TAM251_Chapter7_TransverseShear_prelecture_Johnson

Wednesday, July 7, 2021 1:24 PM

TAM251_Chapter7_TransverseShear_prelecture_Johnson

Chapter 7: Transverse Shear

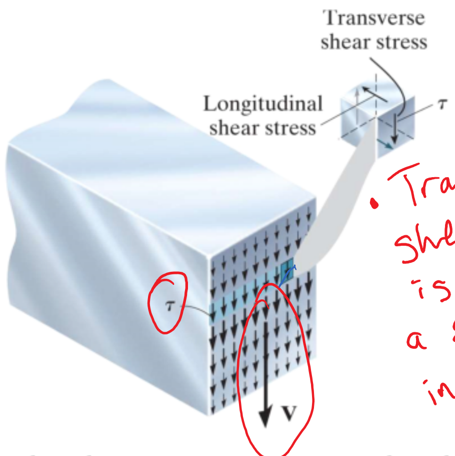
Glulam beam?

Chapter Objectives

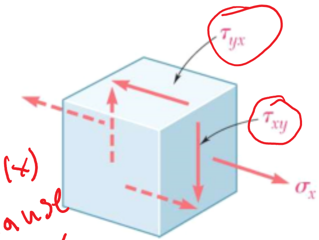
- ✓ Determine shear stress in a prismatic beam ← uniform beam of constant cross-section
- ✓ Determine shear flow in a built-up beam ← beam made of multiple boards, such as a "glulam" beam

Bending failure usually begins near the half-height of the beam. The fracture often runs longitudinally and between boards

Symmetry of shear stresses



• Transverse shear force $V(x)$ is going to cause a shear stress τ in the cross-section of a beam

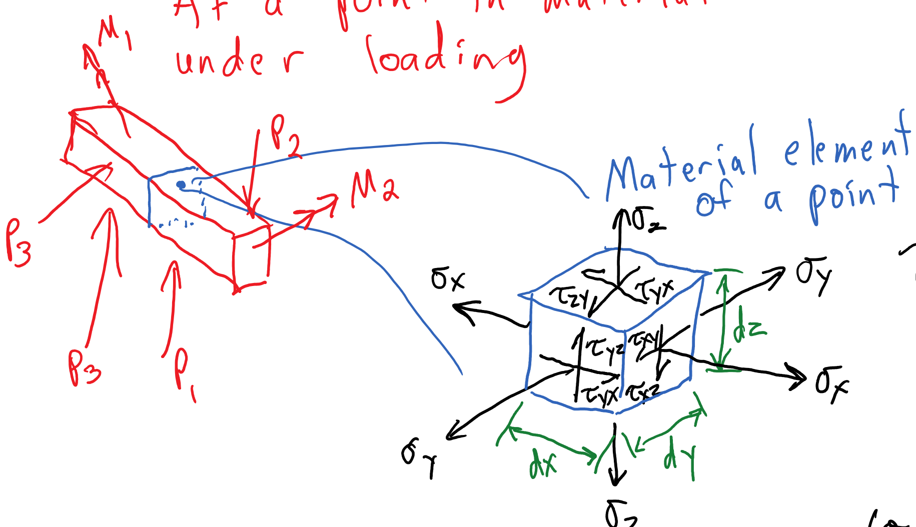


$$V = \int_{area} \tau \cdot dA$$

- When shearing stresses are exerted on the vertical faces of an element, equal stresses must be exerted on the horizontal faces
- Longitudinal shearing stresses must exist in any member subjected to transverse loading.

2

At a point in material under loading



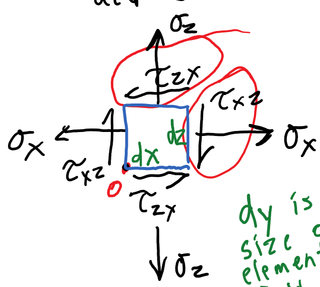
τ_{xz} — z: tells the coordinate direction that the stress acts; x: tells which axis or coordinate is orthogonal to the face

(σ_x is actually σ_{xx})

consider stresses acting on the z-x face

$\Sigma M_0 = 0$

acting on the z-x face



dy is the size of the element out of the page

$\Sigma M_O = 0$

$$\underbrace{\cancel{-\sigma_x \cdot dz\, dy}}_{☆} \cdot \frac{dz}{2} + \underbrace{\cancel{\sigma_x \cdot dz\, dy}}_{☆} \cdot \frac{dz}{2}$$

$$\underbrace{\cancel{-\sigma_z \cdot dx \cdot dy}}_{☆} \cdot \frac{dx}{2} + \underbrace{\cancel{\sigma_z \cdot dx\, dy}}_{☆} \frac{dx}{2} + \underbrace{\tau_{zx} \cdot dx \cdot dy}_{☆}\, dz - \underbrace{\tau_{xz} \cdot dz \cdot dy}_{☆} \cdot dx = 0$$

☆ = forces

divide by dx dy dz

$\Rightarrow \tau_{zx} - \tau_{xz} = 0$

$\Rightarrow \boxed{\tau_{zx} = \tau_{xz}}$

ALWAYS TRUE
- in solids
- in fluids

same can be proved for all shear stresses

$\tau_{xy} = \tau_{yx}$

$\tau_{yz} = \tau_{zy}$

Stress Inventory

Up to 3 normal stresses: σ_{xx} $(= \sigma_x)$, σ_y, σ_z

up to 3 shear stresses:

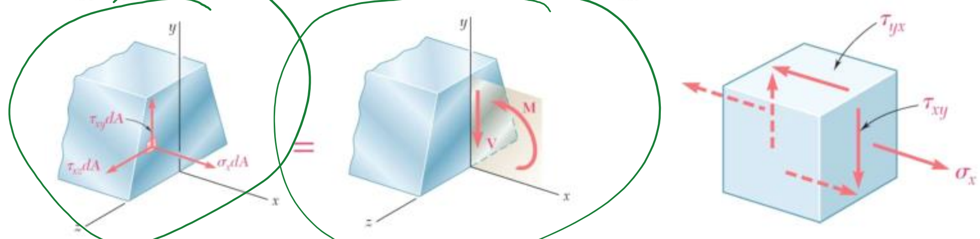
τ_{xy}, τ_{yx} } equal

τ_{xz}, τ_{zx} } equal

τ_{yz}, τ_{zy} } equal

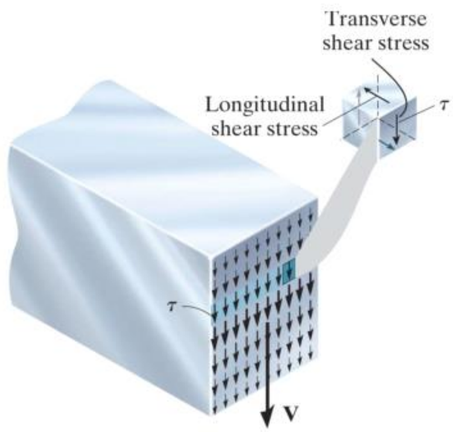
Shear stress in beams

- Transverse loading applied to a beam results in normal and shearing stresses in transverse sections.



- Distribution of normal and shearing stresses satisfies

$$F_x = \int \sigma_x dA = 0 \qquad M_x = \int (y\tau_{xz} - z\tau_{xy})dA = 0$$
$$F_y = \int \tau_{xy} dA = -V \qquad M_y = \int z\sigma_x dA = 0$$
$$F_z = \int \tau_{xz} dA = 0 \qquad M_z = \int (-y\sigma_x) = M$$

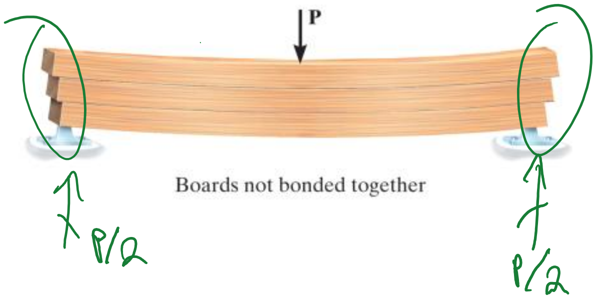


- When shearing stresses are exerted on the vertical faces of an element, equal stresses must be exerted on the horizontal faces
- Longitudinal shearing stresses must exist in any member subjected to transverse loading.

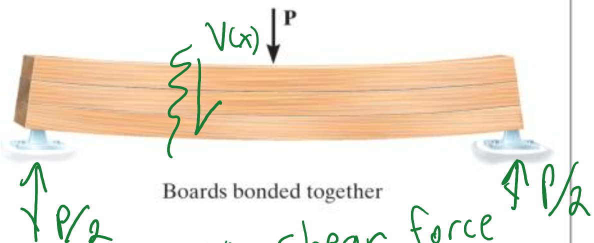
3

Transverse loading of beams

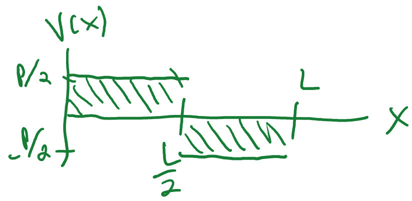
Shear forces due to transverse loading creates corresponding **longitudinal** shear stresses which will act along **longitudinal** planes of the beam.

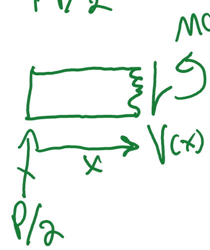


Boards not bonded together



Boards bonded together





shear force carried by the glue joint causes the boards to deform as one beam ⇒ longitudinal shear force

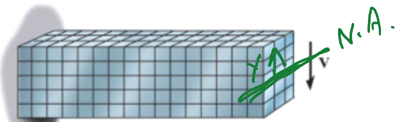
4

Transverse loading of beams

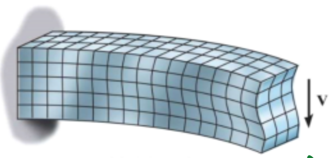
When a transverse shear load is applied, it tends to cause warping of the cross section. Therefore, when a beam is subject to moments and shear forces, the cross section will **not** remain plane as assumed in the derivation of the bending stress formula. $\sigma_x = -\frac{M \cdot y}{I_z}$ max $|\sigma_x|$ is at the top and/or bottom of the beam

However, we can assume that the warping due to the transverse shear stresses is small enough that it can be neglected, which is particularly true for slender beams.

⇒ allows us to continue using $\sigma_x = -\frac{M \cdot y}{I}$ for cases where $M(x) \neq 0$ and $V(x) \neq 0$



(a) Before deformation



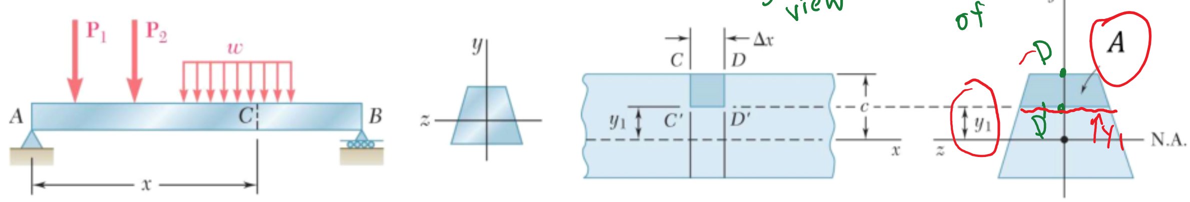
(b) After deformation

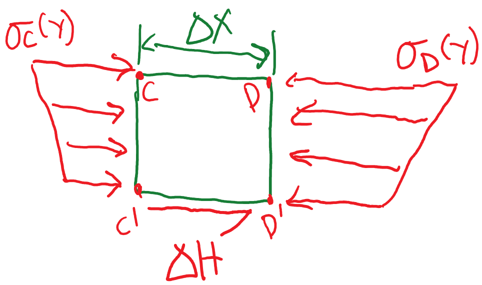
deformations that warp transverse planes here are highly exaggerated

5

Longitudinal shear forces in beams

Consider a prismatic beam subjected to transverse loading





$V(x) = \frac{dM}{dx}$

$\sigma_C(Y) = \frac{-M_c \cdot Y}{I_z} = -\frac{M(x) \cdot Y}{I_z}$

$\sigma_D(Y) = -\frac{M_D \cdot Y}{I_z} = -\frac{M(x+\Delta x)\cdot Y}{I_z}$

$\Sigma F_x = 0$

but the force caused by $\int \sigma_D \cdot dA$ is clearly larger in magnitude than $\int \sigma_C \cdot dA$

$\Rightarrow$ Must be some shear force ΔH on the bottom of this material element.

6

$Q(Y_1)$ is first mom. of area of A w.r.t. the Neutral Axis

$Q(Y_1) = \bar{Y}_1 \cdot A$

$\Sigma F_x = \Delta H + \int \sigma_c \, dA - \int \sigma_D \, dA = 0$

$\Delta H + \int \frac{-M_c \cdot Y}{I} dA - \int \frac{-M_D \cdot Y}{I} dA = 0$

$\Delta H + \frac{-M_c}{I}\int Y dA - \frac{-M_D}{I}\int Y dA = 0$

$\Delta H + \frac{M_D - M_c}{I}\underbrace{\int Y \cdot dA}_{\text{Area}} = 0$

area of the ...

$\Delta H + \frac{\Delta M}{I_z}\underbrace{\int Y \cdot dA}_{Q(Y)} = 0$

$\Rightarrow \Delta H = \frac{\Delta M \cdot Q(Y)}{I_z}$

Divide by ΔX

$\frac{\Delta H}{\Delta X} = \frac{-\Delta M}{\Delta X} \cdot \frac{Q(Y)}{I_z}$

face above y_1

take $\Delta x \to 0$

$$q = \lim_{\Delta x \to 0} \frac{\Delta H}{\Delta x} = -\frac{dM}{dx} \cdot \frac{Q(y)}{I_z} = V(x) \cdot \frac{Q(y)}{I_z}$$

"shear flow"

$$[q] = \frac{\text{force}}{\text{length}}$$

Shear stress

τ_{xy} caused by ΔH

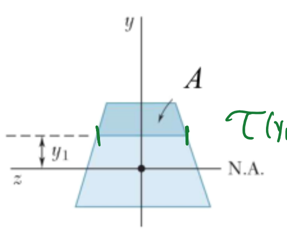
is q / (the width of the beam (out of the page)) at $y = y_1$

$$\boxed{\tau_{xy} = \frac{V(x) \cdot Q(y)}{I_z \cdot t(y)}}$$

Average Shear Stress

q = Shear flow
= force/length acting in the longitudinal axis of the beam

$$q = \frac{\Delta H}{\Delta x} = \frac{V \cdot Q}{I}$$



$\tau(y_1) = \frac{q}{t(y_1)}$

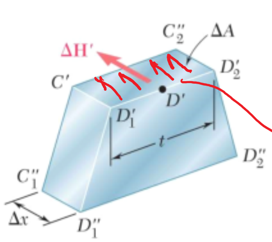
$q = \frac{V \cdot Q}{I}$

Q = 1st mom of area above y_1

V = transverse shear force in the beam

$Q = A \cdot \bar{y}$ of the shaded area above y_1

I = 2nd mom. of area of the entire cross-section

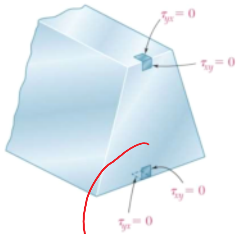


$$\tau_{avg} = \frac{\Delta H}{\Delta A} = \frac{\Delta H}{\Delta x \cdot t(y_1)} = \frac{V \cdot Q}{I \cdot t(y_1)}$$

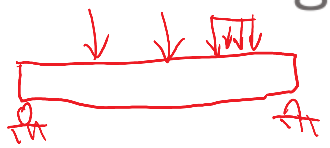
τ_{avg} = average shear stress along the width of the cut plane of interest

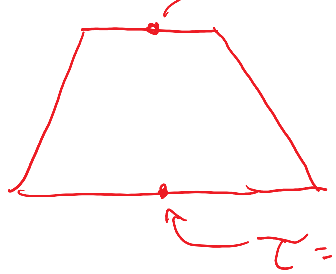
7

Shear stress at free edges



entire cross-section





$\tau = 0$

$\tau = 0$

No shear traction on upper or lower faces

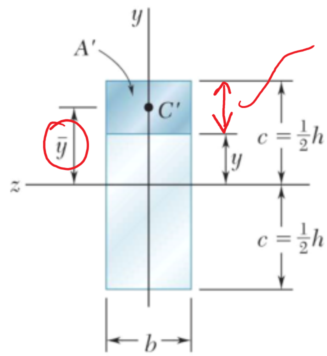
$\Rightarrow \tau_{xy} = \tau_{yx}$

$\Rightarrow \tau = 0$ in the cross-section at top & bottom edges

τ_{max} will occur at the Neutral Axis

8

Shear Stress Distribution (rectangular cross-section)



$\frac{1}{2}h - y$

$$\tau = \frac{V \cdot Q}{I_z t}$$

$V = V(x)$

$I_z = \frac{bh^3}{12}$ } of the entire cross-section

$t = b$

$Q(y) = A \cdot \bar{y}$ ← centroid of A with respect to N.A.

↑ of area above y

$A = \left(\frac{1}{2}h - y\right) \cdot b$

$\bar{y} = y + \frac{1}{2}\left(\frac{1}{2}h - y\right) = y + \frac{h}{4} - \frac{y}{2}$

$= \frac{y}{2} + \frac{h}{4} = \frac{1}{2}\left(y + \frac{h}{2}\right)$

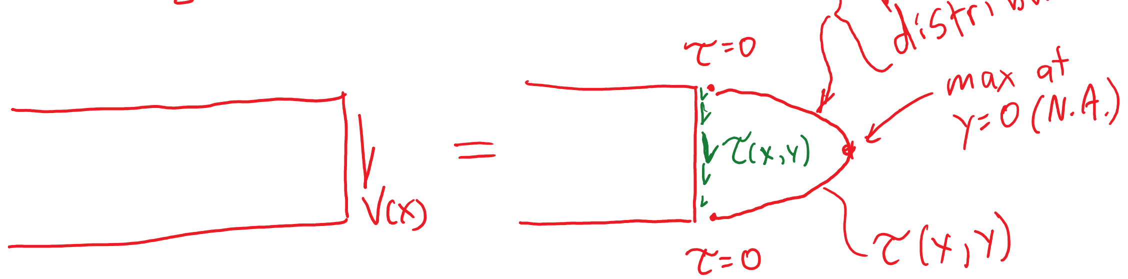
$Q(y) = \left(\frac{1}{2}h - y\right) \cdot b \cdot \frac{1}{2} \cdot \left(\frac{1}{2}h + y\right)$

$Q(y) = \frac{b}{2}\left(\frac{h^2}{4} - y^2\right)$ } max. where $y = 0$ (N.A.)

9

$Q(y)$ = for a rectangular cross section

$$\tau = \frac{V(x) \cdot \overbrace{\frac{b}{2}\left(\frac{h^2}{4} - y^2\right)}^{Q(y)}}{\underbrace{\left(\frac{bh^3}{12}\right)}_{I_z} \cdot \underset{\uparrow t}{b}}$$

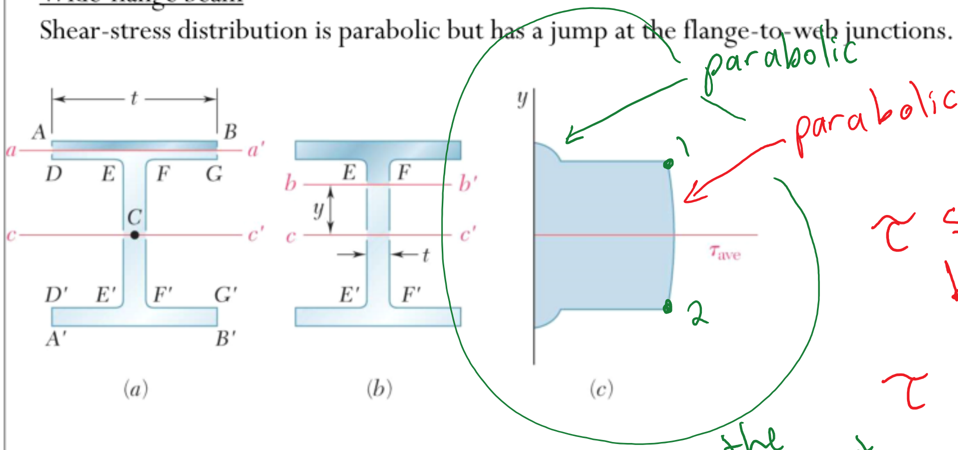


Shear Stress Distribution: American Standard (S-beam) and wide-flange (W-beam) beams

Wide-flange beam

Shear-stress distribution is parabolic but has a jump at the flange-to-web junctions.

$\tau = \frac{q}{t} = \frac{V \cdot Q}{I \cdot t}$



(a) (b) (c)

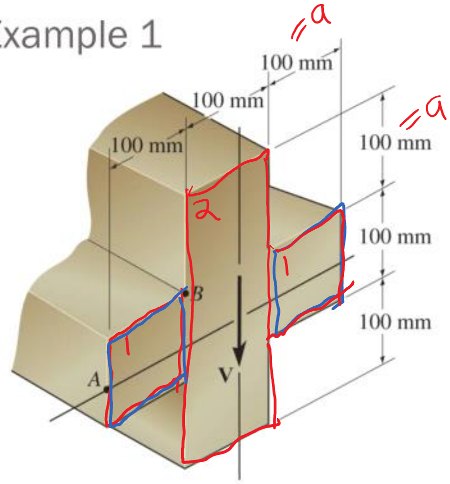
τ small in flanges because t large

τ large in web. b.c. t small

Notice the symmetry about N.A.

$Q_1 = Q_2$

Example 1



Knowing that the vertical shear in the beam is V = 400 N, determine the average shear stress at points A and B.

on the N.A.

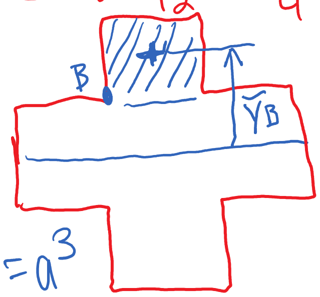
$$\tau = \frac{V \cdot Q}{I \cdot t}$$

$$I = 2I_1 + I_2$$

$$I_1 = \frac{a^4}{12} \qquad I_2 = \frac{a \cdot (3a)^3}{12} = \frac{9a^4}{4}$$

$$I = 2 \cdot \frac{a^4}{12} + \frac{9a^4}{4} = \frac{29a^4}{12}$$

At B : $t = a$



$$Q_B = A_B \cdot \bar{y}_B = a^2 \cdot a = a^3$$

$$A_B = a^2$$

$$\bar{y}_B = \frac{a}{2} + \frac{a}{2} = a$$

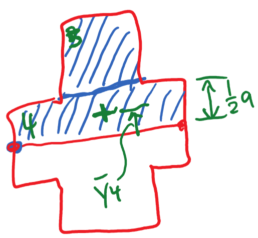
$$\boxed{\tau_B = \frac{V \cdot a^3}{\left(\frac{29}{12}a^4\right) \cdot a} = \frac{12 \cdot V}{29 \cdot a^2}}$$

force

length^2

11

At A



$$Q_A = ? = Q_3 + Q_4$$

$$t_A = 3a$$

$$Q_3 = a^2 \cdot a = a^3 \quad (= Q_B)$$

$$Q_4 = A_4 \cdot \bar{y}_4 = \left(\frac{a}{2} \cdot 3a\right) \cdot \frac{1}{4}a = \frac{3}{8}a^3$$

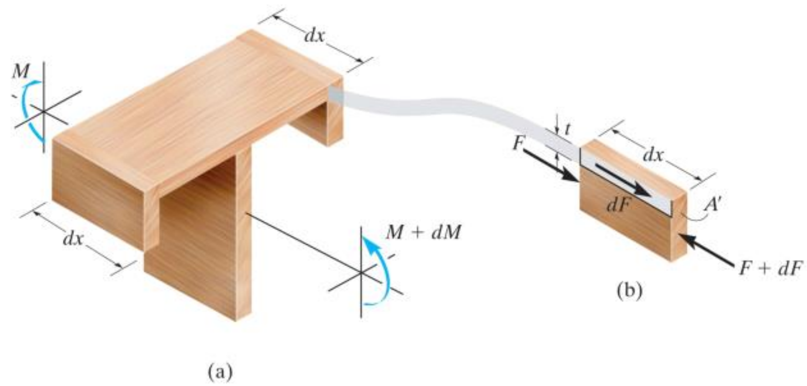
$$Q_A = a^3 + \frac{3}{8}a^3 = \frac{11}{8}a^3$$

$$\boxed{\tau_A = \frac{V \cdot \left(\frac{11}{8}a^3\right)}{\left(\frac{29}{12}a^4\right)(3a)} = \frac{V}{a^2} \cdot \frac{11/8}{29/4} = \frac{11 \cdot V}{58a^2}}$$

Shear Flow in Built-up Beams

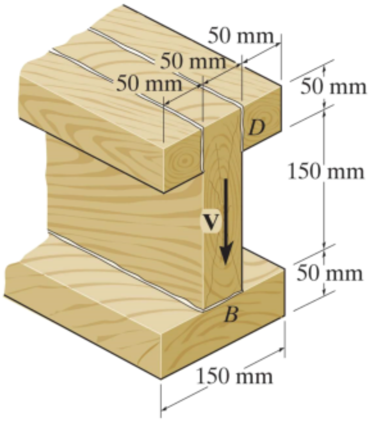
Consider the built-up beam below where the section is composed of 4 rectangular segments glued to one another.

How can we calculate the shear stress in the glued segments?



12

Example 2



A beam is made of four planks glued together. Knowing that the vertical shear in the beam is $V = 500$ N, determine the minimum required shear strength τ_g for the glue.

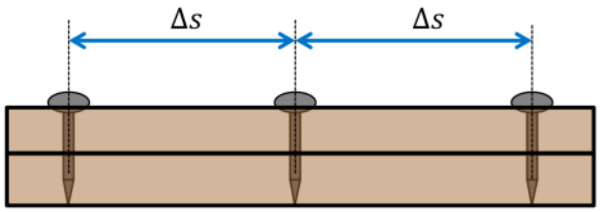
13

Built up beams with fasteners (bolts or nails)

Unlike glue, fasteners supply resistance to longitudinal shear forces at fixed internals.

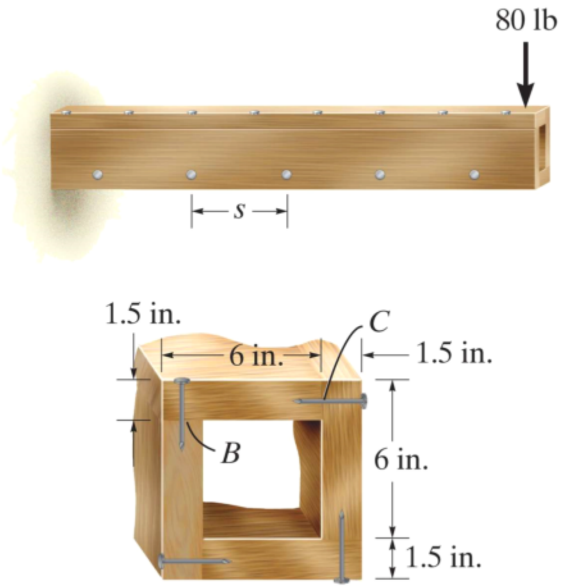
Fasteners are typically spaced at a constant interval Δs along the length of the beam.

If we know the shear flow q, how much load does each fastener carry?



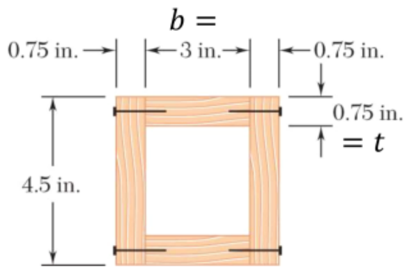
14

Example 3 A beam is made of four planks, nailed together as shown. If each nail can support a shear force of 30 lb, determine the maximum spacing s of the nails at B and at C so that the beam will support the force of 80lb.



15

Example 4



A square box beam is constructed from four planks as shown. Knowing that the spacing between nails is 1.5 in. and the beam is subjected to a vertical shear of magnitude $V = 600$ lb, determine the shearing force in each nail.

16