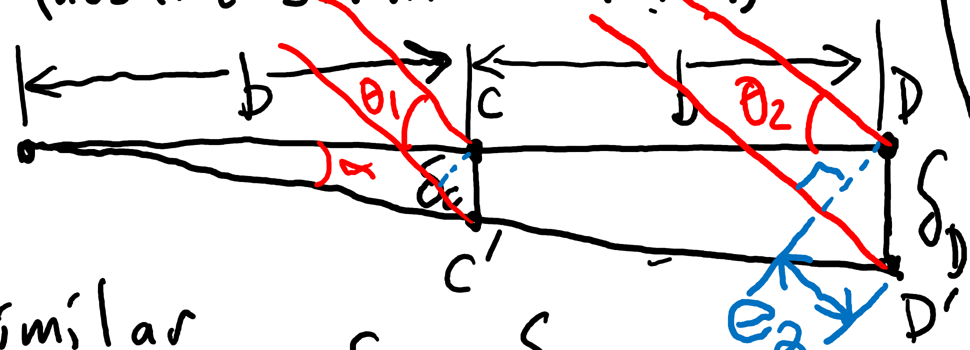
$$\sigma_1 = ?$$
$$\sigma_2 = ?$$
$$\delta_D = ?$$

$\theta_2 = 45^\circ$

$\Downarrow$

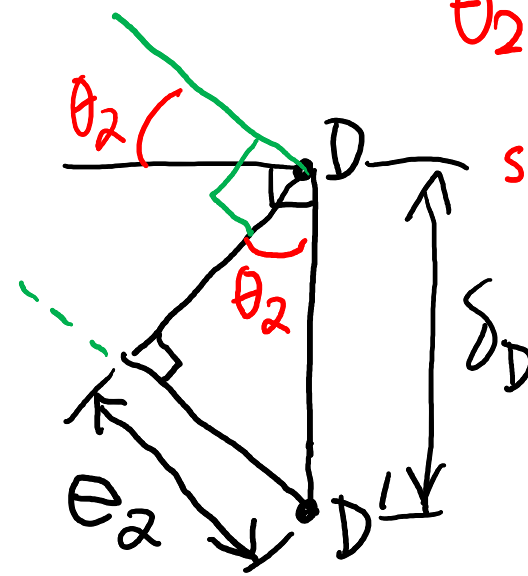
$\sin\theta_2 = \frac{\sqrt{2}}{2}$

3. Geometric Compatibility
(assume small rotation)




similar triangles

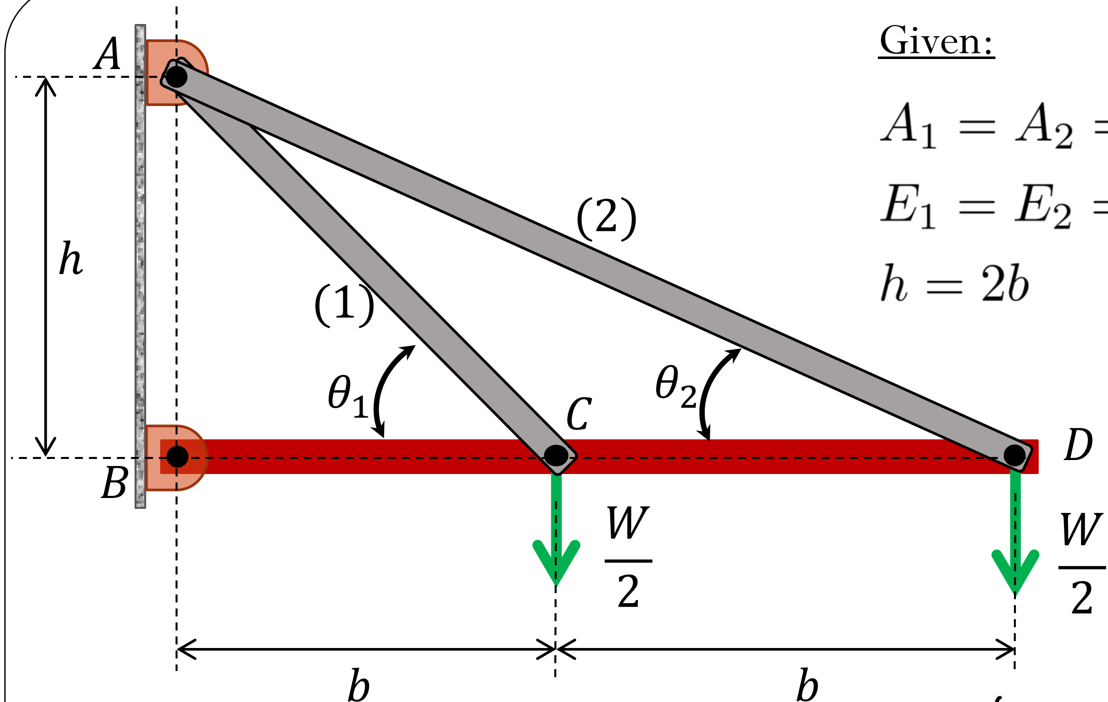
$$\frac{\delta_C}{b} = \frac{\delta_D}{2b} \Rightarrow \delta_D = 2\cdot\delta_C \quad (4)$$




$$\sin\theta_2 = \frac{e_2}{\delta_D} \quad (5)$$

$$\sin\theta_1 = \frac{e_1}{\delta_C} \quad (6)$$

$$\sin\theta_1 = 2/\sqrt{5}$$




**Given:**

$A_1 = A_2 = A$

$E_1 = E_2 = E$

$h = 2b$

**Find:**

$\sigma_1 = ?$

$\sigma_2 = ?$

$\delta_D = ?$

Apply $\delta_D = 2\delta_C$

$$\frac{\cancel{2} F_2 \cancel{b}}{\cancel{EA}} = \cancel{2}\left(\frac{5F_1\cancel{b}}{2\cancel{EA}}\right)$$

$$F_1 = \frac{2}{5}F_2$$

(3) into (5)

$$\frac{F_2\cdot(b\sqrt{2})}{EA\cdot\delta_D} = \frac{\sqrt{2}}{2} \rightarrow \delta_D = \frac{2F_2 b}{EA}$$

(2) into (6)

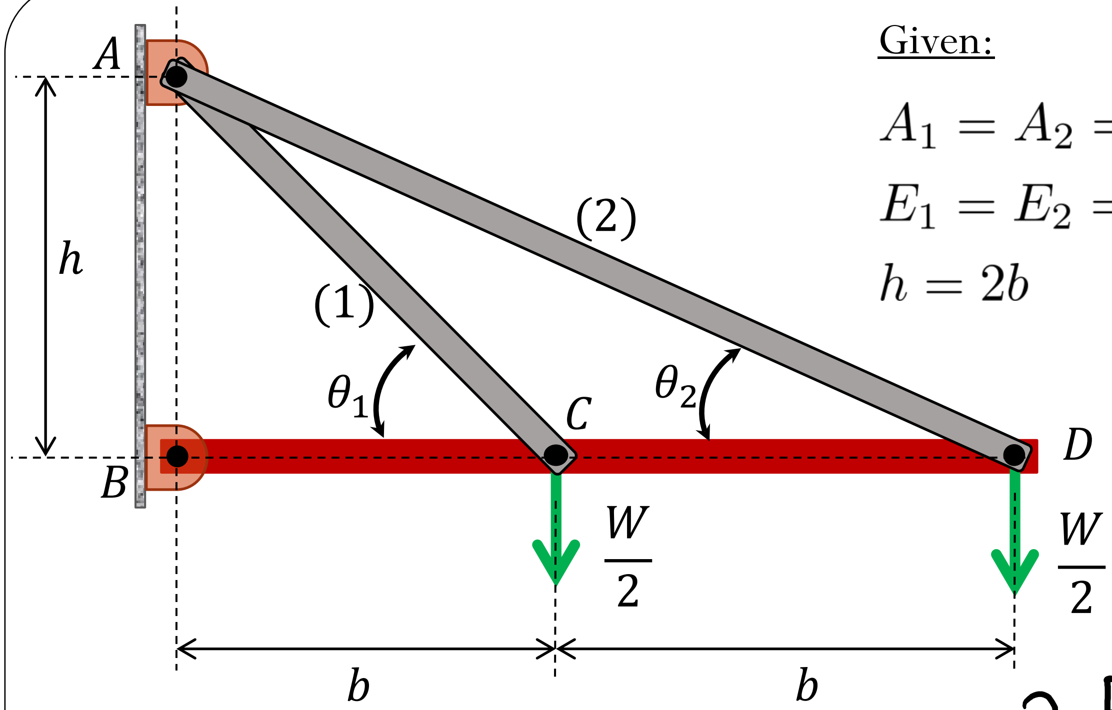
$$\frac{F_1\cdot b\cdot\sqrt{5}}{EA\cdot\delta_C} = \frac{2}{\sqrt{5}} \rightarrow \delta_C = \frac{5F_1 b}{2EA}$$

Plug into (1) and solve

$$\frac{2}{5}F_2\cdot\frac{2}{\sqrt{5}} + 2F_2\frac{\sqrt{2}}{2} = \frac{3W}{2}$$

$$F_2 = \frac{75W}{8\sqrt{5}+50\sqrt{2}}$$

$$F_1 = \frac{15\cdot W}{4\sqrt{5}+25\sqrt{2}}$$




**Given:**

$$A_1 = A_2 = A$$

$$E_1 = E_2 = E$$

$$h = 2b$$

**Find:**

$\sigma_1 = ?$

$\sigma_2 = ?$

$\delta_D = ?$

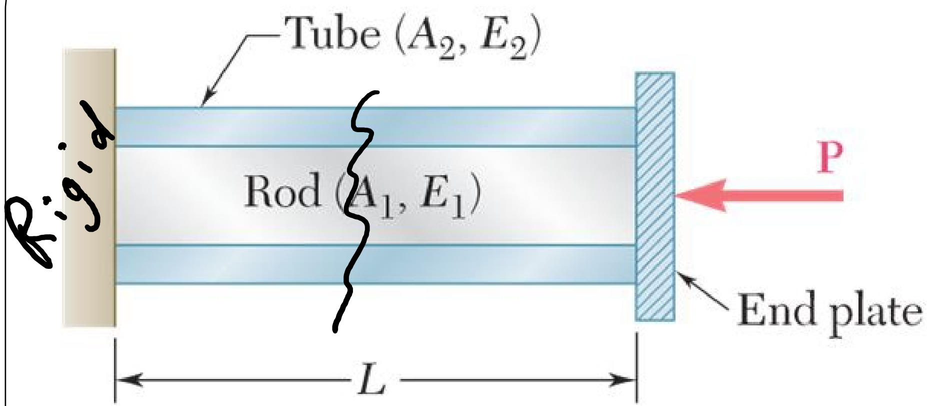
$$\sigma_1 = \frac{F_1}{A}$$

$$\sigma_2 = \frac{F_2}{A}$$

$$\delta_D = \frac{2 \cdot F_2 \cdot b}{E A}$$

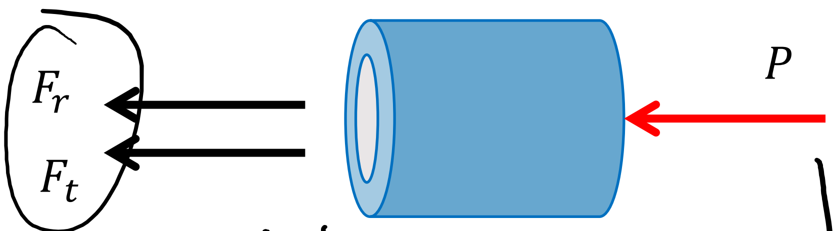
$$= \frac{\cancel{2} b}{E A}\left(\frac{75 W}{\cancel{8}^{\,4}\sqrt{5} + \cancel{50}^{\,25}\sqrt{2}}\right)$$

$$\boxed{\delta_D = \left(\frac{75}{4\sqrt{5} + 25\sqrt{2}}\right)\frac{W \cdot b}{E A}}$$




Find:

- Displacement
- Stress in the rod and tube




1. Equilibrium

$$-F_r - F_t - P = 0$$

$$F_r + F_t = -P$$

2. Force-Elongation

$$e_r = \frac{F_r \cdot L}{E_1 A_1} \quad \bigg| \quad e_t = \frac{F_t \cdot L}{E_2 A_2}$$

3. Geometric Compatibility

$$e_r = e_t$$

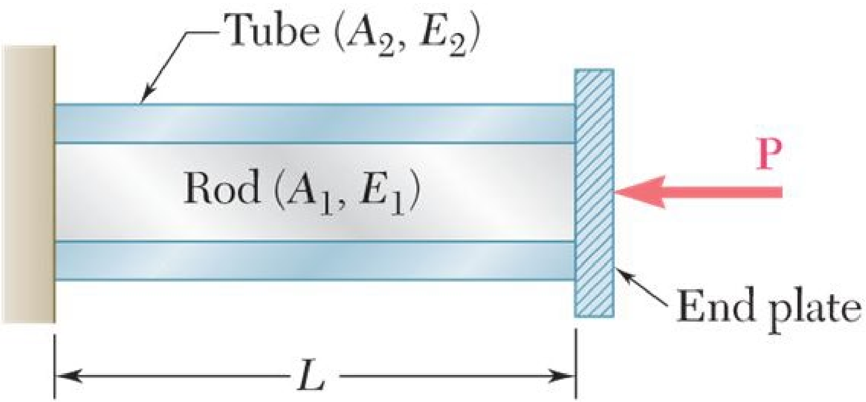
$$\frac{F_r \cancel{L}}{E_1 A_1} = \frac{F_t \cdot \cancel{L}}{E_2 A_2}$$

$$F_r = F_t \cdot \left(\frac{E_1 A_1}{E_2 A_2}\right)$$

$$F_t \cdot \left(\frac{E_1 A_1}{E_2 A_2}\right) + F_t \cdot \left(\frac{E_2 A_2}{E_2 A_2}\right) = -P$$

$$F_t \cdot \left(\frac{E_1 A_1 + E_2 A_2}{E_2 A_2}\right) = -P$$

$$F_t = \frac{-P \cdot E_2 A_2}{E_1 A_1 + E_2 A_2}$$




Find:

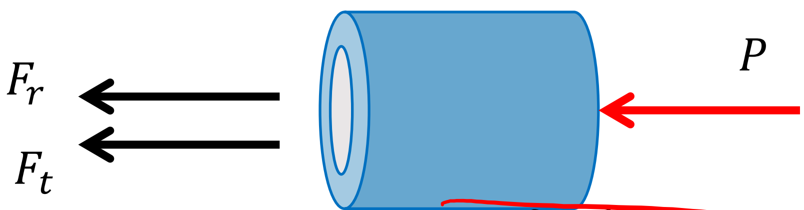
- Displacement
- Stress in the rod and tube ✓

$$F_r + F_t = -P$$

$$F_r + \left(\frac{-PE_2A_2}{E_1A_1+E_2A_2}\right) = -P\cdot\left(\frac{E_1A_1+E_2A_2}{E_1A_1+E_2A_2}\right)$$

$$F_r = \frac{P}{E_1A_1+E_2A_2}\cdot\left[\cancel{E_2A_2} - E_1A_1 - \cancel{E_2A_2}\right]$$

$$F_r = \frac{-P\cdot E_1A_1}{E_1A_1+E_2A_2}$$




$$\boxed{\sigma_r = \frac{F_r}{A_1} = \frac{-PE_1}{(EA)_1+(EA)_2}}$$

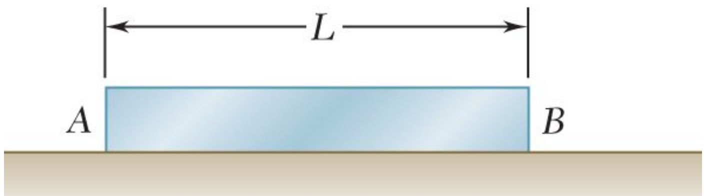
$$\boxed{\sigma_t = \frac{-P\cdot E_2}{(EA)_1+(EA)_2}}$$

$$e = e_t = e_r = \frac{F_t\cdot L}{E_2A_2}$$

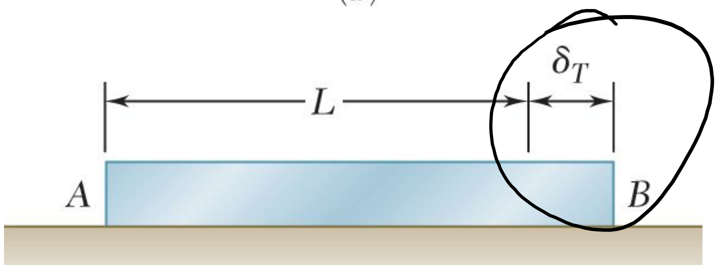
$$\boxed{e = \frac{-P\cdot L}{E_1A_1+E_2A_2}}$$

Displaces to the left

# Problems involving temperature changes




(a)




(b)

- Rod rests freely on a smooth horizontal surface
- Temperature of the rod is raised by $\Delta T$
- Rod elongates by an amount

No friction ⇒ No mech. force applied!

$$\delta_T = \alpha \, L \, \Delta T$$

$$[\alpha] = \frac{1}{\text{temp.}}$$

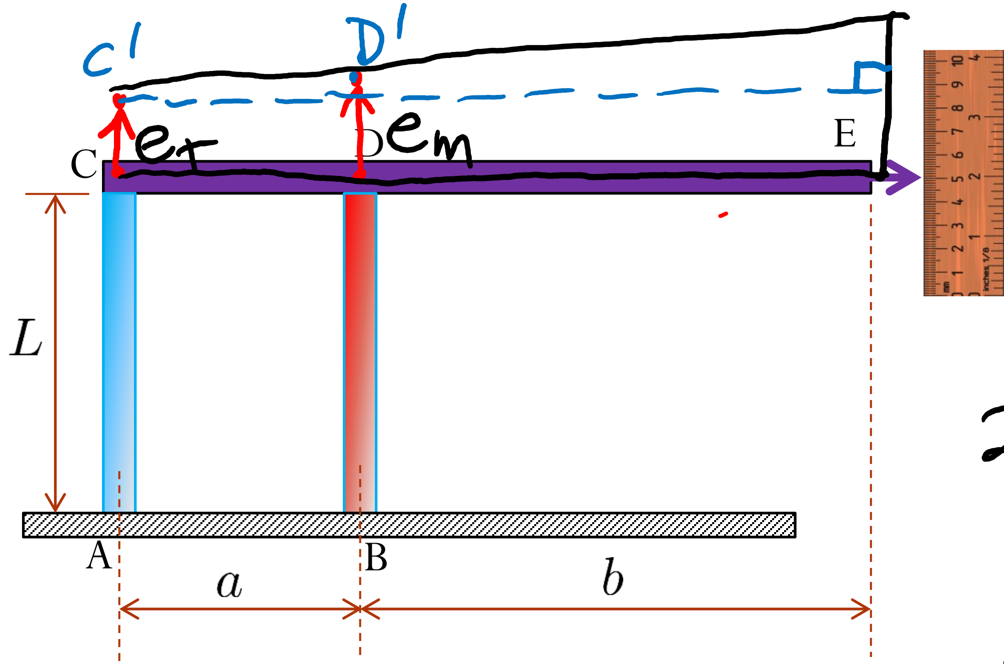
- $\alpha$: coefficient of thermal expansion ($/^oC$)
- This deformation is associated to a thermal strain

$$\epsilon_T = \alpha \, \Delta T \qquad [\alpha \cdot \Delta T] = \frac{\text{temp.}}{\text{temp.}}$$

**NOTE: NO STRESS is associated with the thermal strain**

**Verrazano-Narrows Bridge**: Because of thermal expansion of the steel cables, the bridge roadway is 12 feet (3.66 m) lower in summer than in winter

The device is used to measure a change in temperature. Rod AC and BD are made of Tungsten and Magnesium respectively. At a given temperature $T_o$, the rigid bar CDE is in the horizontal position. Determine an expression for the temperature $T$ as a function of the vertical displacement of point E, $\delta_E$.




- Rod AC: Tungsten $\alpha_t$
- Rod BD: Magnesium $\alpha_m$

$$\alpha_m > \alpha_t$$

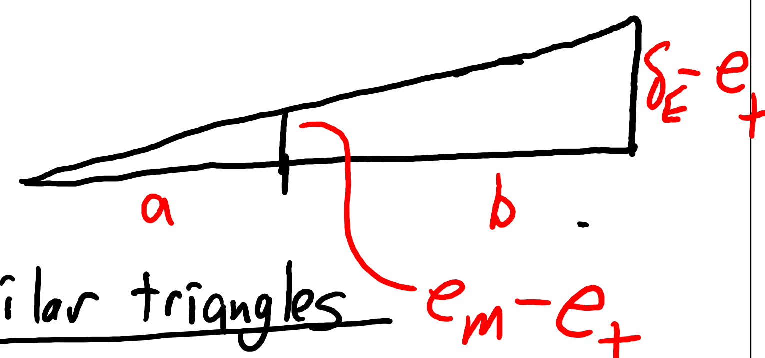
$$\Delta T = T - T_o$$

1. Elongation

Tungsten : $e_t = \alpha_t \cdot L \cdot \Delta T$

Magnesium : $e_m = \alpha_m \cdot L \cdot \Delta T$

2.

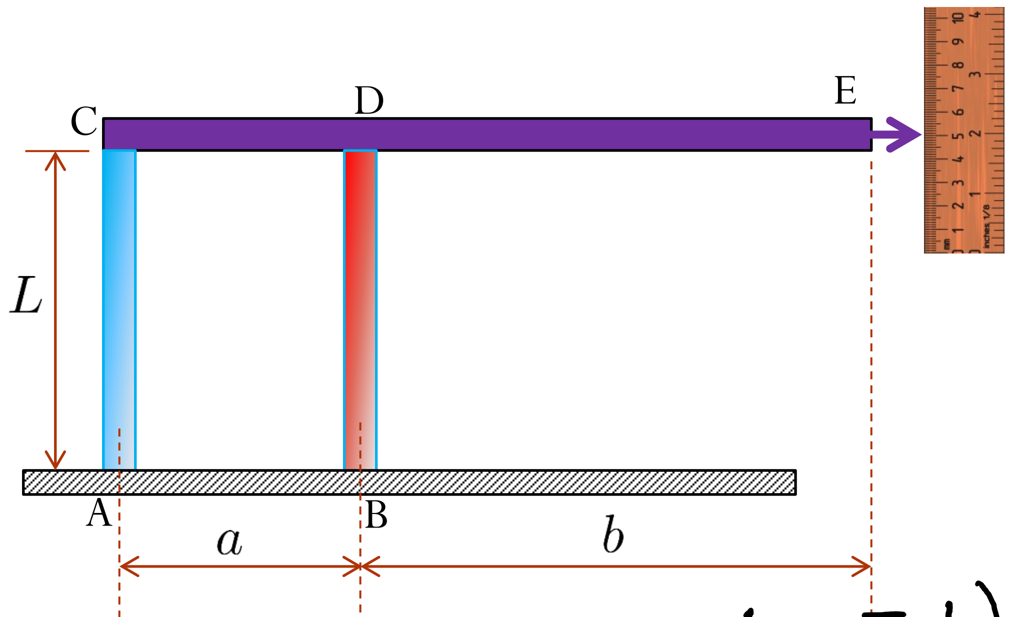



similar triangles

$$\frac{e_m - e_t}{a} = \frac{\delta_E - e_t}{a+b}$$

$$\delta_E = (e_m - e_t)\cdot\frac{a+b}{a} + e_t$$

The device is used to measure a change in temperature. Rod AC and BD are made of Tungsten and Magnesium respectively. At a given temperature $T_o$, the rigid bar CDE is in the horizontal position. Determine an expression for the temperature $T$ as a function of the vertical displacement of point E, $\delta_E$.




- Rod AC: Tungsten $\alpha_t$
- Rod BD: Magnesium $\alpha_m$

$$\alpha_m > \alpha_t$$

$$e_m = \alpha_m \Delta T \cdot L$$

$$e_t = \alpha_t \cdot \Delta T \cdot L$$

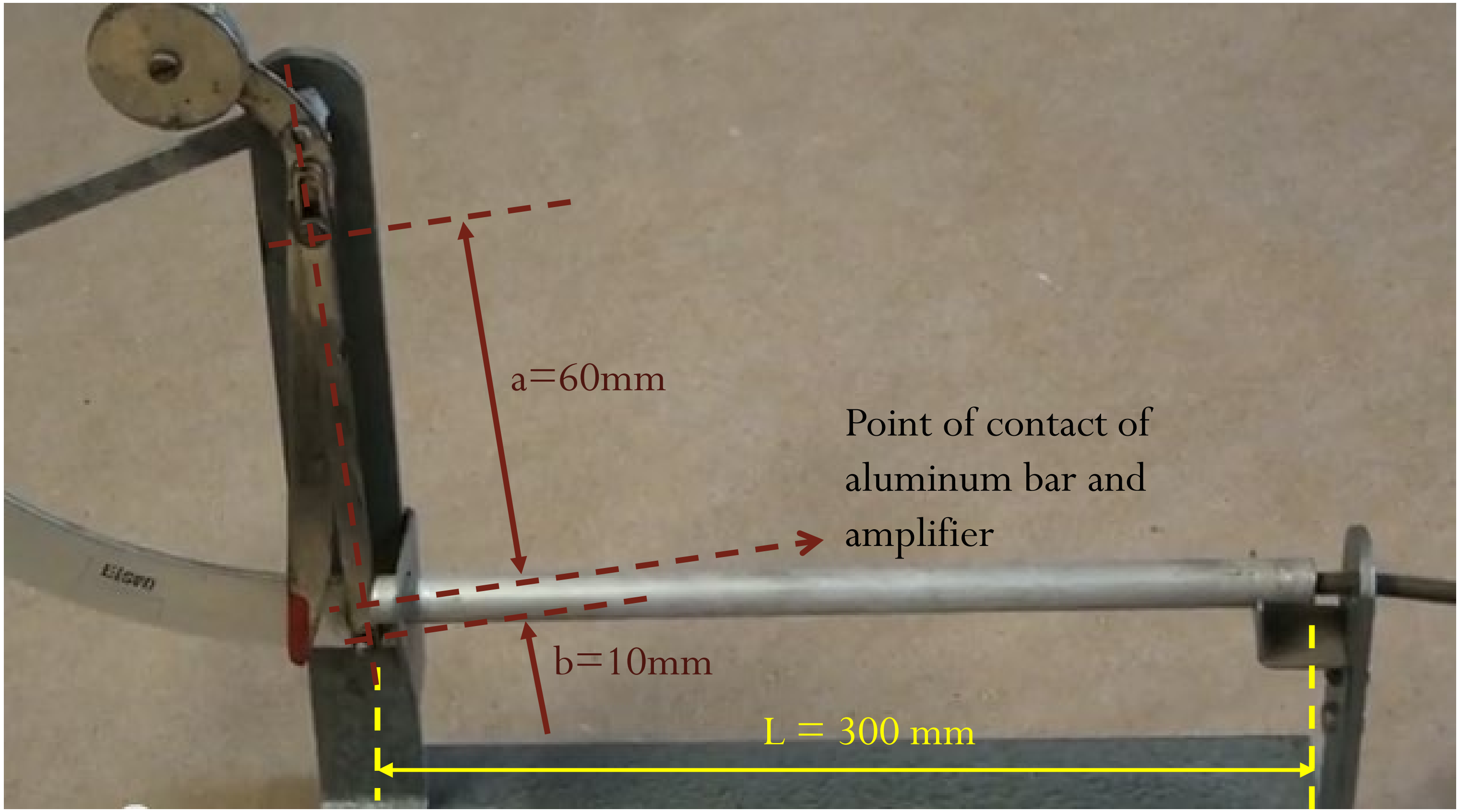
$$\delta_E = (\alpha_m \cdot \Delta T \cdot L - \alpha_t \cdot \Delta T \cdot L)\left(\frac{a+b}{a}\right) + \alpha_t \cdot \Delta T \cdot L$$

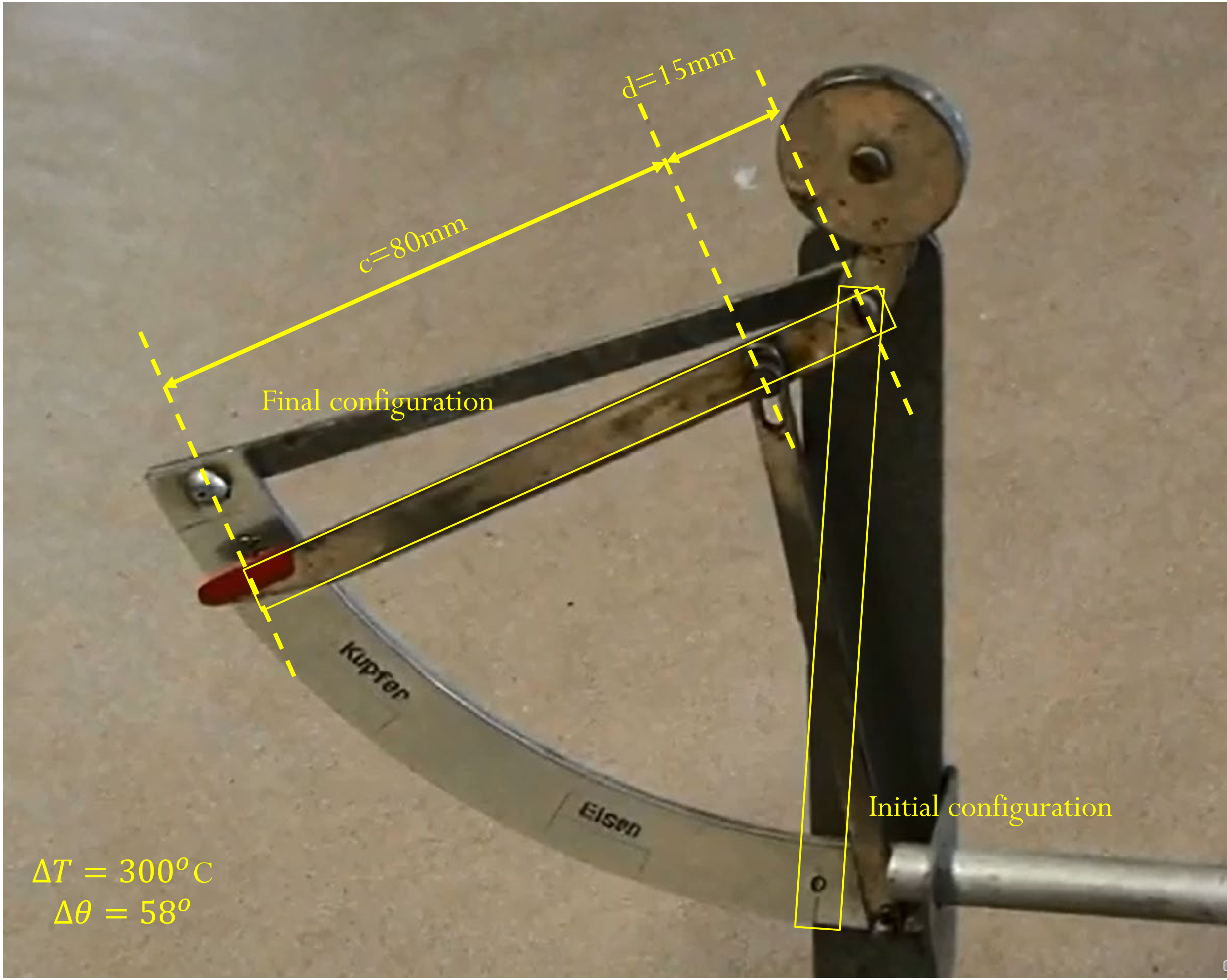
$$= \Delta T \cdot L \left[(\alpha_m - \alpha_t) \cdot \left(\frac{a+b}{a}\right) + \alpha_t\right]$$

$$\boxed{T = T_o + \frac{\delta_E}{L \cdot \left[\alpha_t + (\alpha_m - \alpha_t)\left(\frac{a+b}{a}\right)\right]}}$$

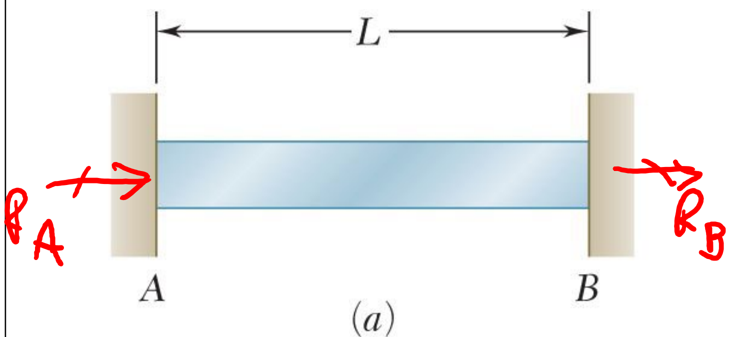
# Measuring the coefficient of thermal expansion

http://www.youtube.com/watch?v=TDnLbjd429M




d=15mm
c=80mm
Final configuration
Kupfer
Eisen
Initial configuration
$\Delta T = 300^o$C
$\Delta\theta = 58^o$

# Statically indeterminate problems

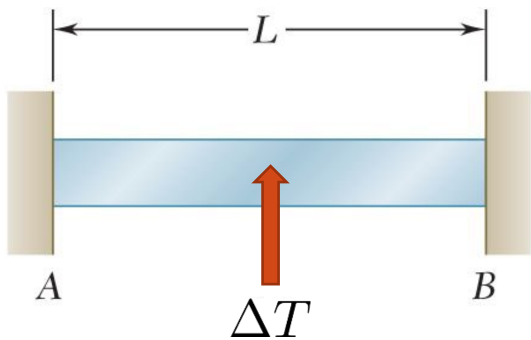




- Initially, rod of length $L$ is placed between two supports at a distance $L$ from each other
- No internal forces $\longrightarrow$ no stress or strain
- Equilibrium: $R_A = -R_B = 0$

$$F = -R_A$$

Statically indeterminate problem!




- After raising the temperature, the <u>total elongation of the rod is still zero!</u>
- The total elongation is given by

$$\delta = \underbrace{\frac{F\,L}{E\,A}}_{\text{mechanical}} + \overbrace{\alpha\,L\,\Delta T}^{\text{thermal}} = 0$$

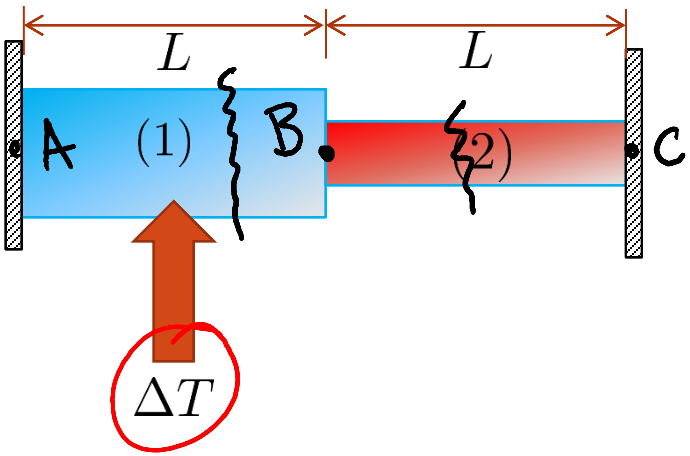
$$\Rightarrow \quad F = \underline{-\alpha\,E\,A\,\Delta T}$$

Rod is under compression


- The stress in the rod due to change in temperature is given by

$$\sigma = -\alpha\,E\,\Delta T$$

} only happens (nonzero therm σ.) if motion constrained!




$E_1 = E_2 = E$

$\alpha_1 = \alpha_2 = \alpha$

$A_1,\ A_2$

Assume perfect insulation at point B.

Find $\sigma_1$ & $\sigma_2$

Find $\delta_B$.

1. Equilibrium

$\Sigma F_x = 0 \Rightarrow F_1 = F_2 = F$

unknown

2. Force - elongation

$$\delta_1 = \frac{F_1 L}{E A_1} + \alpha \cdot \Delta T \cdot L$$

$$\delta_2 = \frac{F_2 L}{E A_2}$$

3. Geom. Compatibility

$$\delta_1 + \delta_2 = 0 \Rightarrow \delta_1 = -\delta_2$$

$$\frac{F \cancel{L}}{E A_1} + \alpha \Delta T \cancel{L} = \frac{-F \cdot \cancel{L}}{E A_2}$$

$$\frac{F}{E}\underbrace{\left(\frac{1}{A_1} + \frac{1}{A_2}\right)}_{\frac{A_1 + A_2}{A_1 A_2}} = -\alpha \cdot \Delta T$$

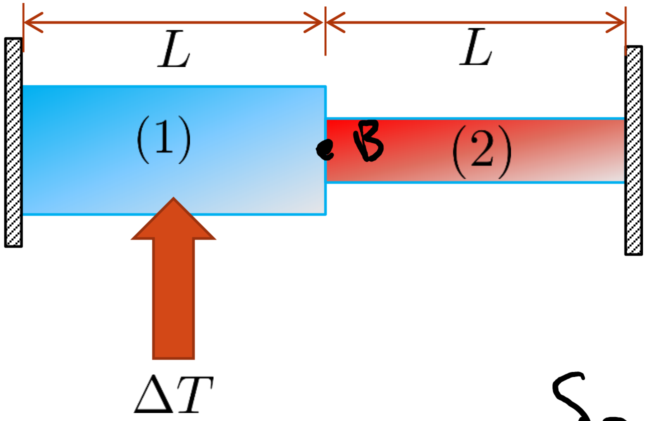
$$\Rightarrow F\left(\frac{A_1 + A_2}{A_1 A_2 E}\right) = -\alpha \Delta T$$

$$F = -\frac{\alpha \cdot \Delta T \cdot A_1 A_2 E}{A_1 + A_2}$$

$\sigma_1 = F/A_1 = \ldots$

$\sigma_2 = F/A_2 = \ldots$

$\delta_B = \delta_1$




$E_1 = E_2 = E$

$\alpha_1 = \alpha_2 = \alpha$

$A_1, A_2$

$$\delta_B = \delta_1 = \frac{F \cdot L}{E A_1} + \alpha \cdot \Delta T \cdot L$$

$$F = \frac{-\alpha \cdot \Delta T \cdot A_1 \cdot A_2 \cdot E}{A_1 + A_2}$$

$$\delta_B = \frac{-\alpha \cdot \Delta T \cdot A_2 \cdot L}{A_1 + A_2} + \alpha \cdot \Delta T \cdot L$$

$$1 = \frac{A_1 + A_2}{A_1 + A_2}$$

$$= \alpha \cdot \Delta T \cdot L \cdot \left(1 - \frac{A_2}{A_1 + A_2}\right)$$

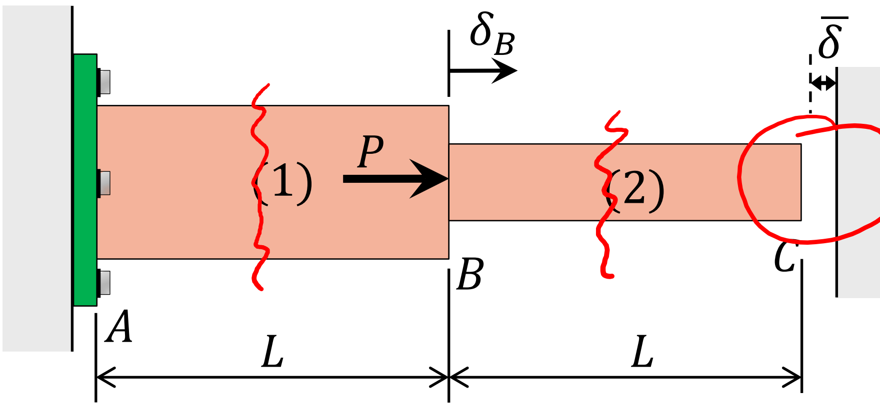
$$= \alpha \cdot \Delta T \cdot L \cdot \left(\frac{A_1 + \cancel{A_2} - \cancel{A_2}}{A_1 + A_2}\right)$$

$$\delta_B = \frac{\alpha \cdot \Delta T \cdot L \cdot A_1}{A_1 + A_2}$$

B displaces to the right

# Misfit problems

$F_1 = k_1 \cdot e_1 \qquad F_2 = k_2 \cdot e_2$

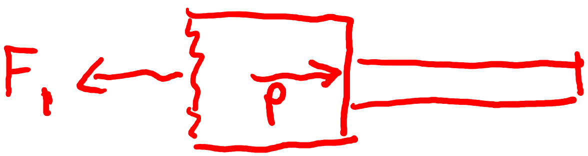



Rods (1) and (2) have stiffnesses $k_1$ and $k_2$, respectively. At end A, the rod is connected to a rigid wall, but at end C there is a small gap $\bar{\delta}$ between the original position of end C and the rigid wall. A force $P$ is applied at point B as shown.

a) What is the minimum required force $P_{\text{min}}$ required to close the gap between end C and the wall at the right?

b) Derive expressions for the stress in each bar if the applied force is $P > P_{\text{min}}$. Which bar(s) are in tension? Which are in compression?


$F_2 = 0$




$F_1 = P$

To close the gap

$e_2 = 0, \quad e_1 = \bar{\delta}$

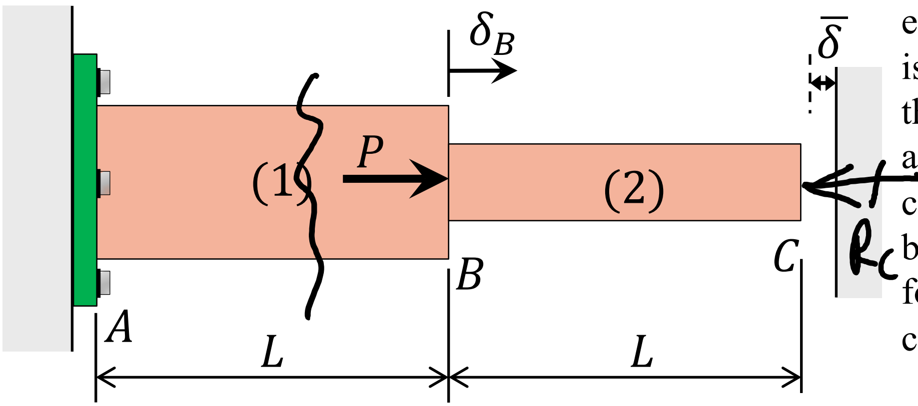
$F_1 = k_1 e_1$

$= k_1 \cdot \bar{\delta}$

$\Rightarrow P = k_1 \cdot \bar{\delta}$

$$\boxed{P_{\text{min}} = k_1 \cdot \bar{\delta}}$$

# Misfit problems

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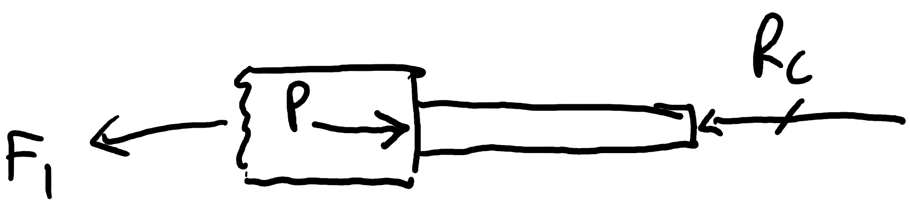
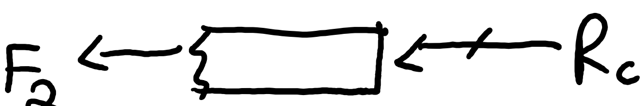



$$-F_1 + P - R_C = 0$$




$$-F_2 - R_c = 0$$

$$-F_2 = +R_c$$

(1)

$$-F_1 + P + F_2 = 0 \Rightarrow F_1 - F_2 = P$$

$$e_1 = \frac{F_1}{k_1} \qquad e_2 = \frac{F_2}{k_2}$$

Geom. Compat.

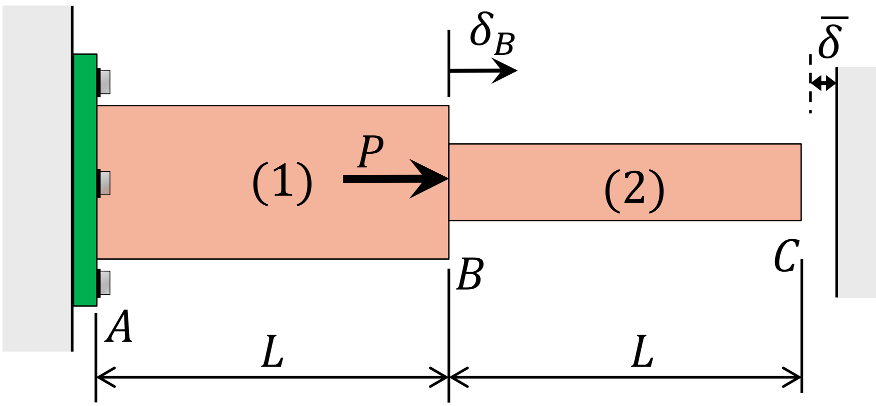
$$e_1 + e_2 = \bar{\delta}$$

$$\frac{F_1}{k_1} + \frac{F_2}{k_2} = \bar{\delta}$$

$$\frac{P + F_2}{k_1} + \frac{F_2}{k_2} = \bar{\delta}$$

$$\Rightarrow (P + F_2)k_2 + F_2 k_1 = \bar{\delta} k_1 k_2$$

# Misfit problems

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$$(P+F_2)k_2 + F_2 \cdot k_1 = \bar{\delta} \cdot k_1 \cdot k_2$$

$$F_2 \cdot (k_1 + k_2) + P \cdot k_2 = \bar{\delta} \cdot k_1 \cdot k_2$$

$$F_2 = \frac{(\bar{\delta} \cdot k_1 - P) \cdot k_2}{k_1 + k_2}$$

$$\sigma_2 = \frac{F_2}{A_2} = \frac{(\bar{\delta} k_1 - P) \cdot k_2}{(k_1 + k_2) \cdot A_2}$$

see that $\bar{\delta} \cdot k_1 = P_{min} \Rightarrow (P_{min} - P) < 0$

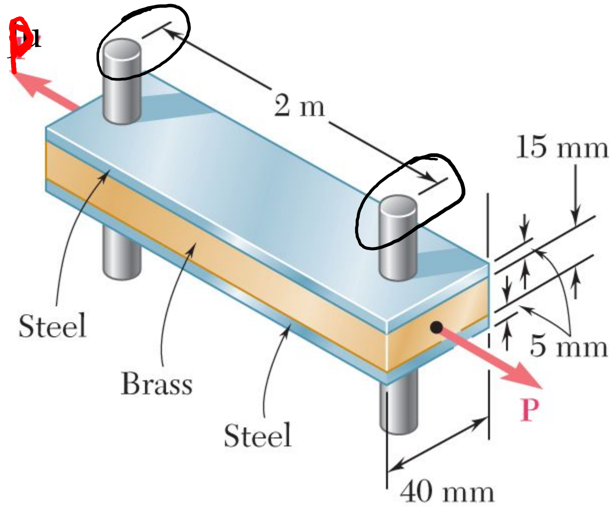
$\Rightarrow$ bar 2 is in compression!

$$F_1 = P + F_2$$

$$= P \cdot \left(\frac{k_1 + k_2}{k_1 + k_2}\right) + \frac{(\bar{\delta} k_1 - P) k_2}{k_1 + k_2}$$

$$\sigma_1 = \frac{(P + \bar{\delta} \cdot k_2) k_1}{(k_1 + k_2) A_1}$$

bar 1 is in tension!




Two steel bars ($E_s$ = 200 GPa and $\alpha_s = 11.7 \times 10^{-6}/°C$) are used to reinforce a brass bar ($E_b$ = 105 GPa, $\alpha_b = 20.9 \times 10^{-6}/°C$) that is subjected to a load $P$ = 25 kN. When the steel bars were fabricated, the distance between the centers of the holes that were to fit on the pins was made 0.5 mm smaller than the 2 m needed. The steel bars were then placed in an oven to increase their length so that they would just fit on the pins. Following fabrication, the temperature in the steel bars dropped back to room temperature. Determine (*a*) the increase in temperature that was required to fit the steel bars on the pins, (*b*) the stress in the brass bar after the load is applied to it.

a) steel bars only

$L = 2\,m$ (desired length)

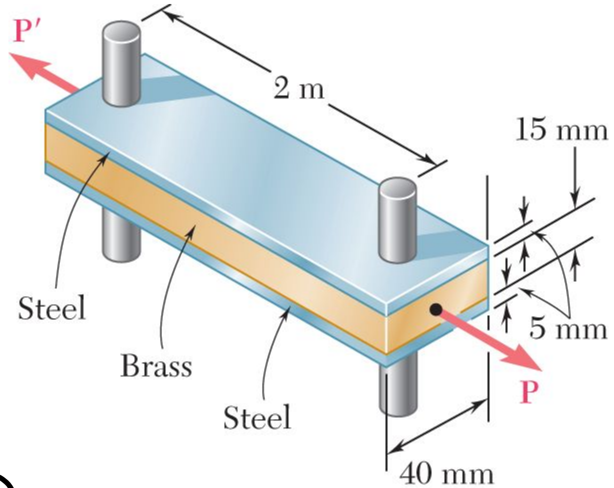
$L_{fab} = L - \delta_m = 2m - 0.0005m = 1.9995m$

$\delta_m$: misfit

want $\delta_T = \alpha_s \cdot \Delta T \cdot L_{fab} = \delta_m \Rightarrow \Delta T = \dfrac{\delta_m}{\alpha_s \cdot L_{fab}}$

$$\boxed{\Delta T} = \frac{0.0005\cancel{m}}{(11.7\times10^{-6}/°C)(1.9995\cancel{m})} = \boxed{21.4°C}$$

b) After steel is pinned to the brass:
1) steel cools to init. temp. (2) steel & brass achieve a new length $\neq L$ ($=2m$) (3) Apply $P$ to the system

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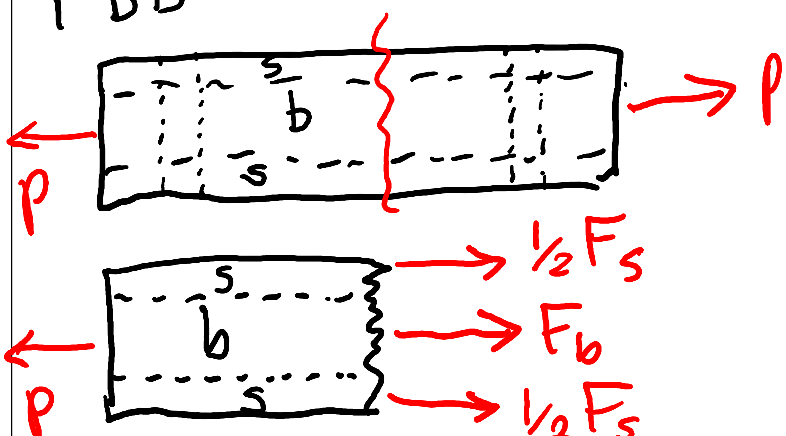
FBD




Equil. $\Sigma F_x = 0$

$$-P + F_b + F_s = 0$$

$$F_b + F_s = P$$

compatibility

$$\delta_s = \delta_b$$

force - elongation

$$\delta_s = \underbrace{-\delta_M}_{\text{return to original length upon cooling}} + \underbrace{\frac{F_s \cdot L}{E_s A_s}}_{\text{change in length due to internal forces}}$$

$$\delta_b = \frac{F_b \cdot L}{E_b A_b} \quad \Big\} \text{ no thermal effects}$$

$$\frac{F_s L}{E_s A_s} - \delta_M = \frac{F_b L}{E_b A_b}$$

$$\boxed{\sigma_b = 3.67 \text{ MPa}}$$