

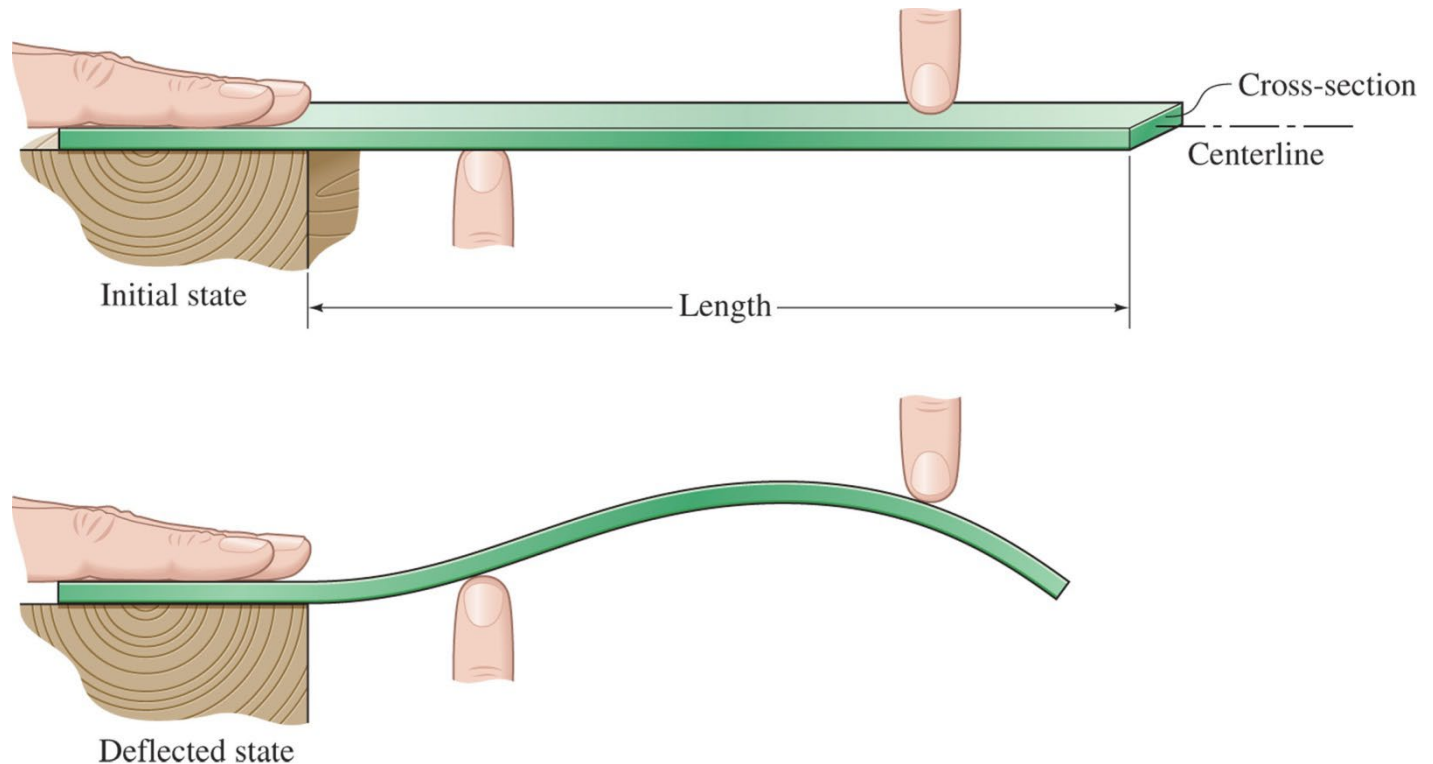
Chapter 6: Bending

Chapter Objectives

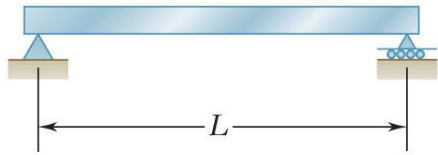
- ✓ Determine the internal moment at a section of a beam
- ✓ Determine the stress in a beam member caused by bending
- ✓ Determine the stresses in composite beams

Bending analysis

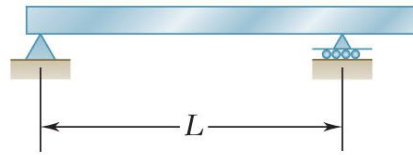
This wood ruler is held flat against the table at the left, and fingers are poised to press against it. When the fingers apply forces, the ruler deflects, primarily up or down. Whenever a part deforms in this way, we say that it acts like a “beam.” In this chapter, we learn to determine the stresses produced by the forces and how they depend on the beam cross-section, length, and material properties.



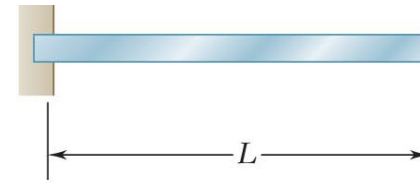
Support types:



(a) Simply supported beam

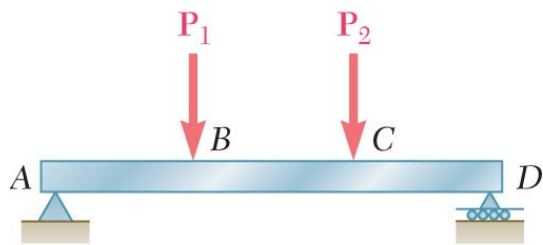


(b) Overhanging beam

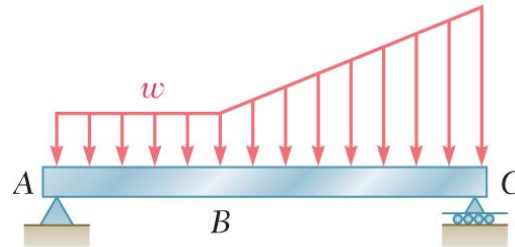


(c) Cantilever beam

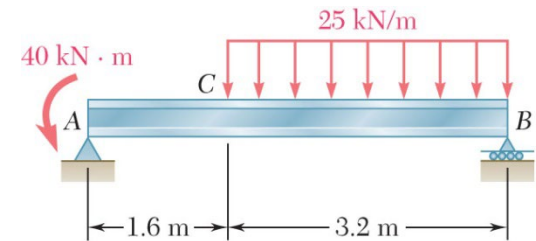
Load types:



Concentrated loads

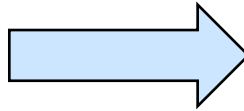
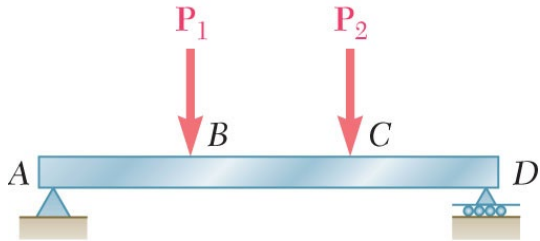


Distributed loads

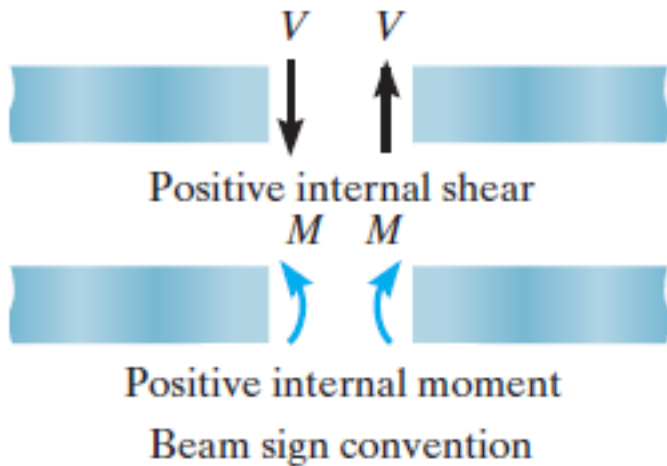


Concentrated moments

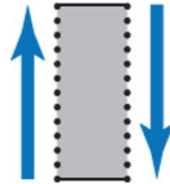
Sign conventions:



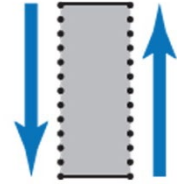
How do we define whether the internal shear force and bending moment are positive or negative?



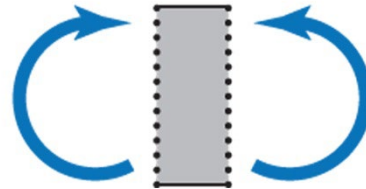
If $V > 0$, we mean



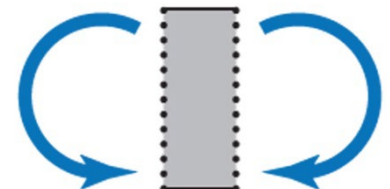
If $V < 0$, we mean



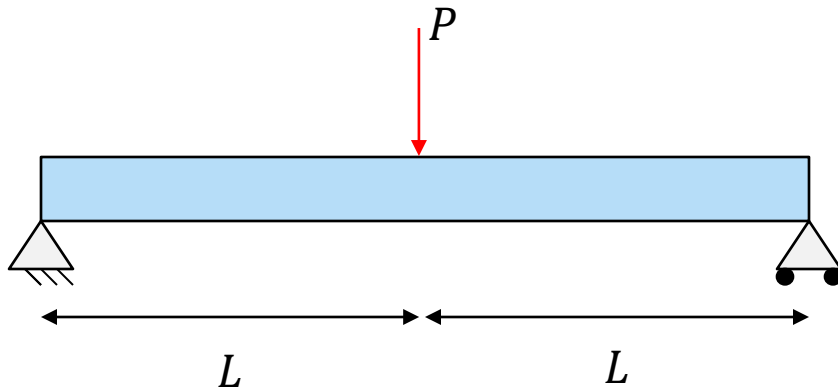
If $M > 0$, we mean



If $M < 0$, we mean



Shear and moment diagrams



Given:

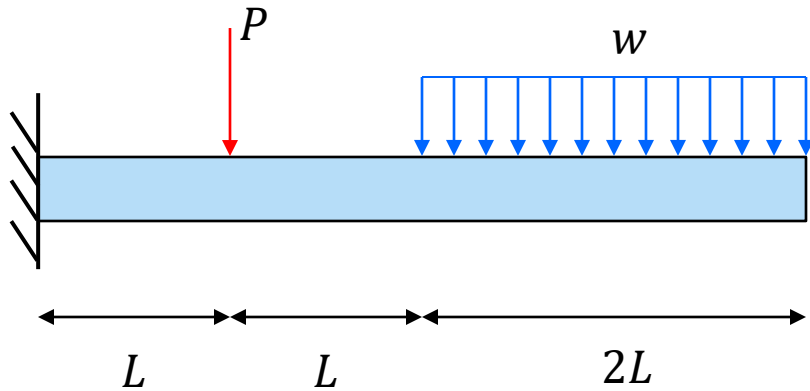
$$P = 2 \text{ kN}$$

$$L = 1 \text{ m}$$

Find:

$V(x)$, $M(x)$, Shear-
Bending moment
diagram along the
beam axis

Example: cantilever beam



Given:

$$P = 4 \text{ kN}$$

$$w = 1.5 \text{ kN/m}$$

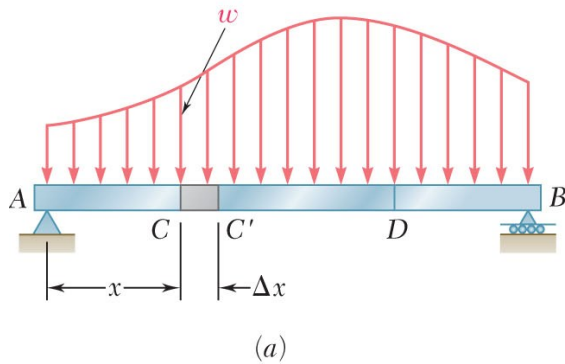
$$L = 1 \text{ m}$$

Find:

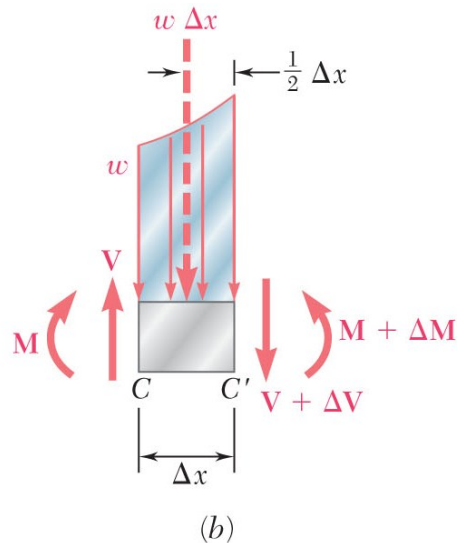
$V(x)$, $M(x)$, Shear-
Bending moment
diagram along the
beam axis

Relations Among Load, Shear and Bending Moments

Relationship between load and shear:

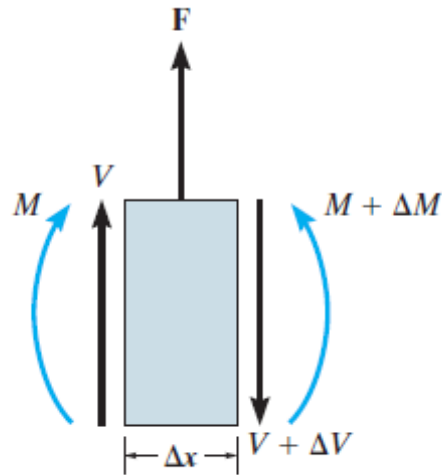


Relationship between shear and bending moment:



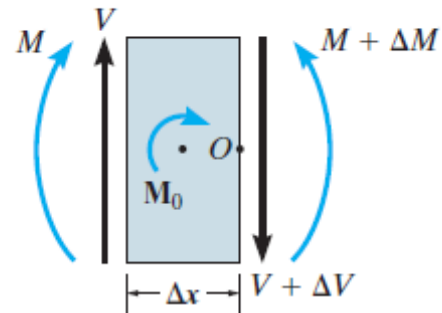
Wherever there is an external concentrated force, or a concentrated moment, there will be a change (jump) in shear or moment respectively.

**Concentrated
force F acting
upwards:**



(a)

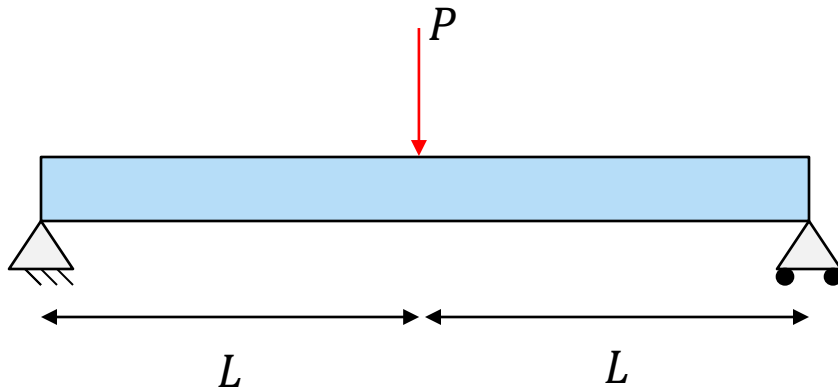
**Concentrated
Moment M_0
acting clockwise**



(b)

Fig. 6-10

Shear and moment diagrams



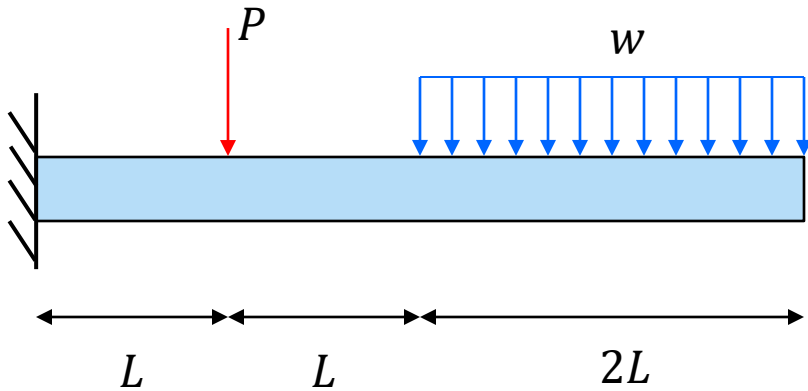
Given:

$$P = 2 \text{ kN}$$

$$L = 1 \text{ m}$$

Use the graphical method to sketch diagrams for $V(x)$ and $M(x)$

Example: cantilever beam



Given:

$$P = 4 \text{ kN}$$

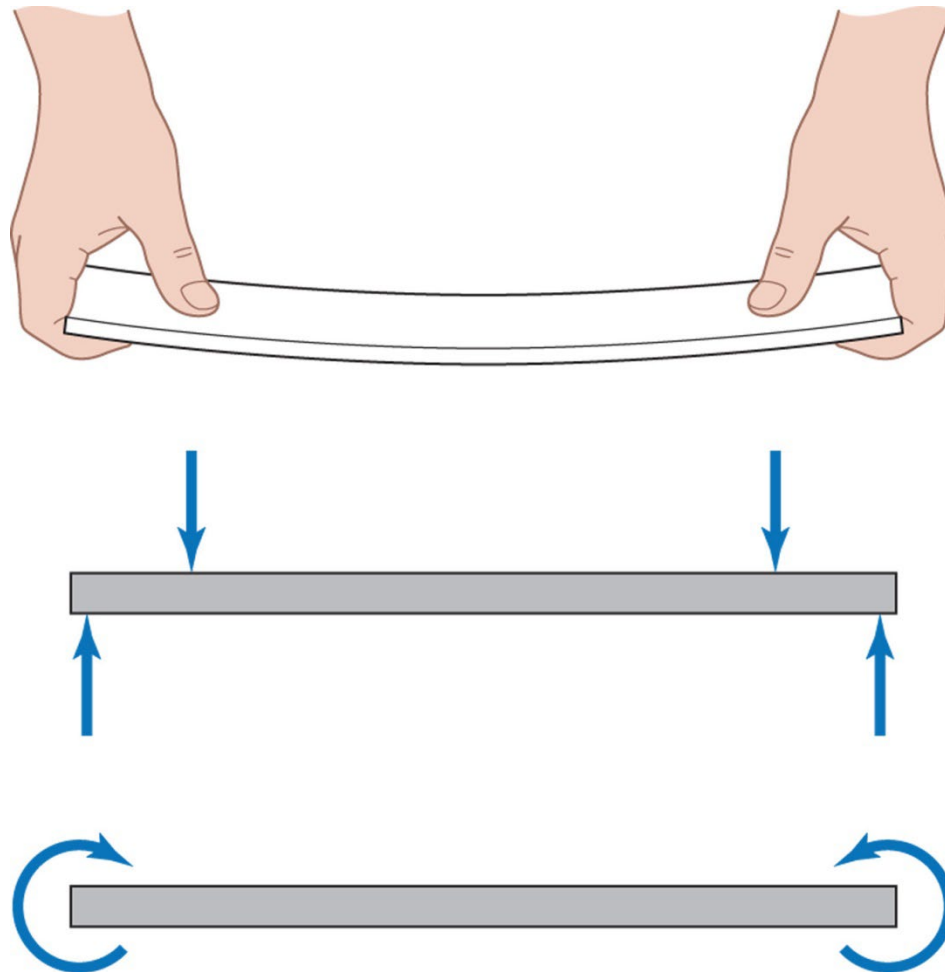
$$w = 1.5 \text{ kN/m}$$

$$L = 1 \text{ m}$$

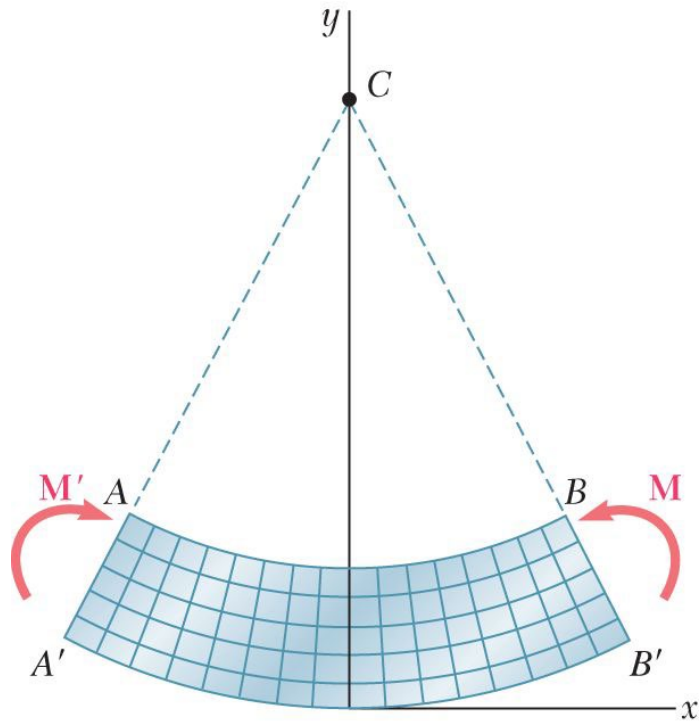
Use the graphical method to sketch diagrams for $V(x)$ and $M(x)$

Pure bending

Take a flexible strip, such as a thin ruler, and apply equal forces with your fingers as shown. Each hand applies a couple or moment (equal and opposite forces a distance apart). The couples of the two hands must be equal and opposite. Between the thumbs, the strip has deformed into a circular arc. For the loading shown here, just as the deformation is uniform, so the internal bending **moment is uniform**, equal to the moment applied by each hand.



Geometry of deformation



(a) Longitudinal, vertical section
(plane of symmetry)

We assume that "plane sections remain plane" → All faces of "grid elements" remain at 90° to each other, hence

$$\gamma_{xy} = \gamma_{xz} = 0$$

Therefore,

$$\tau_{xy} = \tau_{xz} = 0$$

No external loads on y or z surfaces:

$$\sigma_y = \sigma_z = \tau_{yz} = 0$$

Thus, at any point of a slender member in pure bending, we have a **state of uniaxial stress**, since σ_x is the only non-zero stress component

For positive moment, $M > 0$ (as shown in diagram):

Segment AB decreases in length → $\sigma_x < 0$ and $\epsilon_x < 0$

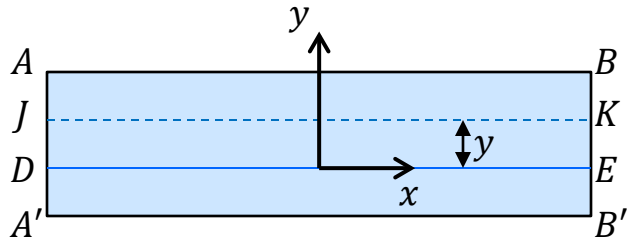
Segment A'B' increases in length → $\sigma_x > 0$ and $\epsilon_x > 0$

→ Hence there must exist a surface parallel to the upper and lower where

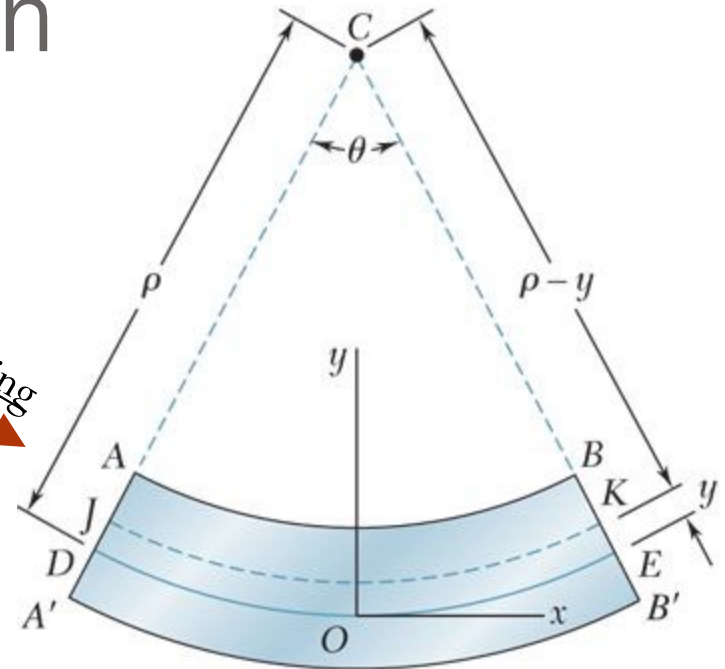
$$\sigma_x = 0 \text{ and } \epsilon_x = 0$$

This surface is called **NEUTRAL AXIS**

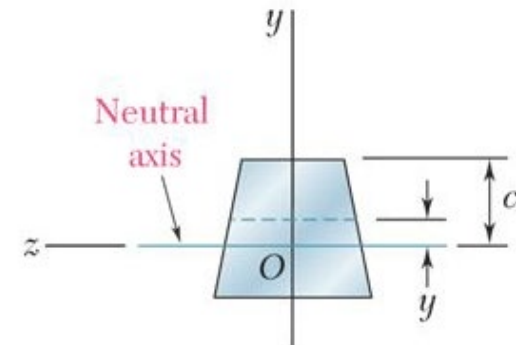
Geometry of deformation



Deformation after bending




(a) Longitudinal, vertical section
(plane of symmetry)

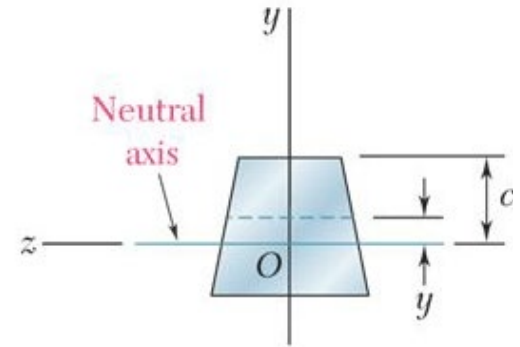


(b) Transverse section

Constitutive and Force Equilibrium

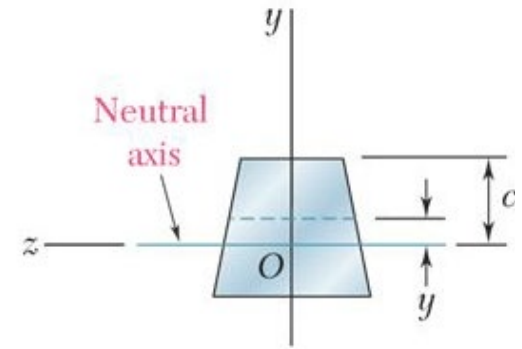
Constitutive relationship: $\epsilon_x = \frac{-y}{\rho}$  $\sigma_x = E\epsilon_x = -\frac{Ey}{\rho}$

Force equilibrium:



(b) Transverse section

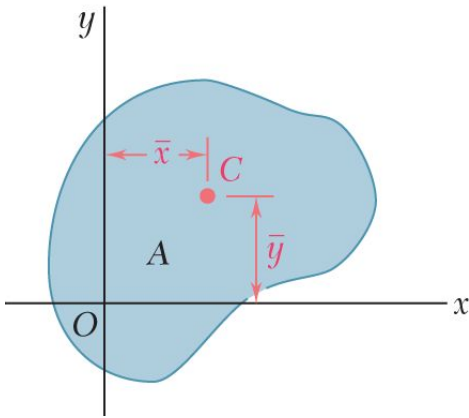
Moment Equilibrium



(b) Transverse section

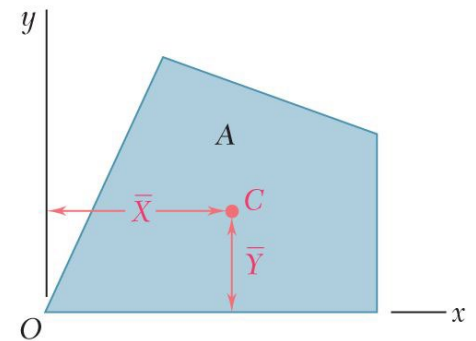
Centroid of an area

- The centroid of the area A is defined as the point C of coordinates \bar{x} and \bar{y} , which satisfies the relation



$$\int_A x dA = A \bar{x}$$

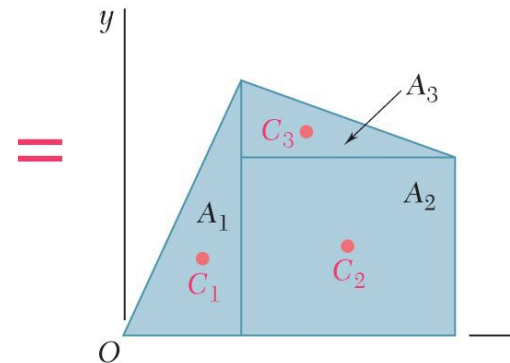
$$\int_A y dA = A \bar{y}$$



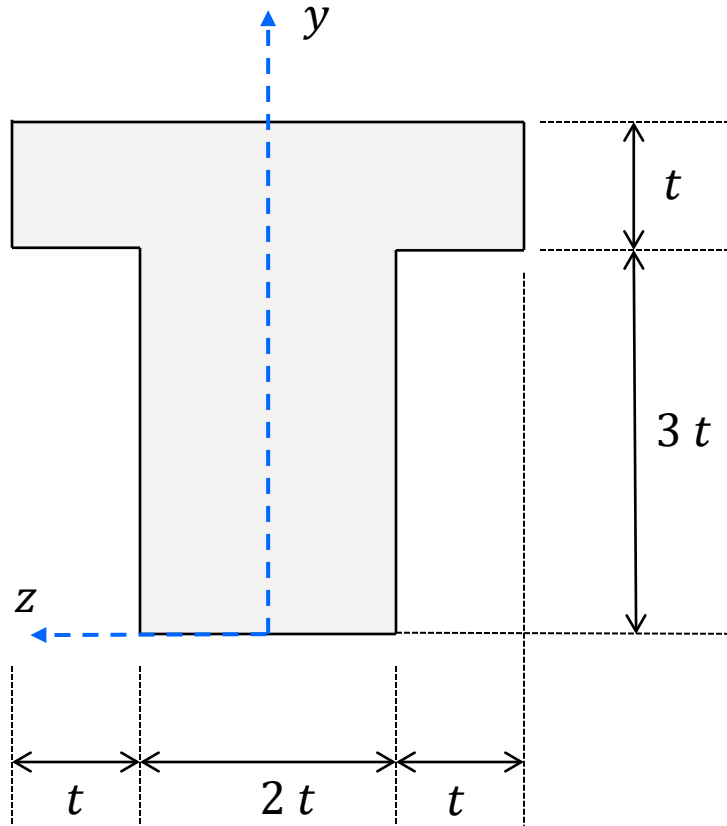
- In the case of a composite area, we divide the area A into parts A_1, A_2, A_3

$$A_{total} \bar{x} = \sum_i A_i \bar{x}_i$$

$$A_{total} \bar{y} = \sum_i A_i \bar{y}_i$$

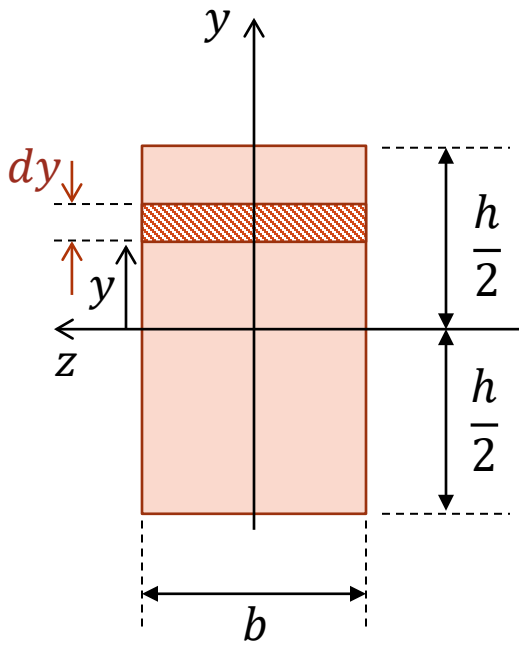


Example: Find the centroid position in the yz coordinate system shown for $t = 20\text{cm}$



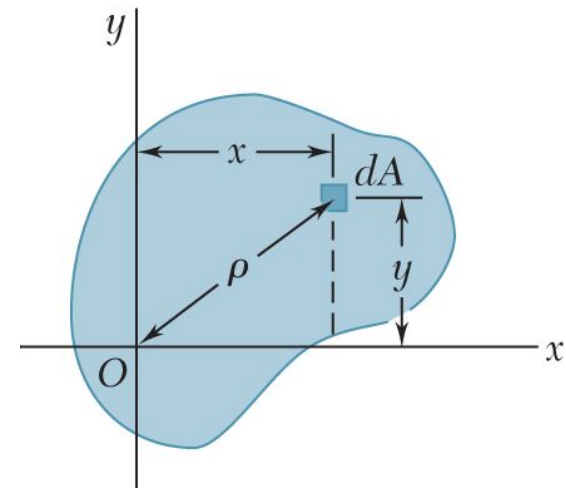
Second moment of area

- The 2nd moment of the area A with respect to the x-axis is given by
- The 2nd moment of the area A with respect to the y-axis is given by
- Example: 2nd moment of area for a rectangular cross section:

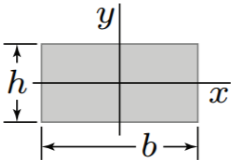
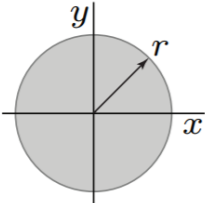
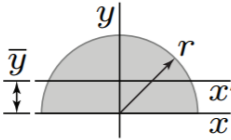


$$I_x = \int_A y^2 dA$$

$$I_y = \int_A x^2 dA$$



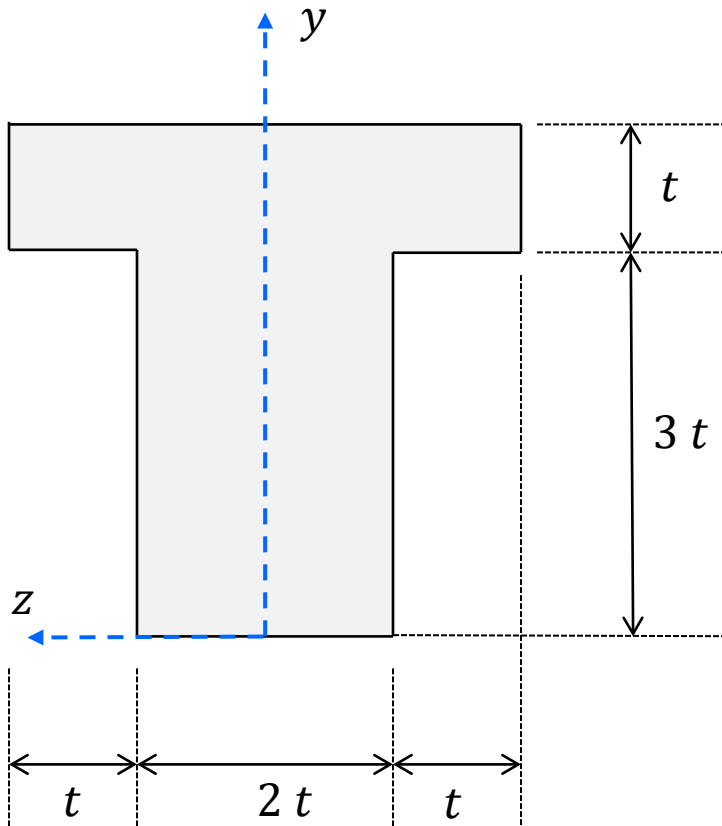
Centroids and area moments of area: Formula sheet

Moments and Geometric Centroids		
	$Q = \bar{y} A$ $I_x = \int_A y^2 dA$ $J_o = \int_A \rho^2 dA$ $\bar{y} = \frac{1}{A} \int_A y dA$	
Rectangle		$I_x = \frac{1}{12} b h^3$
Circle		$I_x = \frac{\pi}{4} r^4$ $J_z = \frac{\pi}{2} r^4$
Semicircle		$I_{x'} = \left(\frac{\pi}{8} - \frac{8}{9\pi} \right) r^4$ $\bar{y} = \frac{4r}{3\pi}$
Parallel Axis Theorem		$I_c = I_{c'} + A d_{cc'}^2$

Parallel-axis theorem: the 2nd moment of area about an axis through C parallel to the axis through the centroid C' is given by

$$I_C = I_{C'} + A d_{CC'}^2$$

Example: Find the 2nd moment of area about the horizontal axis passing through the centroid assuming $t = 20$ cm

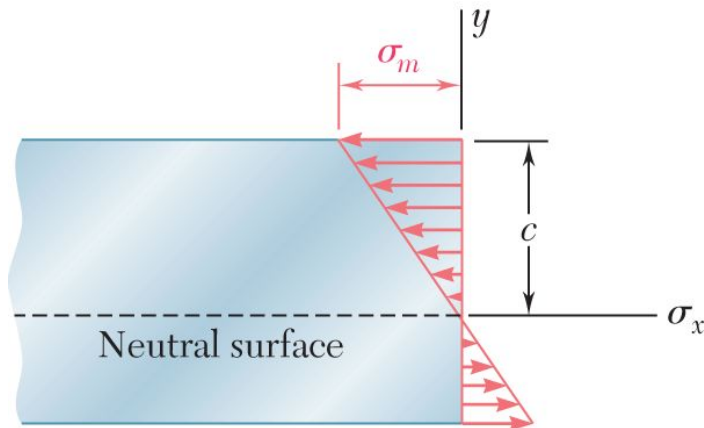


Bending stress formula

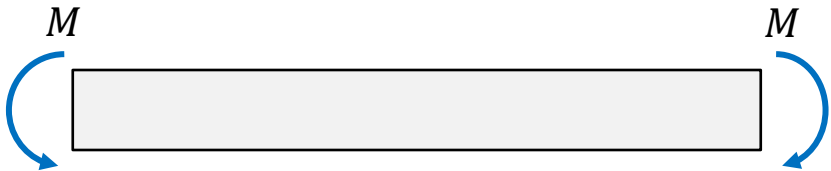
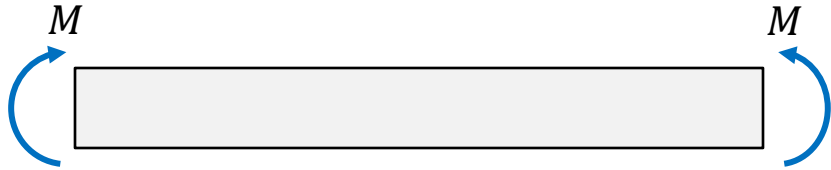
$$\sigma_x(x, y) = -\frac{M(x) y}{I_z(x)}$$

- The maximum magnitude occurs the furthest distance away from the neutral axis. If we denote this maximum distance “c”, consistent with the diagram below, then we can write

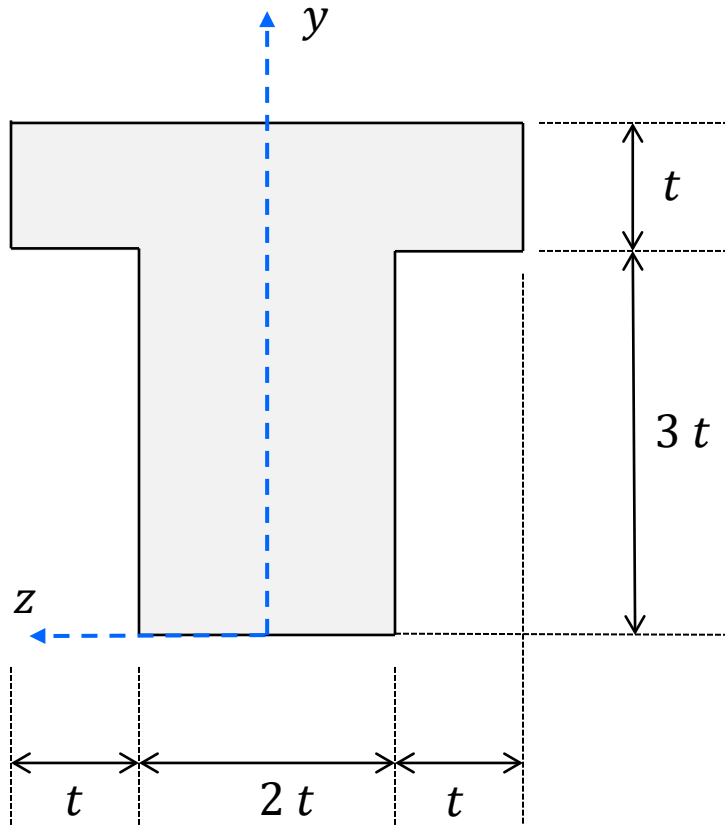
$$\sigma_m = \frac{|M|c}{I_z}$$



Bending stress sign



Example: Find the maximum tensile and compressive stresses in this beam subjected to moment $M_z = 100 \text{ N-m}$ with the moment vector pointing in the direction of the z-axis. Again take $t = 20 \text{ cm}$.



Why I-beams?



<http://studio-tm.com/constructionblog/wp-content/uploads/2011/12/steel-i-beam-cantilevered-over-concrete-wall.jpg>

Summary of bending in beams

- Maximum stress due to bending

$$\sigma = \frac{Mc}{I}$$

- Bending stress is zero at the neutral axis and ramps up linearly with distance away from the neutral axis
- I is the 2nd moment of area **about the neutral axis of the cross section**
 - Be sure to find the cross-section's centroid and evaluate I about an axis passing through the centroid, using the parallel axis theorem if needed

$$I_C = I_{C'} + A d_{C'}^2$$

- To determine stress sign, look at the internal bending moment direction:
 - Side that moment curls **towards** is in **compression**
 - Side that moment curls **away from** is in **tension**

