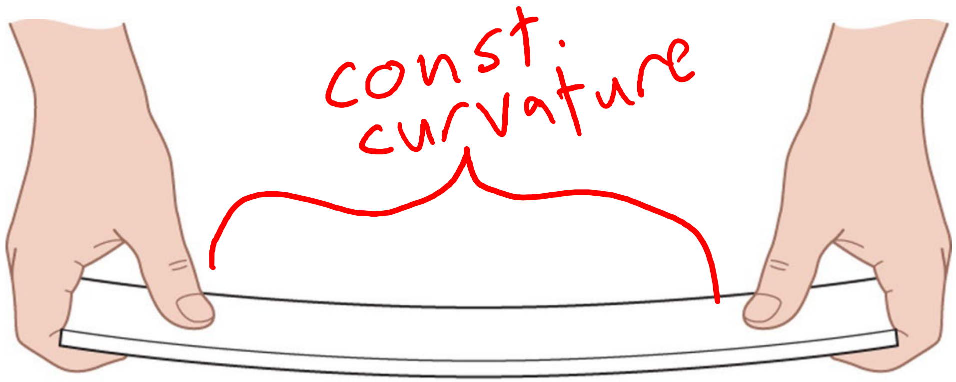
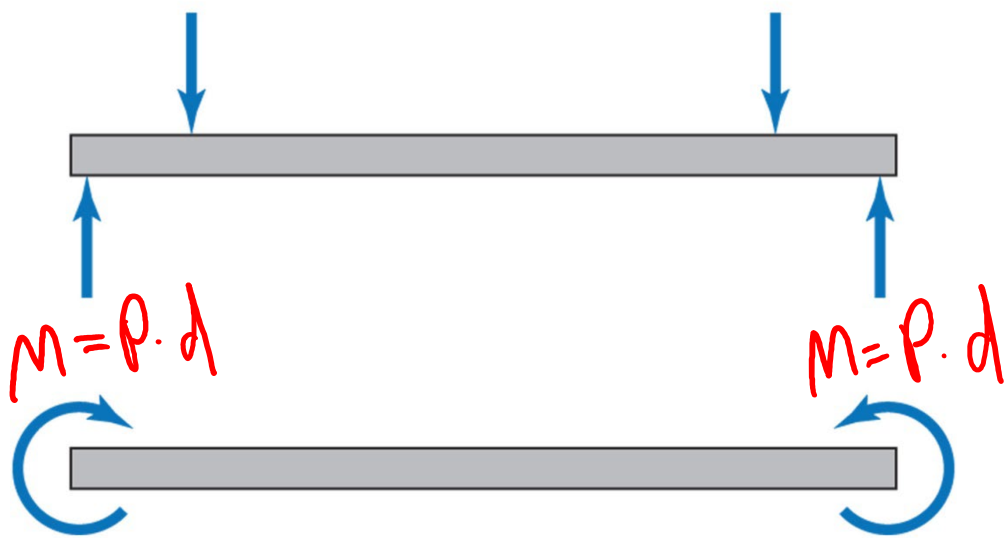
Pure bending

Take a flexible strip, such as a thin ruler, and apply equal forces with your fingers as shown. Each hand applies a couple or moment (equal and opposite forces a distance apart). The couples of the two hands must be equal and opposite. Between the thumbs, the strip has deformed into a circular arc. For the loading shown here, just as the deformation is uniform, so the internal bending **moment is uniform**, equal to the moment applied by each hand.





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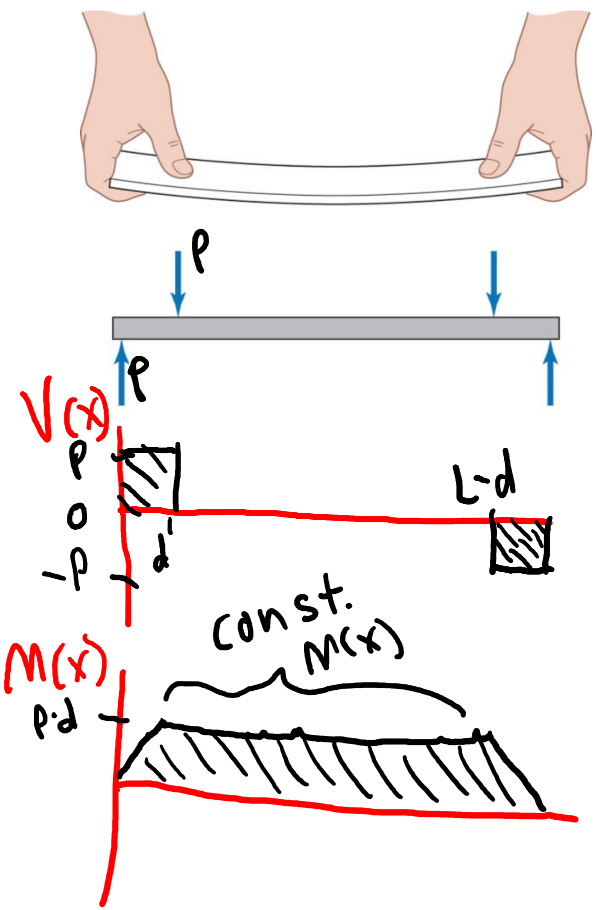
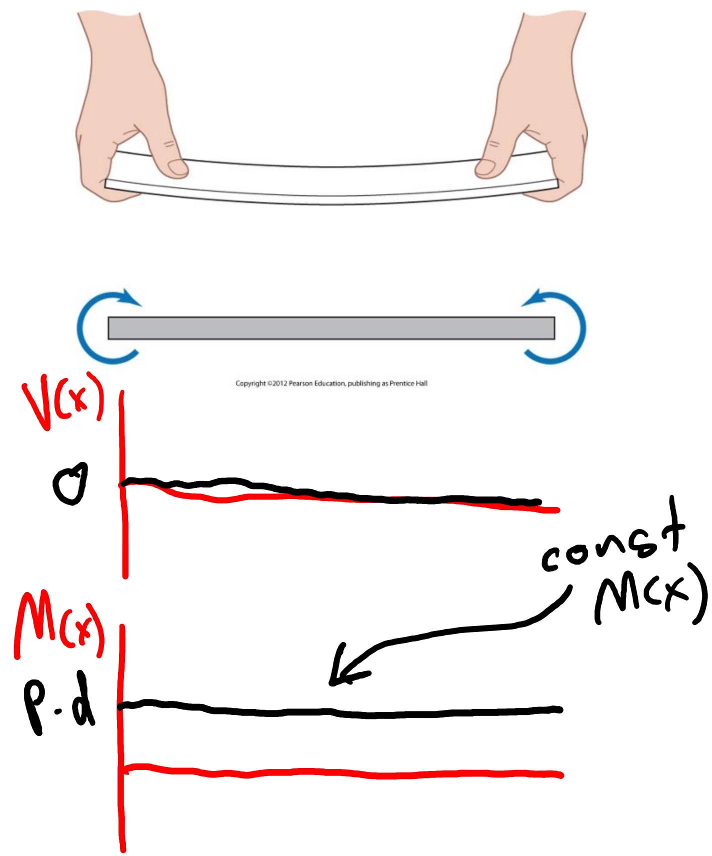
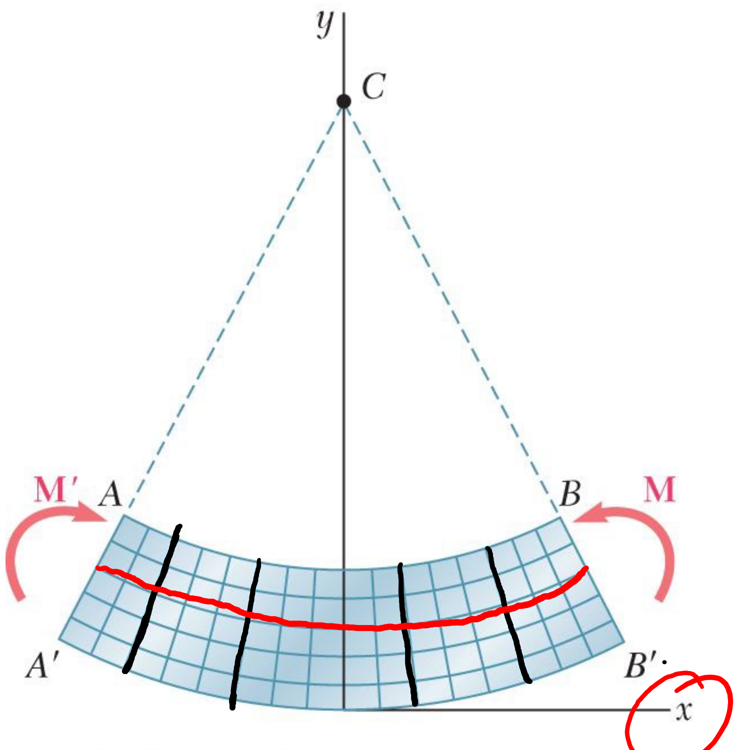




Geometry of deformation (Pure bending $M \neq 0$, $V = 0$)



(a) Longitudinal, vertical section (plane of symmetry)

We assume that "plane sections remain plane" → All faces of "grid elements" remain at 90^o to each other, hence

$$\gamma_{xy} = \gamma_{xz} = 0$$ shear strain

Therefore,

$$\tau_{xy} = \tau_{xz} = 0$$ shear stress

No external loads on y or z surfaces:

$$\sigma_y = \sigma_z = \tau_{yz} = 0$$

Thus, at any point of a slender member in pure bending, we have a **state of uniaxial stress**, since σ_x is the only non-zero stress component

acts parallel to the axis of the beam

For positive moment, $M > 0$ (as shown in diagram):

Segment AB decreases in length → $\sigma_x < 0$ and $\epsilon_x < 0$

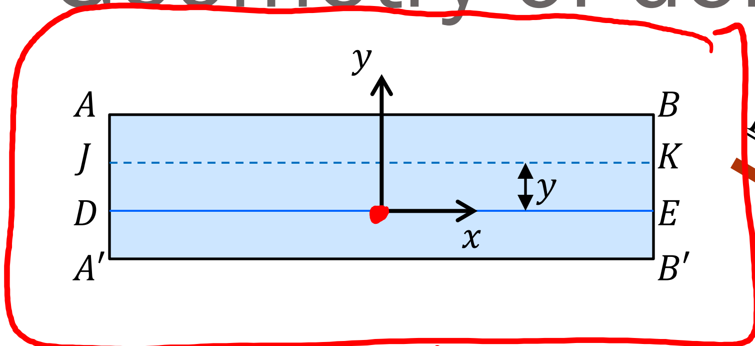
Segment $A'B'$ increases in length → $\sigma_x > 0$ and $\epsilon_x > 0$

→ Hence there must exist a surface parallel to the upper and lower where

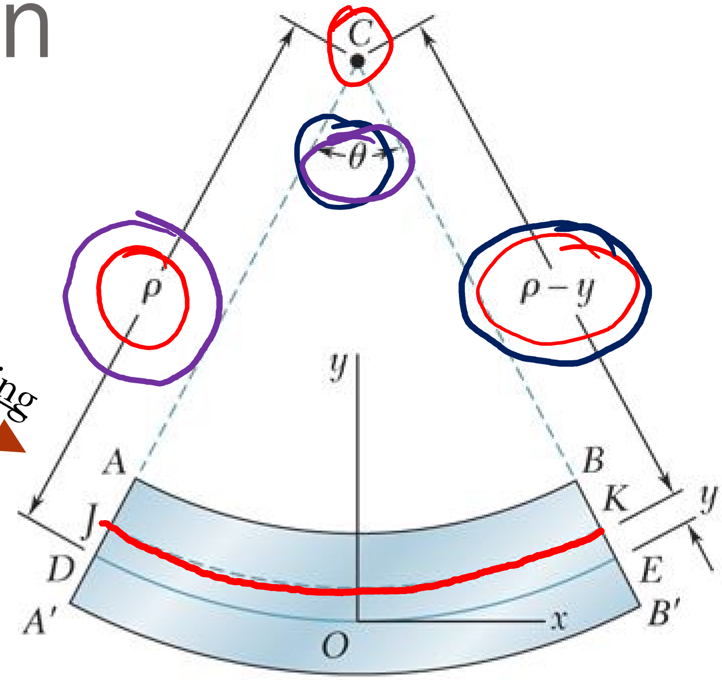
$$\sigma_x = 0 \text{ and } \epsilon_x = 0$$

This surface is called **NEUTRAL AXIS**

Geometry of deformation



Deformation after bending



(*a*) Longitudinal, vertical section (plane of symmetry)

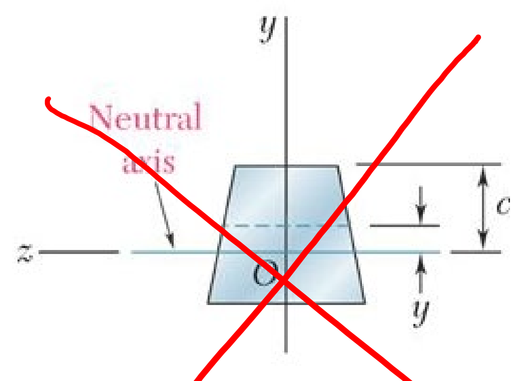
DE is the Neutral Axis ($\varepsilon_x = 0$)

$\varepsilon_x(y) \leftarrow$ strain field as a function of y

L_{JK} = length line JK

$$\varepsilon_x = \frac{\Delta L_{JK}}{L_{JK\,initial}} = \frac{L_{JK\,final} - L_{JK\,initial}}{L_{JK\,initial}}$$

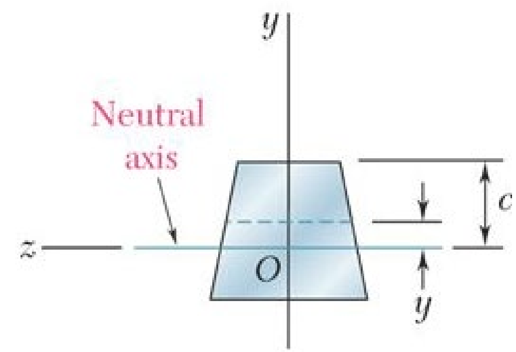
$$\varepsilon_x = \frac{(\cancel{\rho} - y)\theta - \cancel{\rho\theta}}{\cancel{\rho\theta}} \Rightarrow \boxed{\varepsilon_x = \frac{-y}{\rho}}$$



(*b*) Transverse section

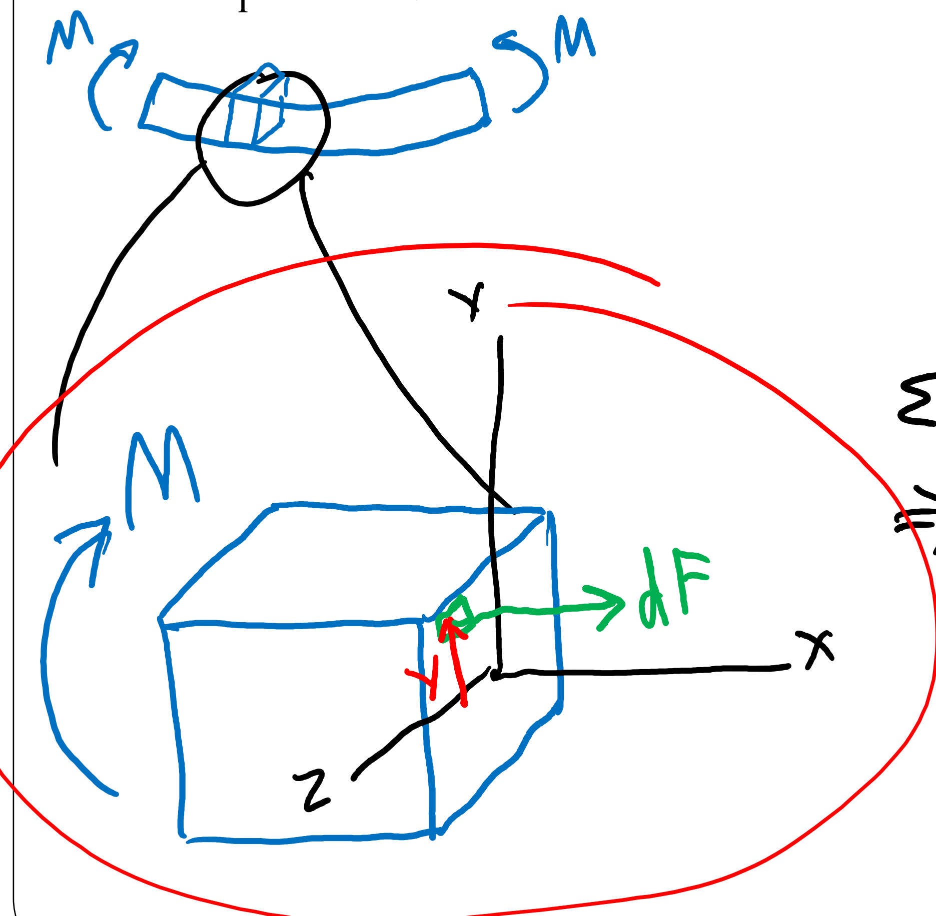
Constitutive and Force Equilibrium

Constitutive relationship: $\boxed{\epsilon_x = \frac{-y}{\rho}}$ $\Rightarrow$ $\boxed{\sigma_x = E\epsilon_x = -\frac{Ey}{\rho}}$



(b) Transverse section

Force equilibrium:



$\Sigma F_x = 0$

$\Rightarrow \int dF = \int \sigma_x \cdot dA = \int -\frac{E \cdot y}{\rho} dA$

$= \cancel{-\frac{E}{\rho}} \boxed{\int y \cdot dA = 0}$

$\Rightarrow$ N.A. is at the centroid ($\bar{y} = 0$)