Chapter 4: Axial Load

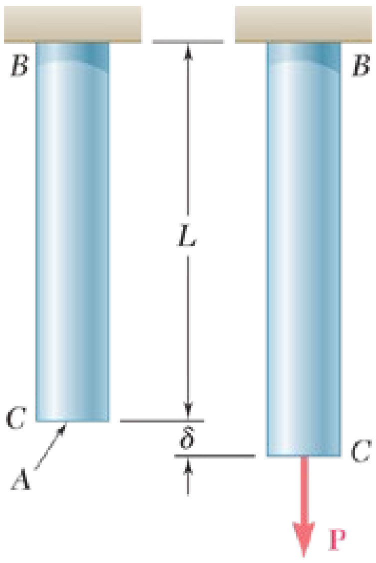
Chapter Objectives

- ✓ Determine the elastic deformation of axially loaded members
- ✓ Apply the principle of superposition for total effect of different loading cases
- ✓ Deal with compatibility conditions
- ✓ Deal with thermal stresses
- ✓ Misfit problems

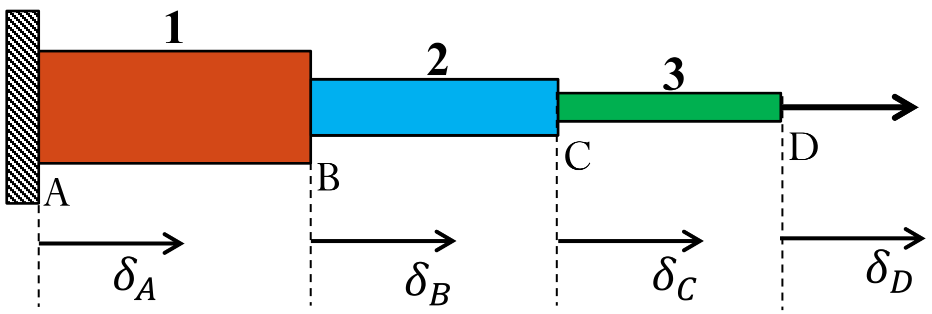
Axial deformation

So far…

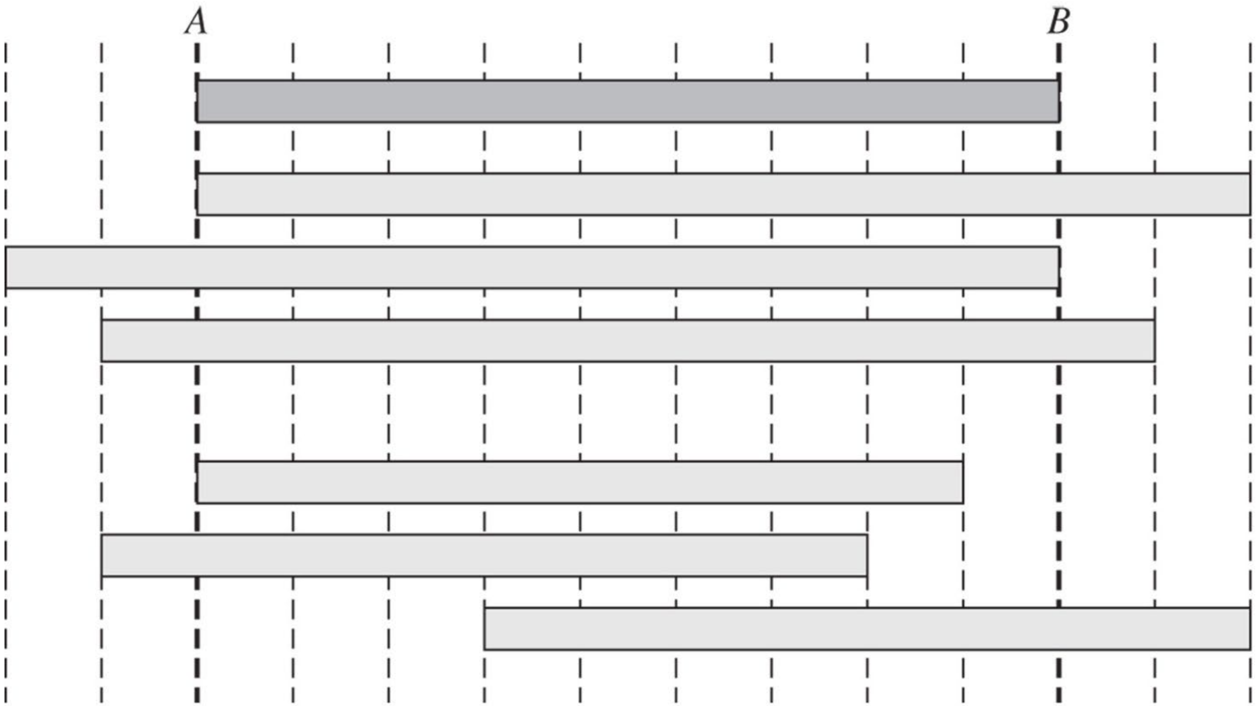
$$\sigma = \frac{F}{A} \qquad \epsilon = \frac{\delta}{L} \qquad \sigma = E\,\epsilon$$



Superposition principle: 1) The loading must be linearly related to the stress or displacement that is to be determined; 2) Small deformation assumption

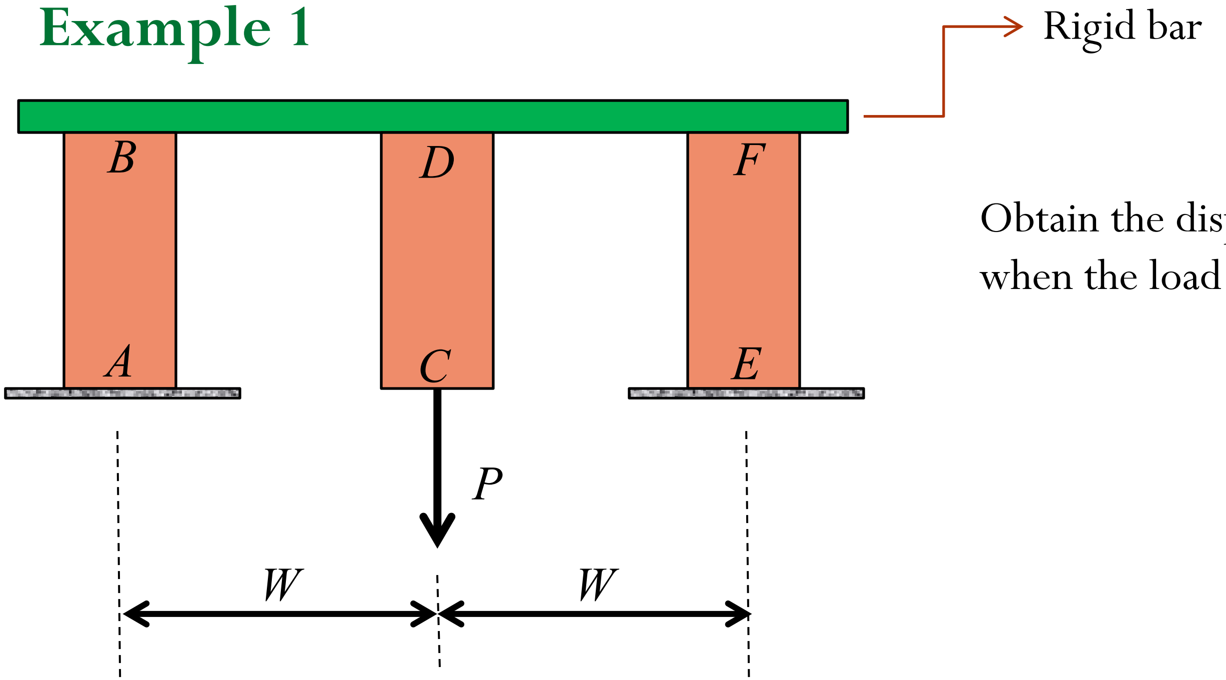


$$\delta = \sum_i \delta_i = \sum_i \frac{F_i\, L_i}{E_i\, A_i}$$



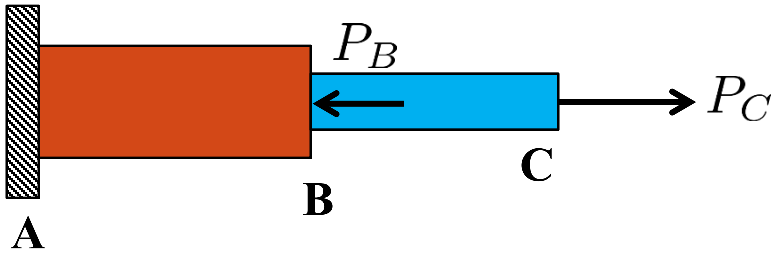
δ_A	δ_B	$\delta = \delta_{B/A}$
0	2	2
-2	0	2
-1	1	2
0	-1	-1
-1	-2	-1

Example 1

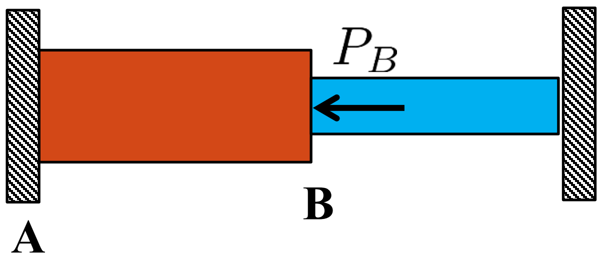


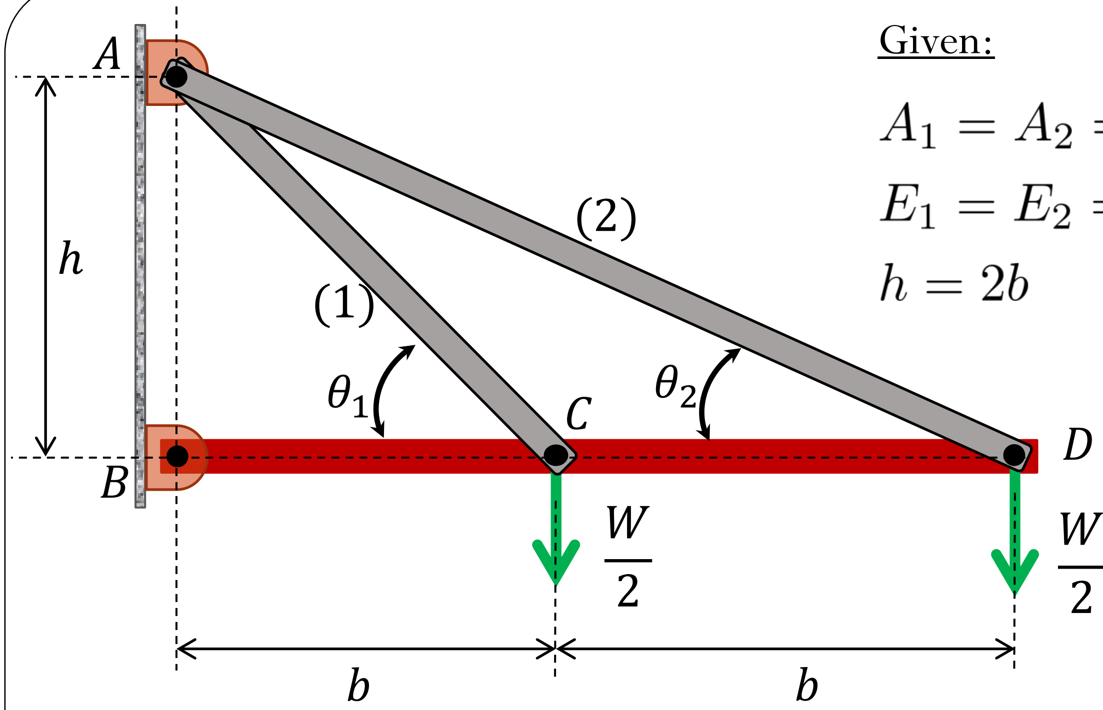
Obtain the displacement of points C and D when the load P is applied

Statically **Determinate** Problems



Statically **Indeterminate** Problems





Given:

$$A_1 = A_2 = A$$

$$E_1 = E_2 = E$$

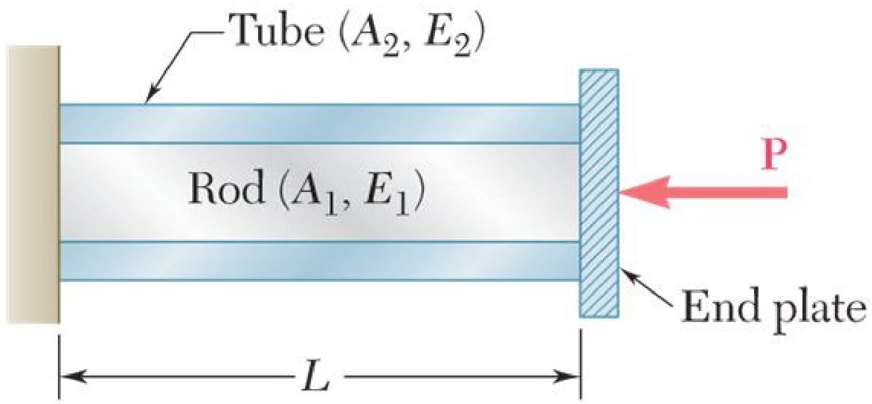
$$h = 2b$$

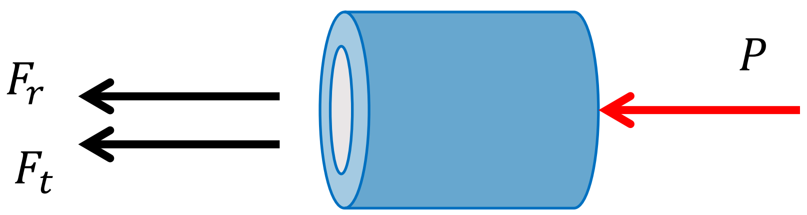
Find:

$$\sigma_1 = ?$$

$$\sigma_2 = ?$$

$$\delta_D = ?$$

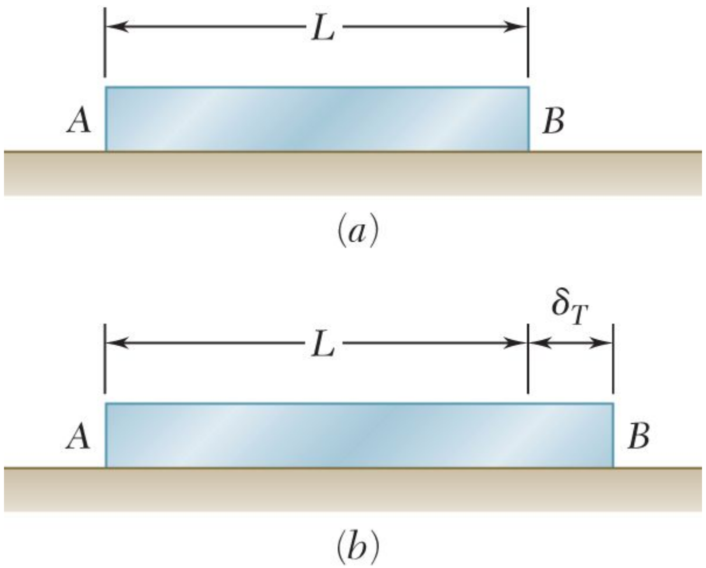




Find:

- Displacement
- Stress in the rod and tube

Problems involving temperature changes



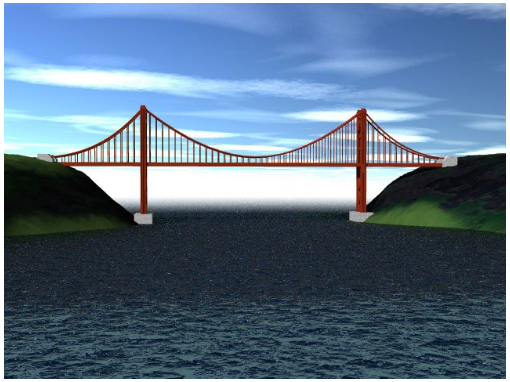
- Rod rests freely on a smooth horizontal surface
- Temperature of the rod is raised by ΔT
- Rod elongates by an amount

$$\delta_T = \alpha \, L \, \Delta T$$

- α: coefficient of thermal expansion ($/^oC$)
- This deformation is associated to a thermal strain

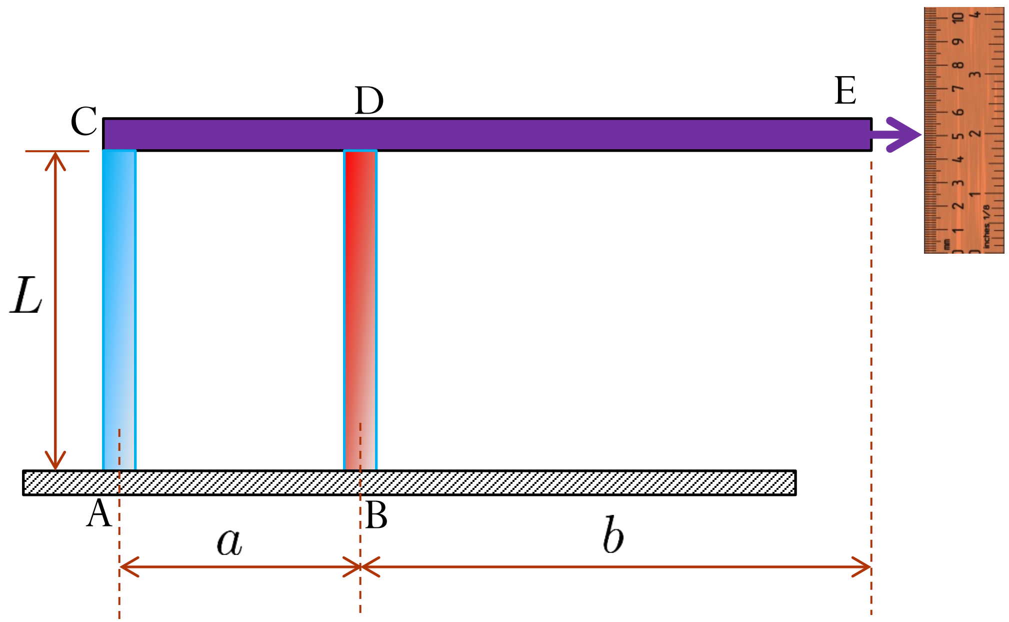
$$\epsilon_T = \alpha \, \Delta T$$

NOTE: NO STRESS is associated with the thermal strain



Verrazano-Narrows Bridge: Because of thermal expansion of the steel cables, the bridge roadway is 12 feet (3.66 m) lower in summer than in winter

The device is used to measure a change in temperature. Rod AC and BD are made of Tungsten and Magnesium respectively. At a given temperature T_o, the rigid bar CDE is in the horizontal position. Determine an expression for the temperature T as a function of the vertical displacement of point E, δ_E.

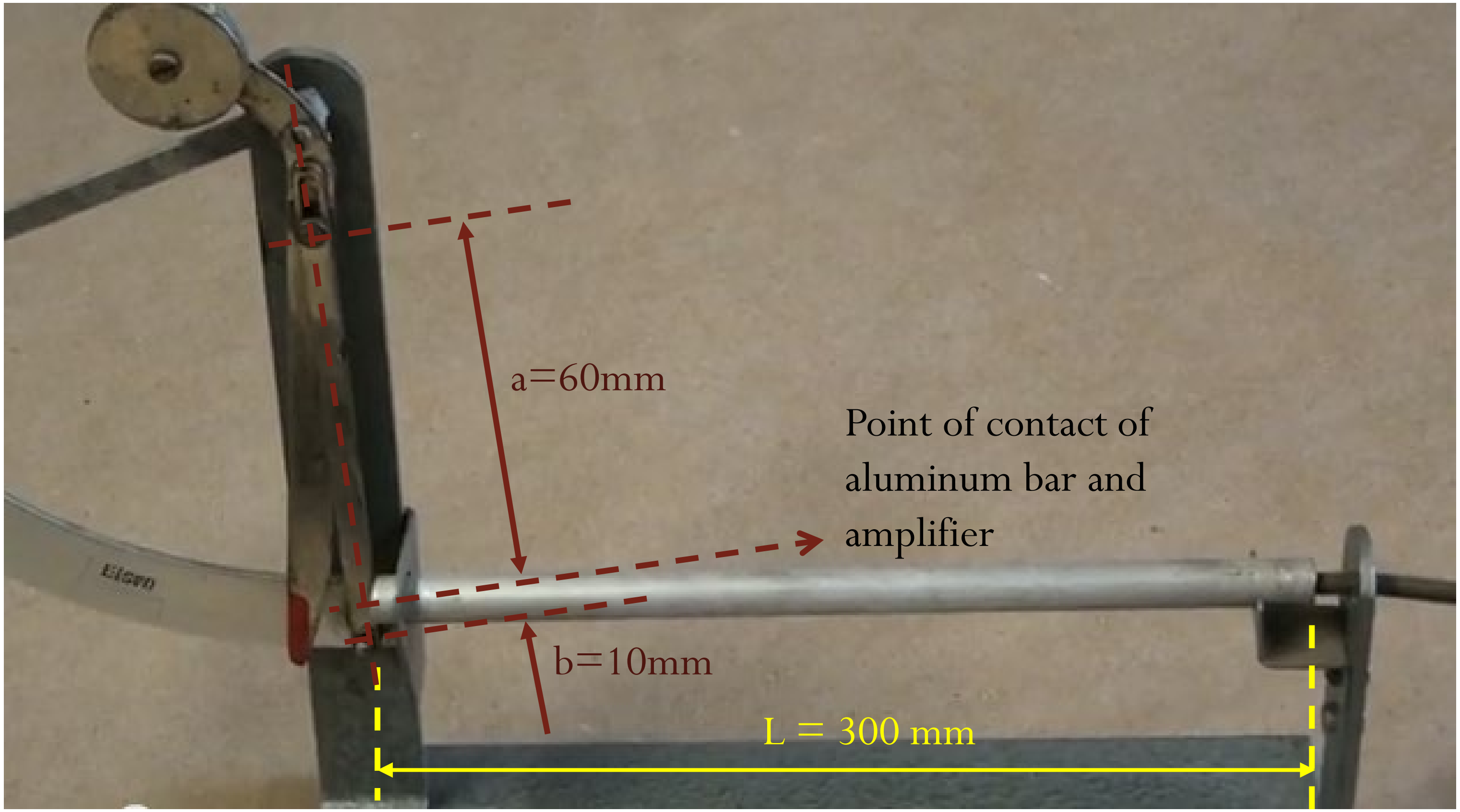


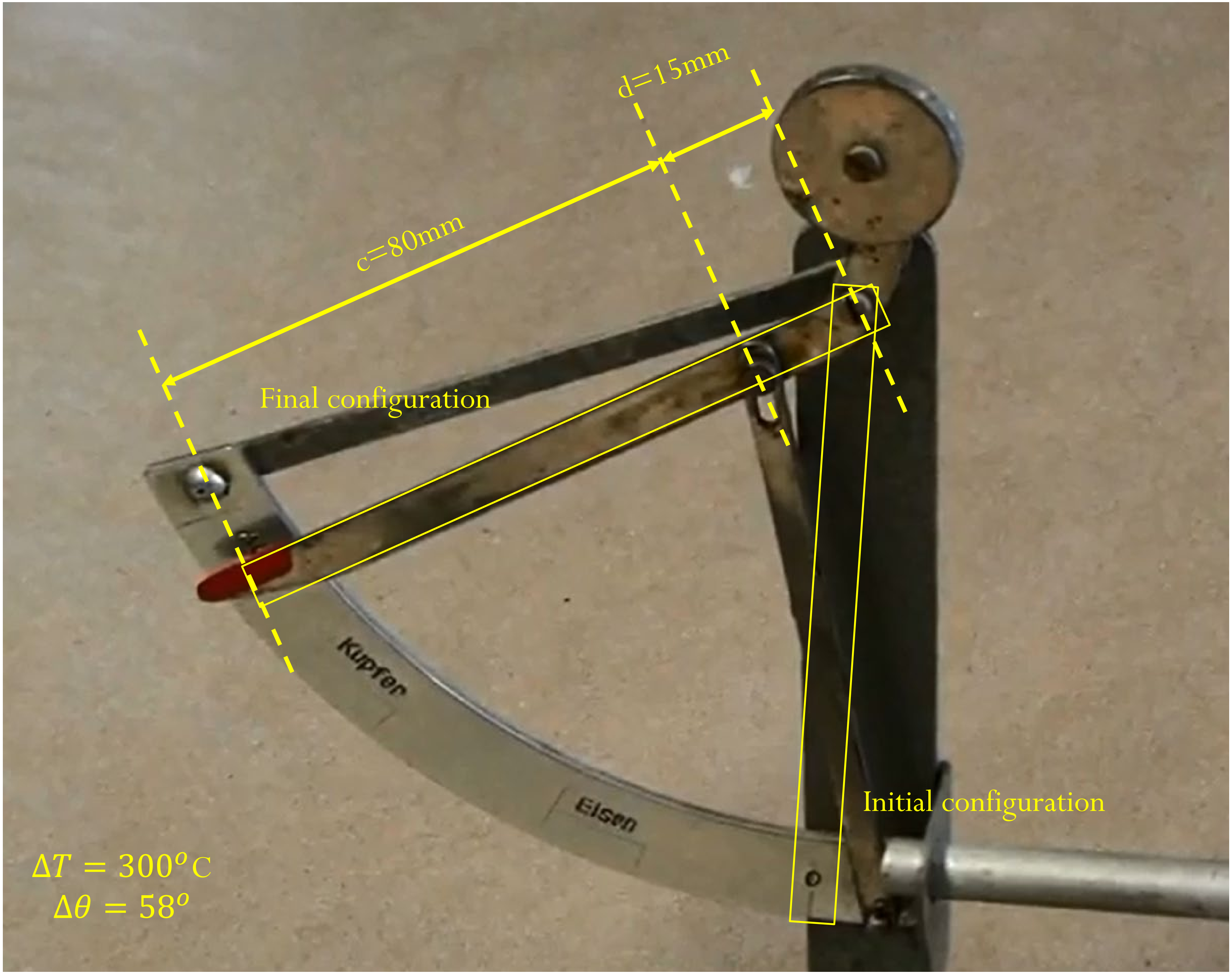
- Rod AC: Tungsten α_t
- Rod BD: Magnesium α_m

$$\alpha_m > \alpha_t$$

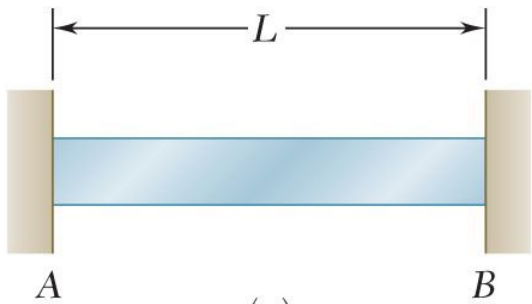
Measuring the coefficient of thermal expansion

http://www.youtube.com/watch?v=TDnLbjd429M




d=15mm
c=80mm
Final configuration
Kupfer
Eisen
Initial configuration
$\Delta T = 300^o$C
$\Delta\theta = 58^o$

Statically indeterminate problems



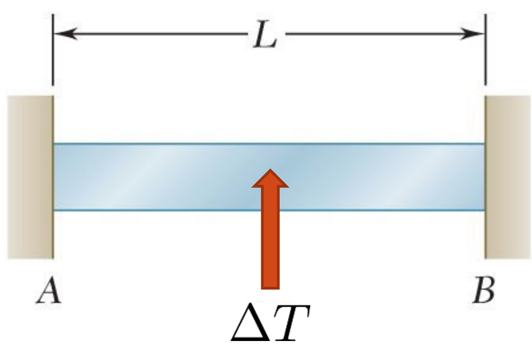
(a)

- Initially, rod of length L is placed between two supports at a distance L from each other
- No internal forces $\longrightarrow$ no stress or strain
- Equilibrium: $R_A = -R_B = 0$

$$F = -R_A$$ Statically indeterminate problem!

- After raising the temperature, the <u>total elongation of the rod is still zero!</u>
- The total elongation is given by

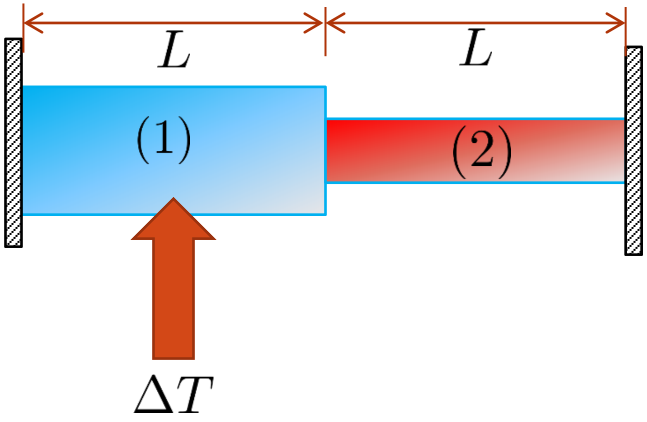
$$\delta = \frac{FL}{EA} + \alpha\, L\, \Delta T = 0$$



$$\Rightarrow \quad F = -\alpha\, E\, A\, \Delta T$$ Rod is under compression

- The stress in the rod due to change in temperature is given by

$$\sigma = -\alpha\, E\, \Delta T$$

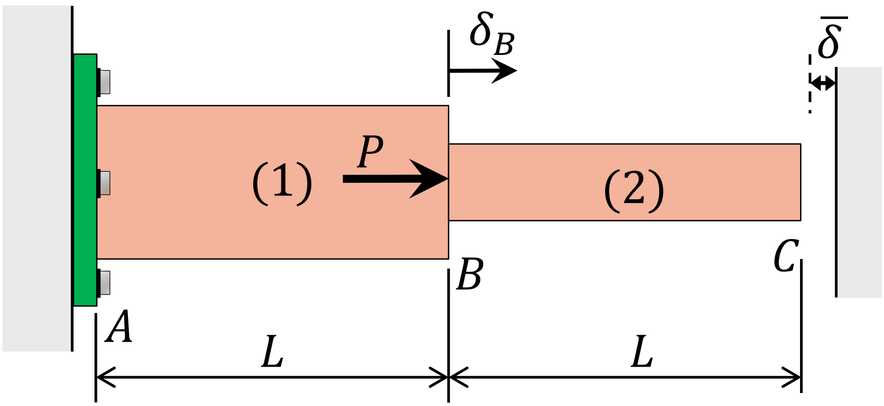


$$E_1 = E_2 = E$$

$$\alpha_1 = \alpha_2 = \alpha$$

$$A_1,\ A_2$$

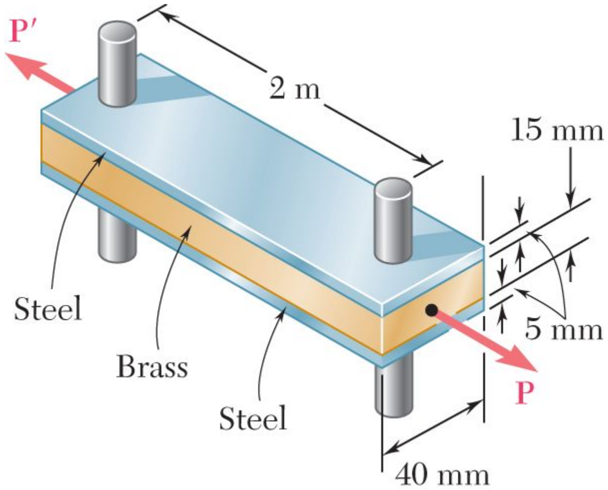
Misfit problems



Rods (1) and (2) have stiffnesses k_1 and k_2, respectively. At end A, the rod is connected to a rigid wall, but at end C there is a small gap $\bar{\delta}$ between the original position of end C and the rigid wall. A force P is applied at point B as shown.

a) What is the minimum required force P_{min} required to close the gap between end C and the wall at the right?

b) Derive expressions for the stress in each bar if the applied force is $P > P_{\text{min}}$. Which bar(s) are in tension? Which are in compression?



Two steel bars ($E_s = 200$ GPa and $\alpha_s = 11.7 \times 10^{-6}/°\text{C}$) are used to reinforce a brass bar ($E_b = 105$ GPa, $\alpha_b = 20.9 \times 10^{-6}/°\text{C}$) that is subjected to a load $P = 25$ kN. When the steel bars were fabricated, the distance between the centers of the holes that were to fit on the pins was made 0.5 mm smaller than the 2 m needed. The steel bars were then placed in an oven to increase their length so that they would just fit on the pins. Following fabrication, the temperature in the steel bars dropped back to room temperature. Determine (*a*) the increase in temperature that was required to fit the steel bars on the pins, (*b*) the stress in the brass bar after the load is applied to it.