TAM251_Chapter8 _CombinedLoading_prelecture_Johnson_Su21

Monday, July 12, 2021 1:35 PM

TAM251_Chapter8_CombinedLoading_prelecture_Johnso...

Chapter 8: Combined Loading

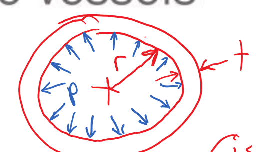
Chapter Objectives

✓ Determine stresses developed in thin-walled pressure vessels

✓ Determine stresses developed in a member's cross section when axial load, torsion, bending and shear occur simultaneously.

Thin-walled pressure vessels

- Inner radius: r
- Wall thickness: t
- Thin walls: $\frac{r}{t} \geq 10$
- Assume that stress distribution in thin wall is uniform
- Pressure vessel contains fluid under pressure p



is valid when variables r_i, r_o, t are used in the thick-wall formulation

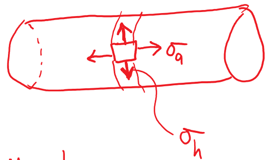
p acts uniformly throughout the vessel

Cylindrical vessels

Spherical vessels

Pressure vessel failures

fracture in cylindrical pressure vessels tends to happen along longitudinal lines



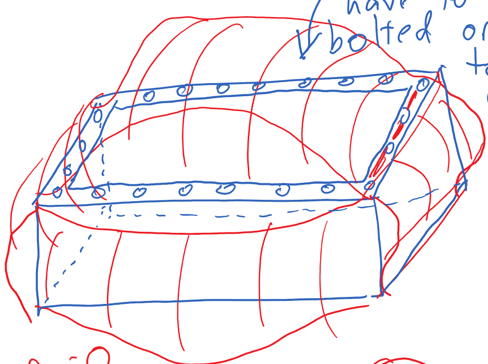
Hoop stress: σ_h

Axial stress: σ_a

We will show that $\sigma_h = 2\cdot\sigma_a \Rightarrow$ failure happens in this direction

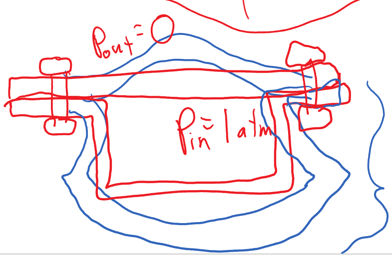
ASIDE

Why do we rarely (never) see rectangular pressure vessels?



lid would have to be bolted or clamped to the container

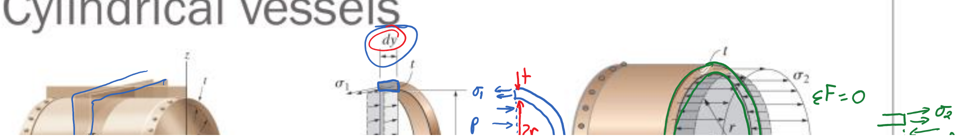
If p is high, the lid must be bolted on with a total force of at least $p \times A$ (area of the lid) ← this total force would be very large even for a small pressure

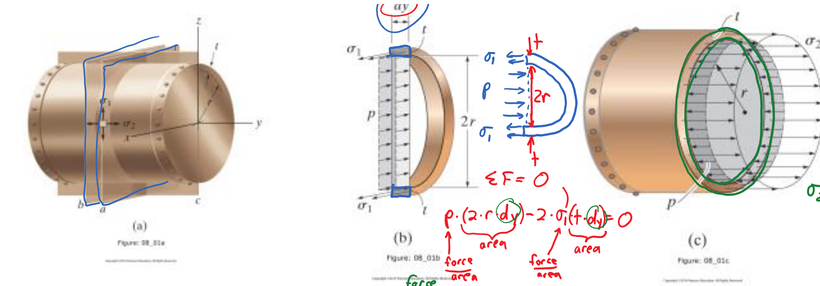


leaks are unlikely to develop in cylindrical pressure vessels under pressure.

deformations would be LARGE!

Cylindrical vessels





$\Sigma F = 0$

$\underbrace{p}_{\frac{\text{force}}{\text{area}}} \cdot \underbrace{(2 \cdot r \cdot dy)}_{\text{area}} - 2 \cdot \underbrace{\sigma_1}_{\frac{\text{force}}{\text{area}}} \underbrace{(t \cdot dy)}_{\text{area}} = 0$

$[\sigma_1] = \text{stress} = \frac{\text{force}}{\text{area}}$

$[p] = \text{stress} = \frac{\text{force}}{\text{area}}$

$[\frac{r}{t}] = \text{dimensionless} = 1$, $[r] = \text{length}$, $[t] = \text{length}$

$\sigma_1 = \frac{p \cdot r}{t}$ = hoop stress, σ_h

$\boxed{\sigma_h = \frac{p \cdot r}{t}}$

$\Sigma F = 0$

$\sigma_2 \cdot (\underbrace{t}_{\text{thickness}} \times \underbrace{2\cancel{\pi} r}_{\text{circumference}}) - p \cdot \overbrace{(\cancel{\pi} \cdot r^{\cancel{2}})}^{\text{area of inner circles}} = 0$

$\sigma_2 = \frac{p \cdot r}{2 \cdot t}$ = axial stress = σ_a

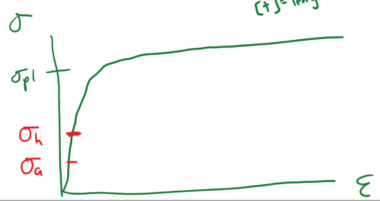
$\boxed{\sigma_a = \frac{p \cdot r}{2 \cdot t}}$

$[p]$ = we know $\frac{\text{force}}{\text{area}}$

$[\sigma_a] = \text{stress} = \frac{\text{force}}{\text{area}}$

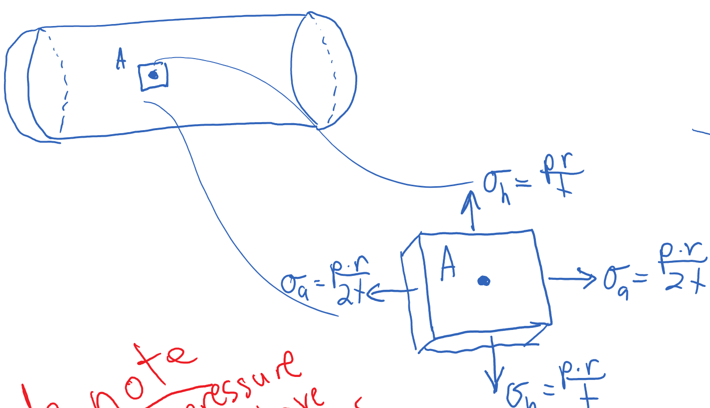
$[r/t] = 1$

$[2] = 1 = \text{dimensionless}$



$\sigma_h = 2 \times \sigma_a$

= why cylindrical vessels tend to fail longitudinally!

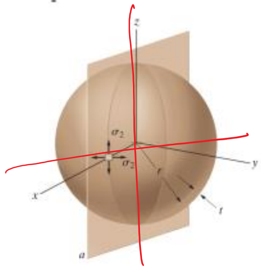


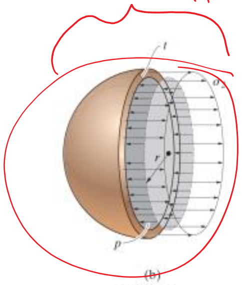
If we were to rotate our choice of material element, the values of the stresses in the thin wall would change. (we won't do that yet, but we will in Ch. 9)

Side note

cylindrical pressure vessels often have hemispherical end caps. In the end caps, the spherical theory applies.

Spherical vessels


(a)
Figure: 08_02a


(b)
Figure: 08_02b

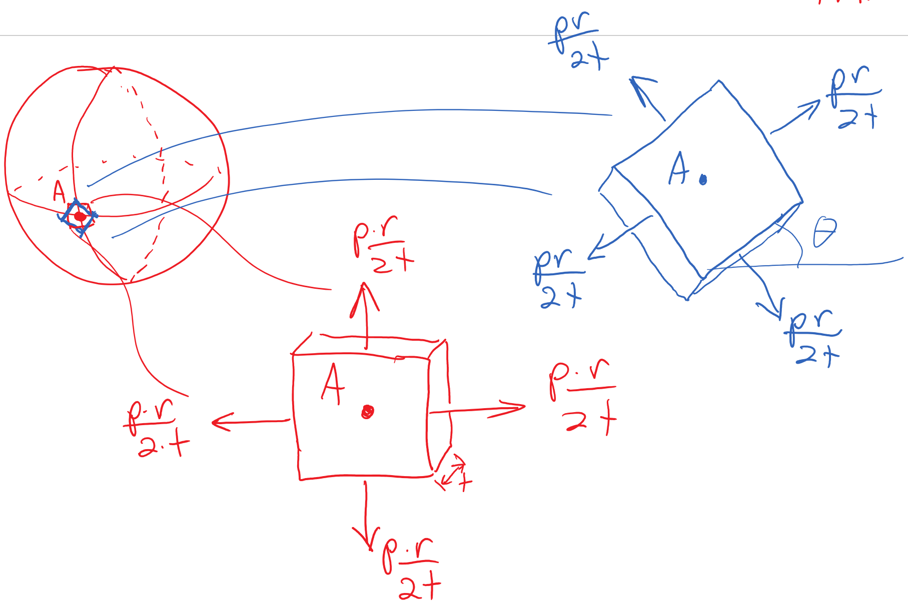
No matter how you "cut" the vessel to derive the stress, you get the same F.B.D. as the axial stress case of the cylindrical vessel (see previous slide)

$$\Sigma F = 0$$

$$\sigma_2 \cdot (t \cdot 2\pi r) - p \cdot (\pi r^2) = 0$$

$$\boxed{\sigma_2 = \frac{p \cdot r}{2 \cdot t}}$$

acts in both directions of any stress element in the thin wall



No matter what value θ has, the normal stresses in the thin wall will equal $p \cdot r / 2 \cdot t$ — only true for spherical pressure vessels

Example: A spherical pressure vessel of D = 900-mm outer diameter is to be fabricated from a steel having an ultimate tensile stress σ_U = 400 MPa. Knowing that a factor of safety of FS=4.0 is desired and that the gage pressure can reach p=3.5 MPa, determine the smallest wall thickness that should be used.

$p = p_{gage} = p_{inner} - p_{outer}$

$1\ MPa = 10^6\ Pa = 10^3\ kPa \approx 10\ atm$

usually atmospheric pressure

$P_{atm} = 1\ atm \approx 100\ kPa \approx 14.7\ psi$

If p_{outer} is 1 atm (absolute), we usually just approximate $p_{outer} = 0$

$p \approx p_{inner}$

spherical: $\sigma_a = \frac{p \cdot r}{2 \cdot t}$ $\qquad \sigma_{allow} = \frac{\sigma_u}{F.S.}$

we want $\sigma_a \le \sigma_{allow} \Rightarrow \frac{p \cdot r}{2 \cdot t} \le \frac{\sigma_u}{F.S.}$

$r = r_o - t = \frac{D}{2} - t \Rightarrow \frac{p}{2t}\left(\frac{D}{2} - t\right) \le \frac{\sigma_u}{F.S.}$

$$\frac{p \cdot D}{4t} - \frac{p}{2} \le \frac{\sigma_u}{F.S.}$$

$$4t\left(p\frac{D}{4t}\right) \le \left(\frac{\sigma_u}{FS} + \frac{p}{2}\right)4t$$

$$pD \le \left(\frac{\sigma_u}{FS} + \frac{p}{2}\right) \cdot 4t$$

$$\Rightarrow t \ge \frac{p \cdot D}{4\left(\frac{\sigma_u}{FS} + \frac{p}{2}\right)} \quad \therefore t \ge 7.7\,mm$$

Review of stress analysis

- **Normal force P** Ch. 4

  leads to uniform normal stress $\sigma = \frac{P}{A}$

  $\sigma = P/A$ (uniform)

- **Torsional moment T (circular shafts)**

  leads to shear stress varying in the cross-section $\tau = \frac{T\rho}{J}$

  $\tau = \frac{T\cdot\rho}{J}$

- **Shear force V** (beams)

  leads to shear stress varying in the cross-section $\boxed{\tau = \frac{VQ(y)}{I\cdot t}}$

  Q(y) largest at N.A.

  $\tau = \frac{V\cdot Q}{I\cdot t}$

- **Bending moment M** M>0 comp / tension; M<0 tension / comp.

  leads to normal stress varying in the cross-section $|\sigma| = \frac{Mc}{I}$

  $\sigma_x = \frac{-M\cdot y}{I_z}$

  comp. tension

- **Internal pressure p in a spherical or cylindrical vessel**

  leads to hoop stress $\sigma_h = \frac{pr}{t}$ and axial stress $\sigma_a = \frac{pr}{2t}$

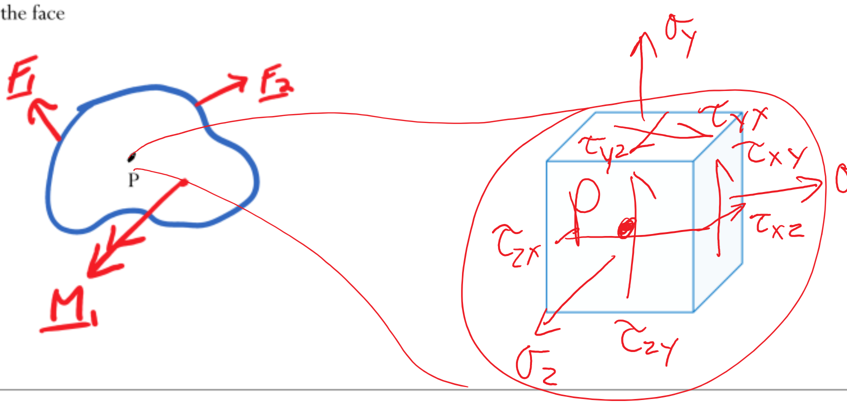
  cylindrical only

State of stress in a solid

Consider a solid body subjected to externally applied loads

Suppose we wish to know about the stresses being caused at a particular point of interest, point P.

We can draw a small (infinitesimal) cube of material around the point of interest and note the infinitesimal forces that act on each face. In general, the force acting on each face can be broken into 3 vector components, one of which is normal to the face and two of which act in the plane of the face



σ_x
σ_y
σ_z
$\tau_{xy} = \tau_{yx}$
$\tau_{xz} = \tau_{zx}$
$\tau_{yz} = \tau_{zy}$

all of the stresses that may act at a point

Stress in 2 and 3 dimensions

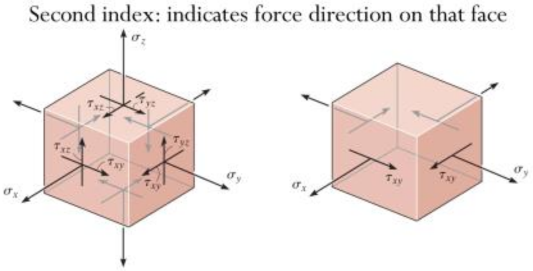
Normal stresses: $\sigma_x, \sigma_y, \sigma_z$

Shear stresses: $\tau_{xy} = \tau_{yx}, \tau_{yz} = \tau_{zy}, \tau_{xz} = \tau_{zx}$

→ equalities due to requirement of zero net moment in equilibrium

First index: indicates face normal direction

Second index: indicates force direction on that face

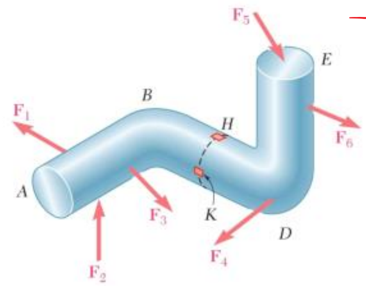


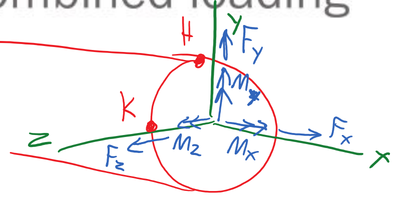
General state of stress
(a)

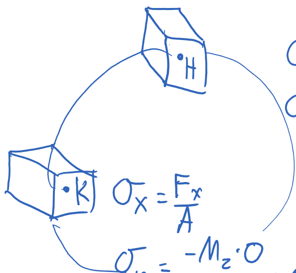
Plane stress
(b)

Figure: 09_01

Stresses under combined loading







$\sigma_x = F_x/A$ $\qquad \tau_{xz} = \dfrac{M_x \cdot \rho}{J}$ (Torsion)

$\sigma_x = -\dfrac{M_z \cdot y}{I}$ (Bending) $\qquad \tau_{xz} = \dfrac{V \cdot Q}{I t} = 0$ (shear)

etc....

$\sigma_x = \dfrac{F_x}{A}$

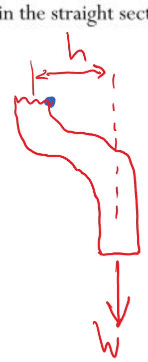
$\sigma_x = \dfrac{-M_z \cdot 0}{I} = 0$

$\tau_{xy} = \dfrac{M_x \cdot \rho}{J}$ (Tonsion)

$\tau_{xy} = \dfrac{V \cdot Q}{I \cdot t} \neq 0$ (on N.A.)

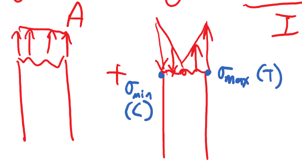
Jet Ski Gondola

A gondola on a ski lift is supported by a bent arm, as indicated in the figure. The arm has a square cross-sectional area with side length b and is offset by a distance h from the line of action of the weight force W. Find an expression for the minimum and maximum tensile stresses in the straight section of the arm.



stresses in the cross-section

$\sigma = \dfrac{W}{A}$ $\qquad \sigma = \dfrac{-(W \times h)\cdot y}{I}$



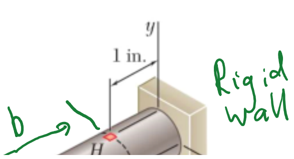
$\sigma_{min} = \dfrac{W}{A} - \left| \dfrac{-(W \times h)\cdot y_{max}}{I} \right|$

$\sigma_{max} = \dfrac{W}{A} + \left| \dfrac{-(W \times h)\cdot y_{max}}{I} \right|$

Example

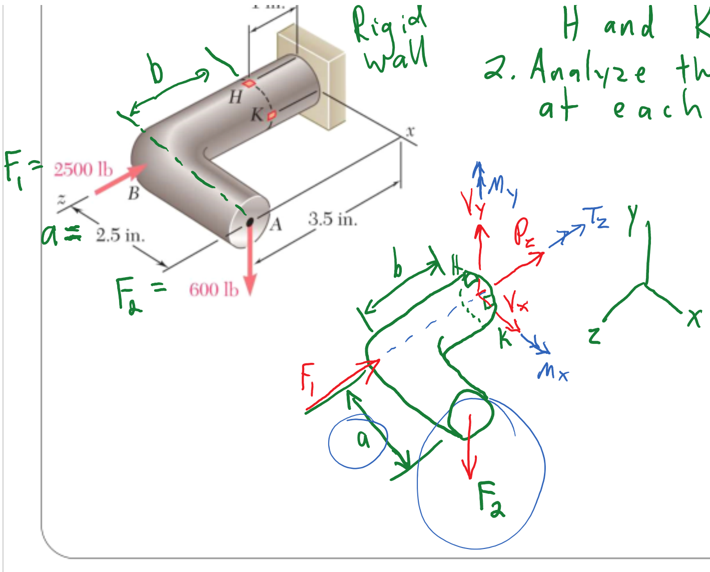
Find the state of stress at points H and K

1. Find the internal forces in the cross-section at H and K.
2. Analyze the stresses acting



H and K.

2. Analyze the stresses acting at each point H & K.



V_x, V_y, P_z are internal forces at the cut.

$\Sigma F_x = \boxed{V_x = 0}$

$\Sigma F_y = V_y - F_2 = 0 \Rightarrow \boxed{V_y = F_2}$

$\Sigma F_z = -F_1 - P_z = 0 \Rightarrow \boxed{P_z = -F_1}$

$\Sigma M_x = M_x + F_2 \cdot b = 0 \Rightarrow \boxed{M_x = -F_2 \cdot b}$

$\Sigma M_y = \boxed{M_y = 0}$

$\Sigma M_z = -T_z - F_2 \cdot a = 0 \Rightarrow \boxed{T_z = -F_2 \cdot a}$

Point H

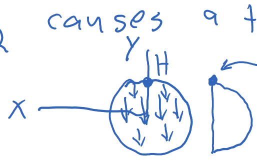
$\sigma_z = \frac{-F_1}{A}$ } uniaxial compression

$\sigma_z = \left|\frac{M_x \cdot y}{I_x}\right| = \frac{F_2 \cdot b \cdot d/2}{I_x}$ } Tensile stress due to bending at H

Tension at H

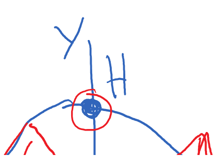
$$\boxed{\sigma_z = \frac{F_2 \cdot b \cdot d/2}{I_{\cancel{z}}} - \frac{F_1}{A}}$$

$V_y = F_2$ causes a transverse shear stress τ_{yz}

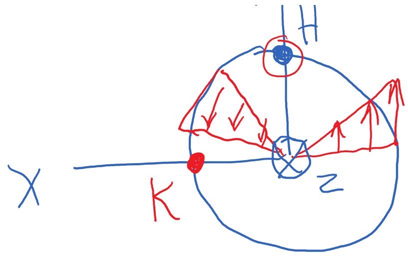


$V_x = 0 \Rightarrow \tau_{xz} = 0$ at H

$T_z = -F_2 \cdot a$ causes a shear stress due to torsion

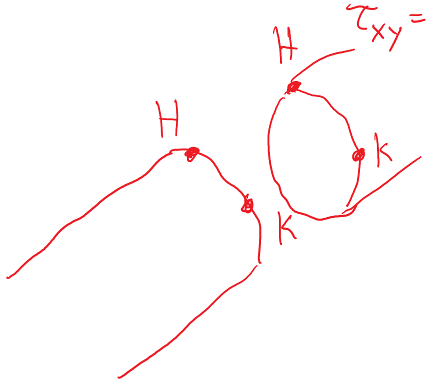


at H $\tau = \frac{T \cdot \rho}{J}$ gives: $\tau_{xz} = \frac{F_2 \cdot a \cdot d/2}{J}$

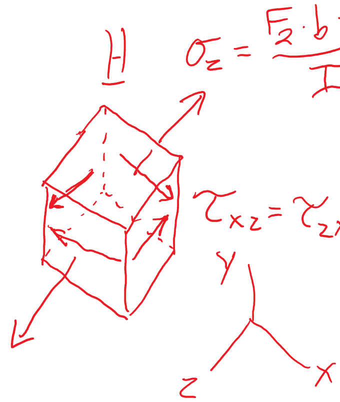


at H $\tau = \frac{T\cdot\rho}{J}$ gives: $\tau_{xz} = \frac{F_2\cdot a\cdot d/2}{J}$

at K



$\tau_{xy} = \frac{F_2\cdot a\cdot d/2}{J}$



$\sigma_z = \frac{F_2\cdot b\cdot d/2}{I_x} - \frac{F_1}{A}$

$\tau_{xz} = \tau_{zx} = \frac{F_2\cdot a\cdot d/2}{J}$

$\sigma_z =$

Point K

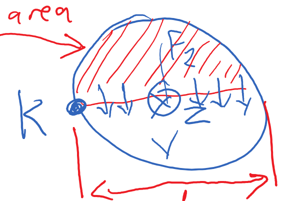
F_1: compressive stress

$$\sigma_z = -\frac{F_1}{A}$$

F_2: $\tau_{yz} = \frac{-V_2\cdot Q}{I\cdot t} = \frac{-F_2\cdot Q}{I\cdot d}$

Q due to this area

$t = d$



τ_{yz} due to the transverse force $V_y = -F_2$ is maximized at the N.A.

$F_2\cdot b$: Bending moment, but K is on the N.A.
$\Rightarrow$ bending stress $= 0$ due to $F_2\cdot b$

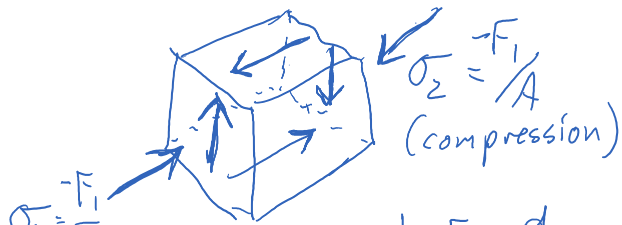
$F_2\cdot a$: Torque $\Rightarrow$ $\tau_{yz} = \frac{F\cdot a\cdot d/2}{J}$

Total the stresses at K

$\sigma = -F_1/A$

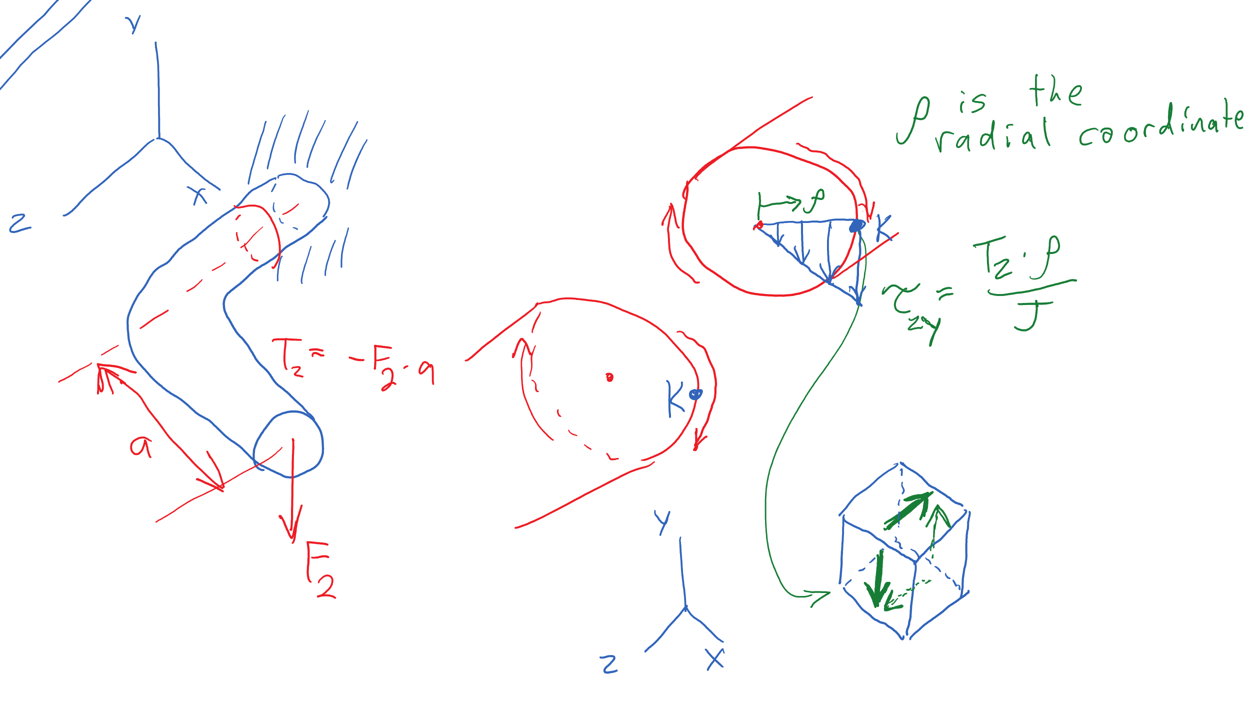
$$\sigma_z = -F_1/A$$

$$\tau_{yz} = \frac{-F_2 \cdot a \cdot d/2}{J} - \frac{F_2 \cdot Q}{I \cdot d}$$

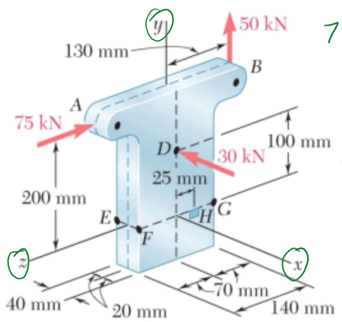


$$\tau_{yz} = \tau_{zy} = \left| \frac{-F_2 \cdot a \cdot \frac{d}{2}}{J} - \frac{F_2 \cdot Q}{I \cdot d} \right|$$

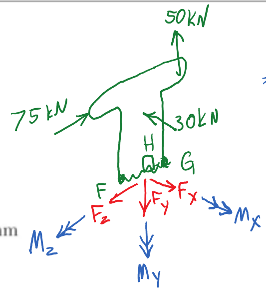
Student question about shear from torsion



Example



Three forces are applied to a short steel post as shown. Determine the state of stress at point H on the outer surface of the post.



$\Sigma F_x = 0$

$\Rightarrow \boxed{F_x = 30\,kN}$

$\Sigma F_y = 0$

$-F_y + 50\,kN = 0$

$\boxed{F_y = 50\,kN}$

$\Sigma F_z = 0 \Rightarrow \boxed{F_z = 75\,kN}$

$\Sigma M_x = 0 \Rightarrow M_x + (50kN)(130mm) - (75kN)(200mm) = 0$

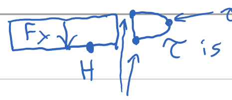
$\boxed{M_x = 21.5\,kN\cdot m}$

$\Sigma M_y = 0 \Rightarrow \boxed{M_y = 0}$

$\Sigma M_z = 0 \Rightarrow M_z + (30kN)(100mm) = 0$

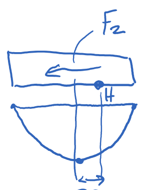
$\boxed{M_z = -3\,kN\cdot m}$

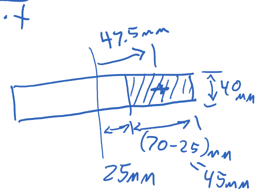
F_x causes transverse shear but the resulting shear stress is zero at H



$F_y \Rightarrow$ Axial load : $\boxed{\sigma_y = \frac{F_y}{A} = \frac{50kN}{(40\times140)mm^2}} = 8.92\,MPa$

$F_z \Rightarrow$ Trans. Shear : $\tau_{yz} = \frac{V\cdot Q}{I\cdot t}$





$Q = (45mm \times 40mm)(47.5mm)$

$= 90\times10^3\,mm^3$

$I_x = \frac{(40mm)(140mm)^3}{12} = 9.15\times10^6\,mm^4$

$$\boxed{\tau_{yz}\Big|_H = \frac{(75\times10^3 N)(90\times10^{-6} m^3)}{(9.15\times10^{-6} m^4)(0.040m)} = 18.4\,MPa}$$

M_x: puts H in compression in the y-direction

$$\boxed{\sigma_y = -\left|\frac{M_x\cdot z_H}{I_x}\right| = \frac{-(21{,}500\,N\cdot m)(0.025m)}{9.15\times10^{-6}m^4} = -117.5\,MPa}$$

$M_y = 0$ (Torsion)

$M_z = -3kN\cdot m$ puts H into tension in the y-direction

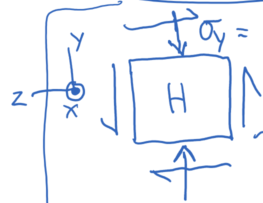
$$\boxed{\sigma_y = \left|\frac{M_z\cdot x_H}{I_z}\right| = \frac{(3000\,N\cdot m)(0.020m)}{\frac{1}{12}(0.140m)(0.040m)^3} = 80.4\,MPa}$$

Total them all: $\sigma_y = 8.92\text{ MPa} - 117.5\text{ MPa} + 80.4\text{ MPa}$

$\boxed{\sigma_y = -28.2\text{ MPa}}$

$\boxed{\tau_{yz} = -18.4\text{ MPa}}$

$\sigma_y = 28.2\text{ MPa}$ (compression)



$\tau_{yz} = 18.4\text{ MPa}$