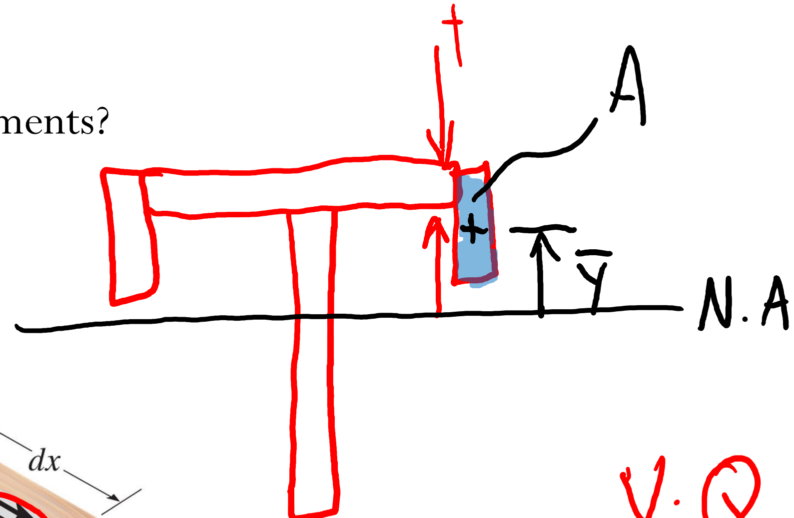
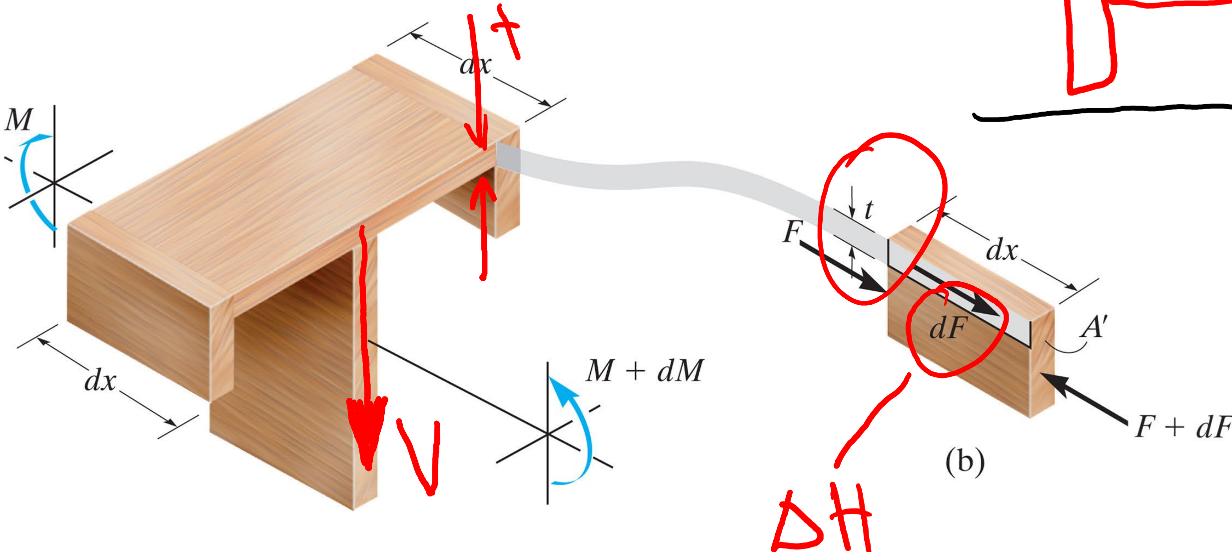
Shear Flow in Built-up Beams

Consider the built-up beam below where the section is composed of 4 rectangular segments glued to one another.

How can we calculate the shear stress in the glued segments?





(a)

$$q = \frac{V \cdot Q}{I}$$

$$Q = A \cdot \bar{y}$$ of the full cross-section

$$\tau = \frac{\Delta H}{t \cdot \Delta x} = \frac{q}{t} = \frac{V \cdot Q}{I \cdot t}$$

t = thickness of the cut surface

Q is for the shaded area, found relative to the neutral axis of the full cross-section

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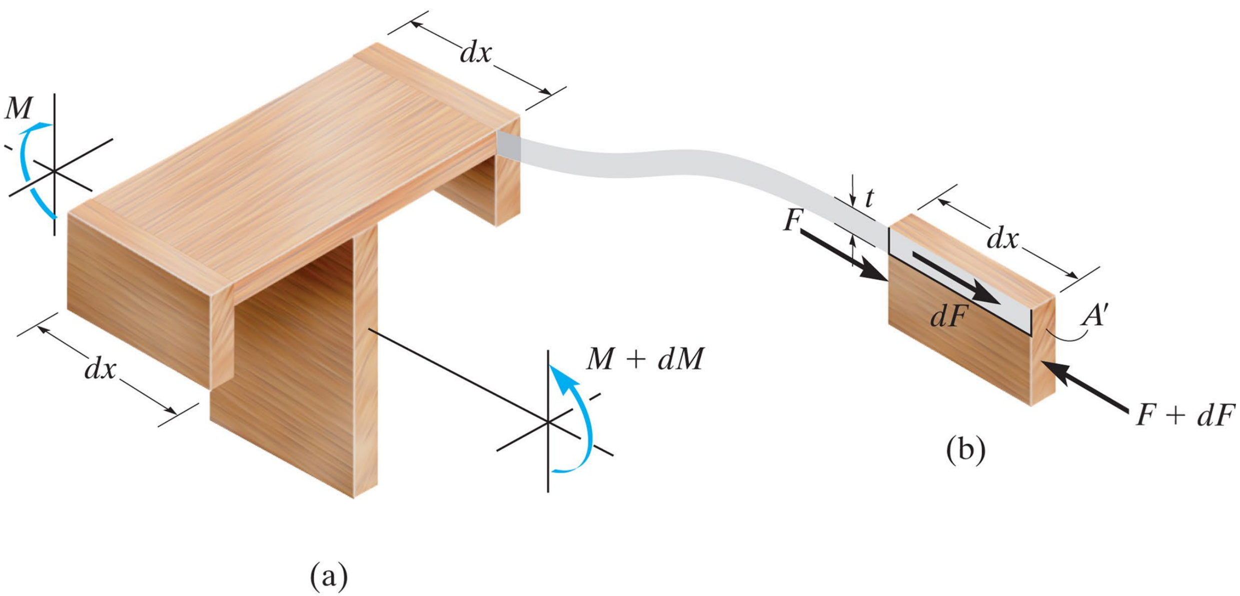
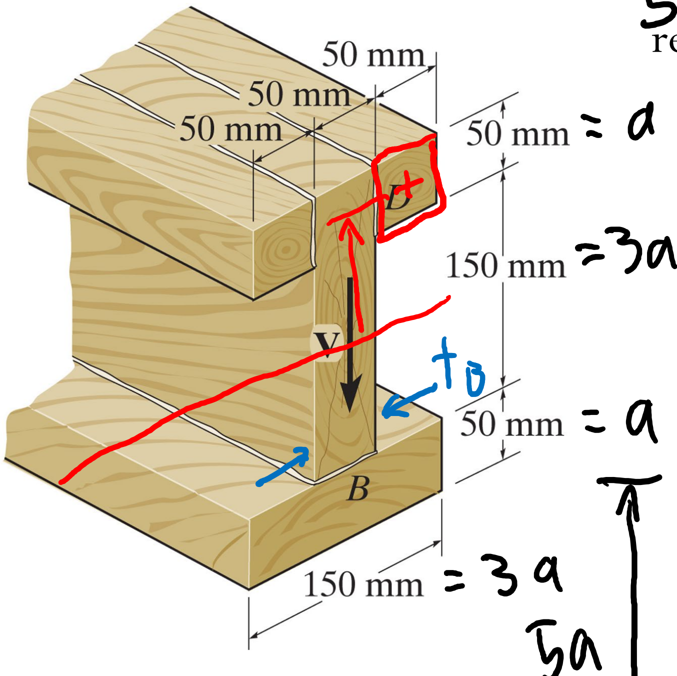
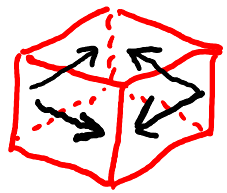


Example 2

A beam is made of four planks glued together. Knowing that the vertical shear in the beam is $V = 500$ N, determine the minimum required shear strength τ_g for the glue.

Find the required shear strength of the glue, τ_{max}

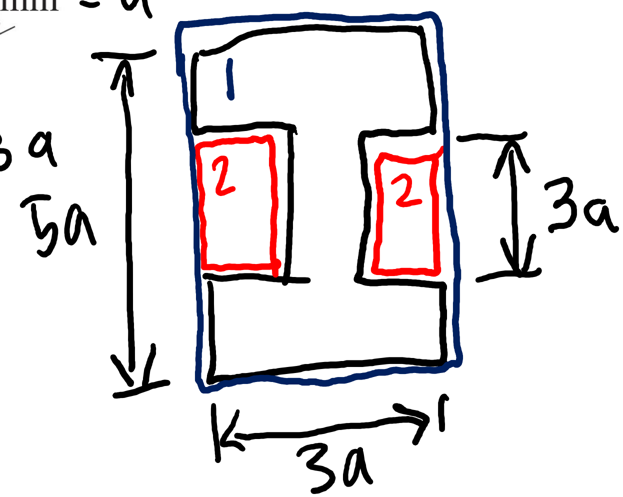




At D

Let $a = 50$ mm

$$\tau = \frac{V \cdot Q}{I \cdot t} \qquad t_B = t_D = a$$

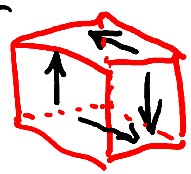


$$I = I_1 - 2 \cdot I_2$$

$$I_1 = \frac{(\cancel{3}a)(5a)^3}{4\,\cancel{12}} = \frac{125}{4} a^4$$

$$I_2 = \frac{a \cdot (3a)^3}{12} = \frac{9}{4} a^4$$

$$I = \frac{125 - 18}{4} a^4 = \frac{107}{4} a^4$$



At B

Find Q_D

$\bar{Y}_D = 1.5a + 0.5a$; $A_D = a^2$
$= 2a$

$$Q_D = 2a^3$$

$$\boxed{\tau_D = \frac{V \cdot 2\cancel{a^3}}{\left(\frac{107}{4} \cancel{a^4}\right) a^2} = \frac{8V}{107a^2}}$$

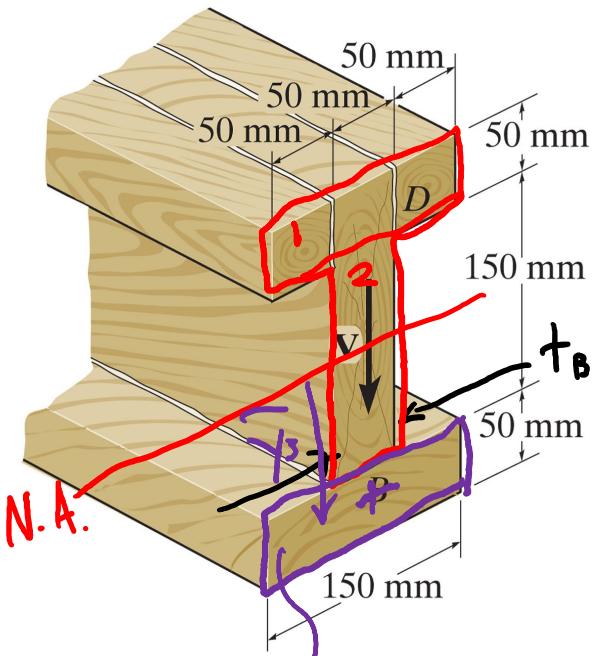
Example 2

$$I = \frac{107}{4}a^4$$

$$\tau_D = \frac{8}{107}\cdot\frac{V}{a^2}$$

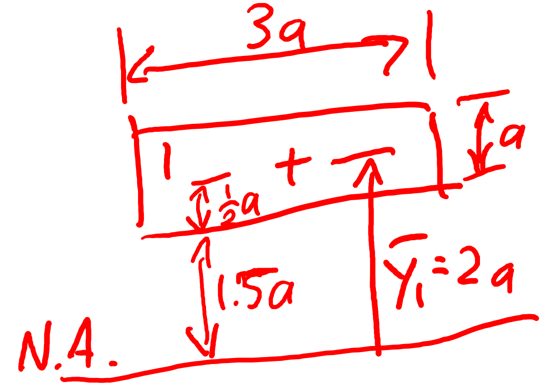
Find $\tau_B = \dfrac{V\cdot Q_B}{I\cdot t_B}$

Q(y) is symmetric about the N.A.



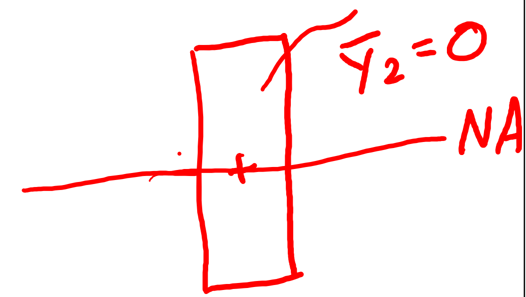
$t_B = a$

$a = 50$ mm



$$Q_B = Q_1 + Q_2$$
$$= (3a^2)(2a) + (3a^2)(0)$$
$$= 6a^3$$

$A_2 = 3a^2$



$$Q_B = \underbrace{A_3}_{3a^2}\cdot\underbrace{\bar{y}_3}_{2a} = 6a^3$$

$$\tau_B = \frac{V\cdot(6a^3)}{\frac{107}{4}a^4\cdot a}$$

$$\tau_B = \frac{24}{107}\cdot\frac{V}{a^2}$$

$\tau_B > \tau_D$ by factor of 3!

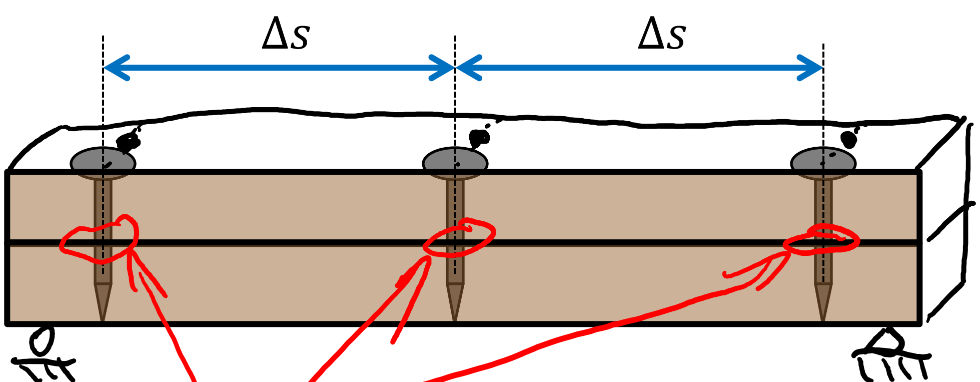
Find glue that can hold $\frac{24}{107}V/a^2$

Built up beams with fasteners (bolts or nails)

Unlike glue, fasteners supply resistance to longitudinal shear forces at fixed inter~~n~~vals.

Fasteners are typically spaced at a constant interval Δs along the length of the beam.

If we know the shear flow q, how much load does each fastener carry?



ΔH is the longitudinal shear force that must be carried by the nails

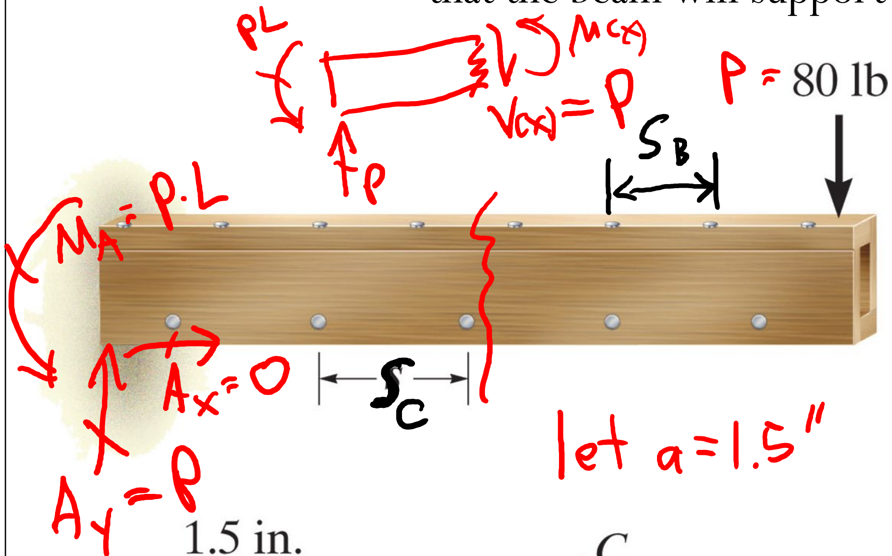
each nail carries

$$\Delta H = q \cdot \Delta S = \frac{V \cdot Q}{I} \cdot \Delta S$$

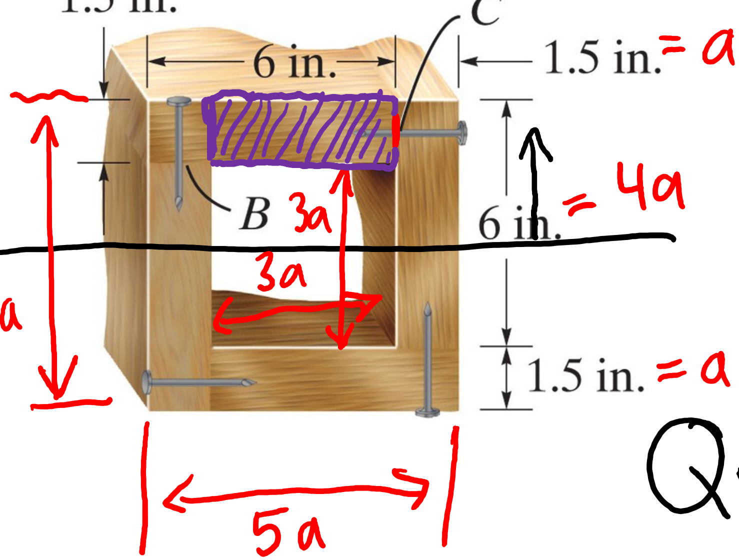
$$Q = A \cdot \bar{y}$$

Example 3

A beam is made of four planks, nailed together as shown. If each nail can support a shear force of 30 lb, determine the maximum spacing s of the nails at B and at C so that the beam will support the force of 80lb. Find V. and reactions.



let a = 1.5"



$$V = P$$

$$F_{nail} = 30 \text{ lb}$$

$$\cancel{F_{nail} = q \cdot \Delta s}$$

$$q = \frac{VQ}{I}$$

$$I = I_{big} - I_{small}$$

$$I = \frac{(5a)^4}{12} - \frac{(3a)^4}{12}$$

$$I = \frac{a^4}{12}(5^4 - 3^4)$$

$$\boxed{I = \frac{a^4}{12}(544) = \frac{136a^4}{3}}$$

$$F_{nail} = q_B \cdot \Delta S_B$$

$$F_{nail} = q_C \cdot \Delta S_C$$

$$\boxed{\Delta S_B = F_{nail}/q_B \qquad \Delta S_C = F_{nail}/q_C}$$

$$Q_B = A_B \cdot \bar{y}_B = (5a^2)(2a) = 10a^3$$

$$Q_C = A_C \cdot \bar{y}_C = (3a^2)(2a) = 6a^3$$

Example 3

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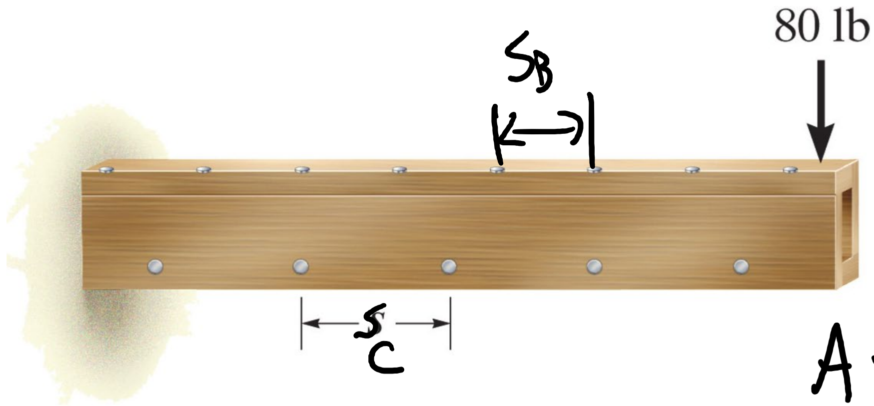
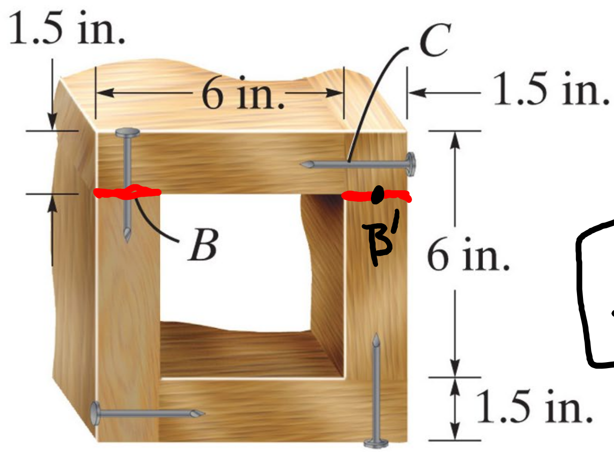
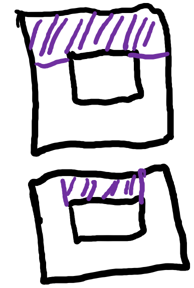






$$Q_B = 10a^3 \qquad I = \frac{136a^4}{3}$$

$$Q_c = 6a^3$$

$$F_{nail} = \Delta S_B \cdot q_B = \Delta S_C \cdot q_C$$

At C: $q_{total} = q_C + q_{C'} = 2q_C = \frac{VQ_C}{I}$

$$q_C = \frac{1}{2}\frac{VQ_C}{I}$$

At B: $q_{total} = q_B + q_{B'} = 2q_B = \frac{VQ_B}{I}$

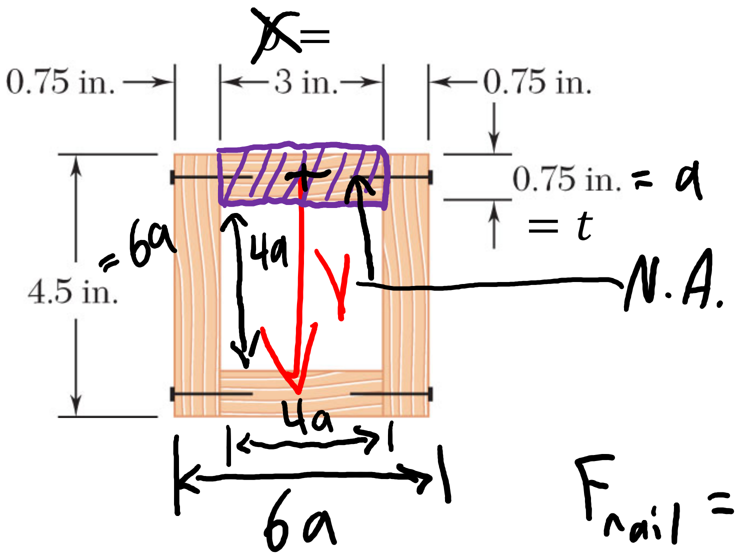
$$\Rightarrow q_B = \frac{1}{2}\frac{VQ_B}{I}$$

$$\boxed{\Delta S_C = \frac{F_{nail}}{q_C}} = \frac{F_{nail} \cdot 2 \cdot I}{V \cdot Q_C} = \frac{2 \cdot (30\cancel{lb}) \cdot \left(\frac{136}{\cancel{3}}\right)(1.5'')^{\cancel{4}}}{(80\cancel{lb}) \cdot 6 \cdot \cancel{(1.5'')^3}} = \dots = \boxed{8.5''}$$

$$\boxed{\Delta S_B = \frac{F_{nail}}{q_B}} = \frac{2(30\cancel{lb})\left(\frac{136}{\cancel{3}}\right)(1.5'')^{\cancel{4}}}{10 \cdot (80\cancel{lb})\,\cancel{(1.5'')^3}} = \dots = \boxed{5.1''}$$

Example 4

$a = 0.75''$



A square box beam is constructed from four planks as shown. Knowing that the spacing between nails is 1.5 in. and the beam is subjected to a vertical shear of magnitude $V = 600$ lb, determine the shearing force in each nail.

$$I = \frac{(6a)^4}{12} - \frac{(4a)^4}{12} = \frac{1296-256}{12}a^4$$

$$= \frac{1040}{12}a^4 = \frac{260}{3}a^4$$

$$F_{nail} = q \cdot \Delta s \qquad q_{total} = q_{left} + q_{right} = 2q$$

$$2q = \frac{V \cdot Q}{I} \Rightarrow q = \frac{1}{2} \cdot \frac{V \cdot Q}{I}$$

$$\boxed{F_{nail} = \frac{1}{2} \cdot \frac{VQ}{I} \cdot \Delta s} \qquad Q = (4a^2)\left(\frac{5}{2}a\right) = 10a^3$$

$$= \frac{1}{2} \frac{(600\text{ lb}) \cdot 10 \cdot (0.75'')^3}{\left(\frac{260}{3}\right)(0.75'')^4}(1.5'') = \boxed{69.2\text{ lb}}$$